

HARVARD UNIVERSITY
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Mathematical Picture Language Seminar

Tuesday, June 1, 2021, at 10:00 (Boston)
15:00 (UK/Eire) 16:00 (C.Europe) 22:00 (China)

Zoom at: <https://harvard.zoom.us/j/779283357?pwd=MitXVm1pYUJlVzZqT3lwV2pCT1ZUQT09>

“Fractal properties of the Hofstadter's butterfly and singular continuous spectrum of the critical almost Mathieu operator”

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Abstract: Harper's operator—the 2D discrete magnetic Laplacian—is the model behind the Hofstadter's butterfly and Thouless theory of the Quantum Hall Effect. We present a result (with I. Krasovsky) that proves one half of the Thouless' "one half" conjecture from the early 80s: that Hausdorff dimension of the spectrum of Harper's operator is bounded by $1/2$ for all irrational fluxes. The model reduces to the critical almost Mathieu family, indexed by phase, and we will also present a complete proof of singular continuous spectrum for this family, for all phases, finishing a program with a long history. The proof is based on a simple Fourier analysis and a new duality-type transform that is also underlying the solution of the Thouless problem. We will also explain how these ideas provide for a very simple proof of zero measure of the spectrum of Harper's operator, a problem previously solved by sophisticated dynamical systems techniques. Finally, we discuss recent progress towards the Thouless Catalan conjecture.

