Fundamental bound on time signal generation

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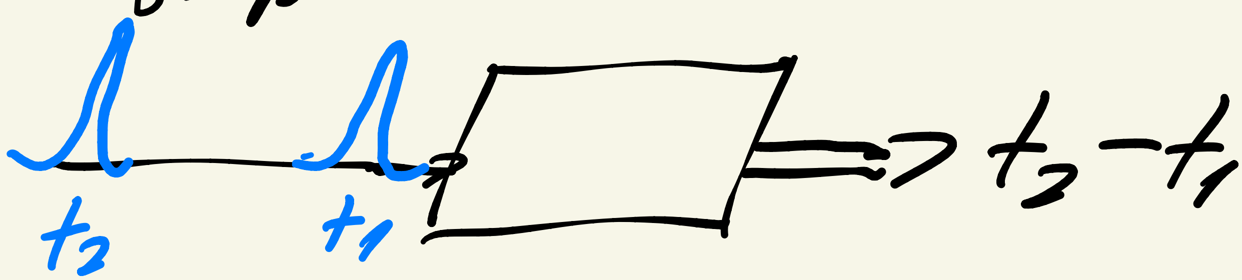
joint work with Yuxiang Yang
(arXiv: 2004.07857)

$$\sigma_x \cdot \sigma_p \geq \frac{\hbar}{2}$$

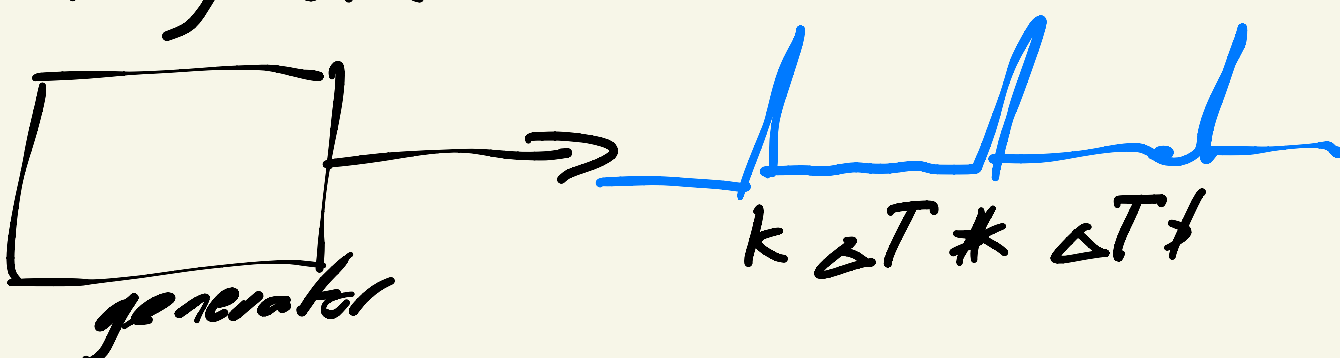
$$H(X) + H(P) \geq \log(e\pi)$$

Two types of "clock":

- stopwatch

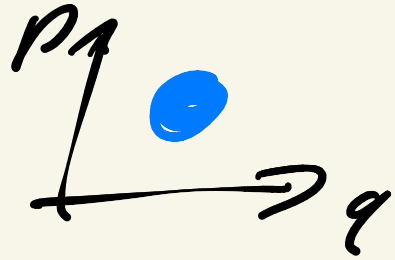


- ticking clock

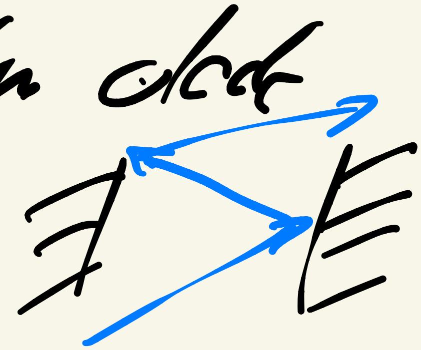


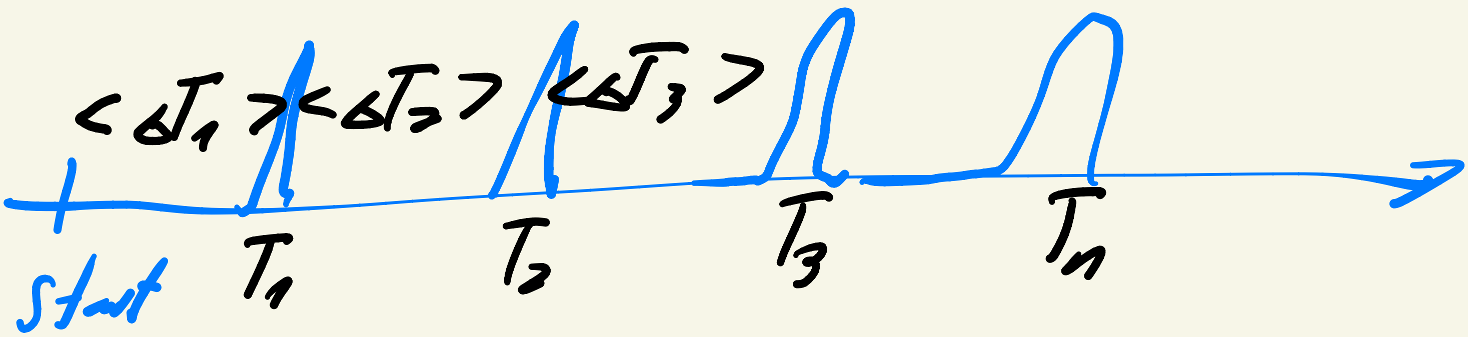
Examples:

- classical pendulum clock



- optical clock





$$T_1 = \Delta T_1 + \Delta T_2 + \dots + \Delta T_n$$

$$\sigma_{T_n}^2 = n \underbrace{\sigma_{\Delta T}}_{\sigma}$$

$$\mu = \langle \Delta T_i \rangle$$

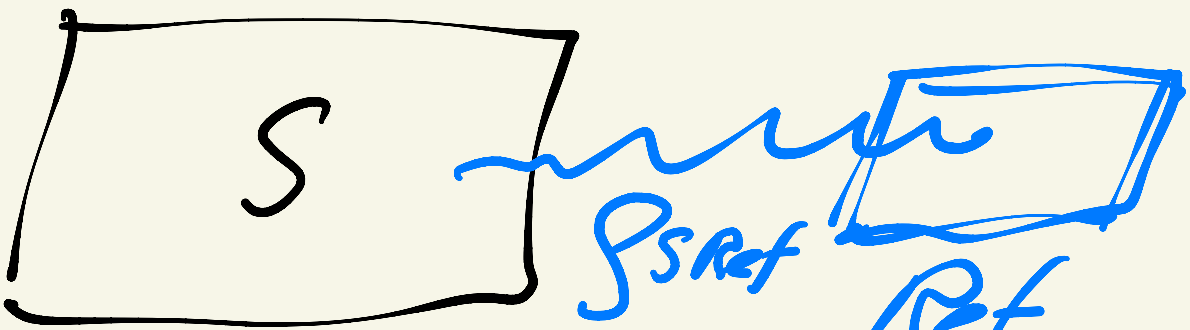
$$\sigma = \langle (\Delta T_i - \mu)^2 \rangle$$

$$\sigma_{T_n} = \sqrt{n}\,\sigma$$

$\sigma\sqrt{n} \geq \mu$

$n\sigma^2 \geq \mu^2$

$n \geq \frac{\mu^2}{\sigma^2} := R$



generator

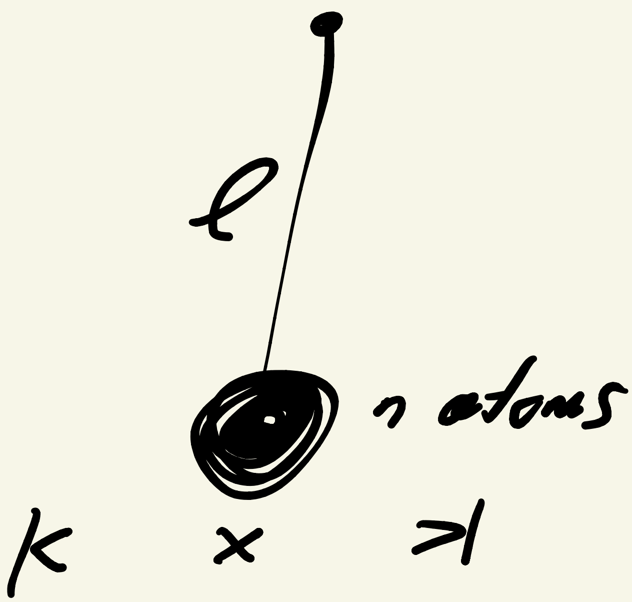
Idea: $d_{ctrl} \hat{=}$ number of dist. states that S admits during its timeevolutions.

Formally:

$$d_{ctrl} = \max_{\rho_{S\,Ref}} 2^{I(Ref:S)}$$

$$\rho_{S\,Ref} = \int dt\, \rho_S(t) \otimes |t\rangle\langle t|_{Ref}$$

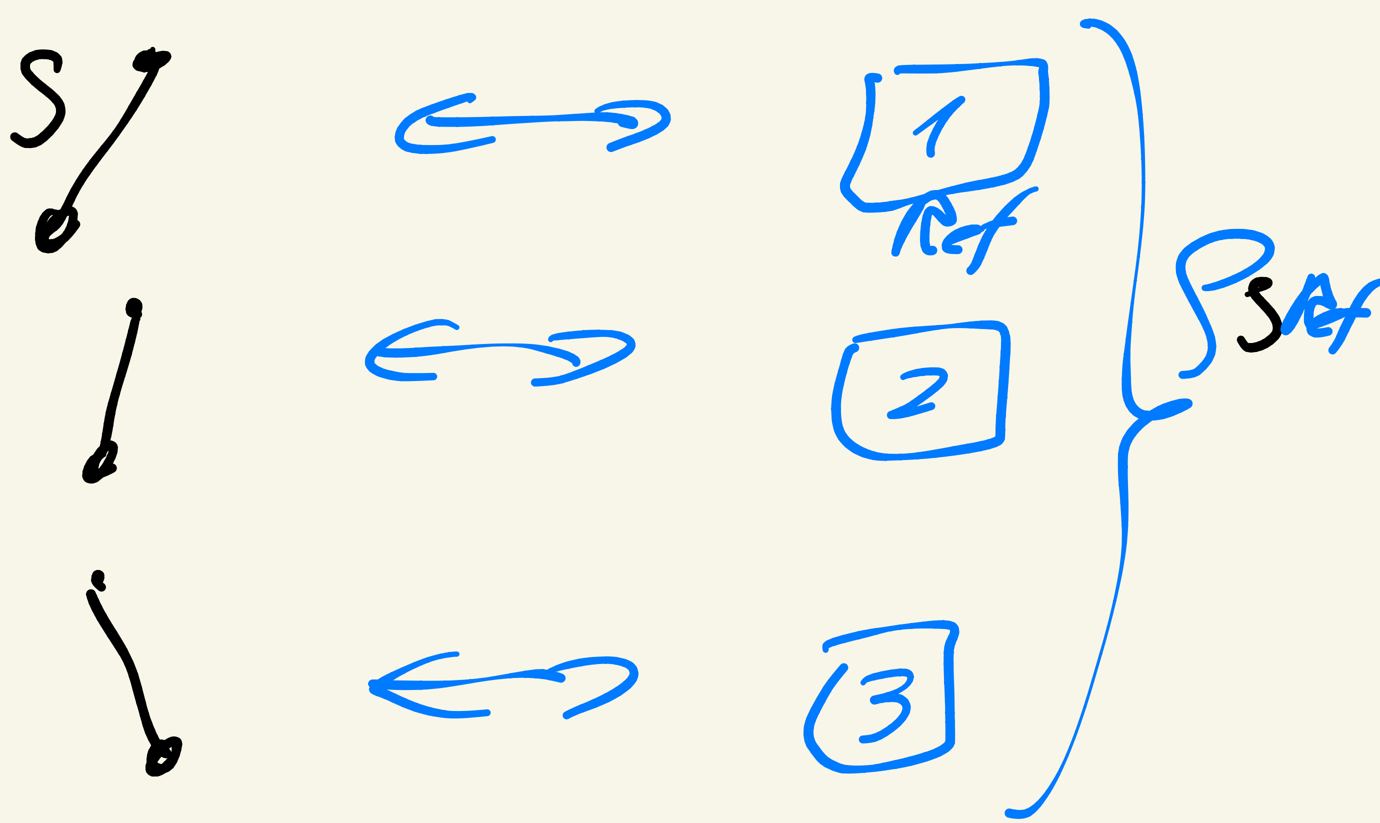
$I(Ref:S) = H(S) - H(S|Ref)$



$x \sim \ell$

$\Delta x \sim \frac{\ell}{\sqrt{n}}$

dist states $\sim \frac{x}{\Delta x} \sim \ell\sqrt{n}$



$$v \sim \sqrt{\frac{g}{\ell}}$$

$$\iff \ell \sim \frac{g}{v^2}$$

$$\implies d_{ctrl} \sim \sqrt{n}\, \frac{g}{v^2}$$

Optical clock: n photons

$d_{ctrl} \sim \sqrt{n}$ for classical light

$d_{ctrl} \sim n$ for squeezed light.

<u>Thm:</u> $R \leq 2\pi e\, d_{ctrl}^2$

Conversely, the bound is approximately achievable!

Remark about proof:

Data Processing Inequality

$$\begin{array}{ccc} A & \longrightarrow & A' \\ B & \longrightarrow & B' \end{array}$$

$$I(A:B) \geq I(A':B') \quad \square$$

Conclusions: To build a good signal generator (ticking clock) one has to look for a system with large d_{clk}, e.g., a fully controlled quantum device.

For n-qubit device, $d_{clk} = 2^n$

$$\Rightarrow R \sim 2^{2n}$$