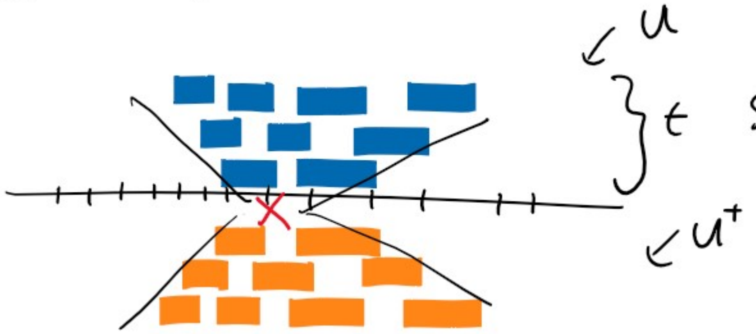
Review of Quantum Circuits

Thursday, June 17, 2021 11:03 AM

Review of Q circuits:

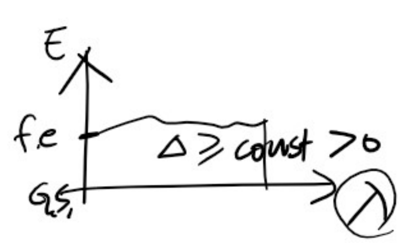



$$\mathrm{Sup}(U \underline{\underline{X}} U^\dagger) \subseteq N_t(\mathrm{supp}(X))$$

locality is preserved.

$$\Longleftrightarrow \quad \| [ e^{itH} X e^{-itH}, Y ] \| \le \dots e^{-\underline{\underline{\mu}}(\mathrm{dist}(X,Y) - \underline{v} t)}$$

USE??




$$GS_{\lambda=1} = \underline{\underline{U_1}}(GS_{\lambda=0})\, \underline{\underline{U_1^\dagger}}$$

$$\partial_t U_t = i\, \underline{D(t)}\, U_t$$

$\Sigma$ local terms

" top. phase of matter

$\Longleftrightarrow$ eq. class of states up to Q. circuits "

# Walker-Wang models

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Walker - Wang "model" ( Crane - Yetter TQFT)

( alg. data 2+1 d anyon theory ) $\Rightarrow$ ( comm. H on 3-mfld, G.S. )

$|\phi\rangle + \alpha|\ \rangle + |\ \rangle + \cdots$

amplitude

" 3-fermion "

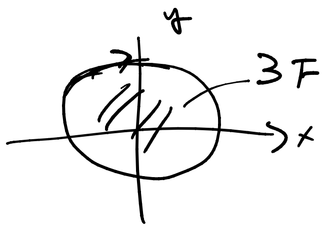
$\{1, a, b, c\}$

$\theta_a = \theta_b = \theta_c = -1$

$S = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$, $\Rightarrow e^{2\pi i c_-/8}$

$= \frac{1}{D}\sum_j d_j^2 \theta_j = -1$

$\Rightarrow$ $c_- \neq 0$




3F W-W model $\boxed{H}$

- G.S. nondegenerate
- $\nexists$ top. charge

det $S \neq 0$

Disentangle?

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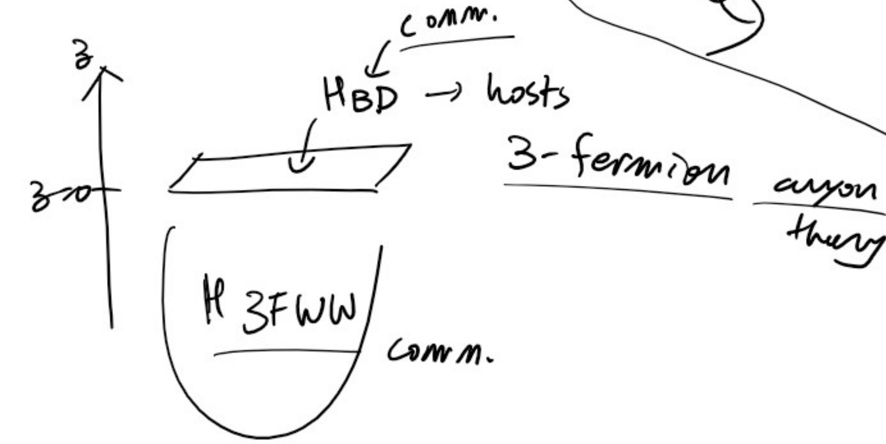
Q) Can you disentangle (G.S.) $\boxed{H_{3FWW}}$? **Maybe.**

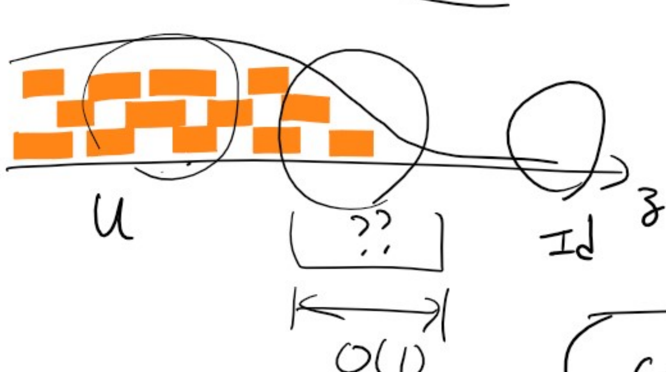
full spectrum.

No. not by circuit.

(i)




(ii) Any circuit U blends into Id.

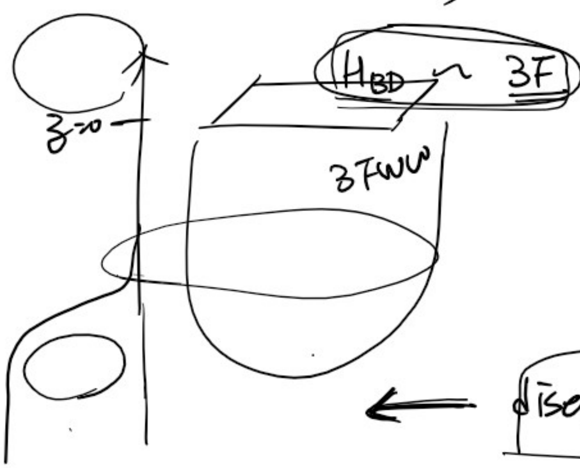



$V(z \ll 0) = \underline{\underline{U}}$

$V(z \gg 0) = \underline{\underline{Id}}$

$(-\infty, 0] \times \mathbb{R}^2 =$ Space

(iii)




← disentangled. $H_{trivial} = \sum_i \sigma_i^x$

$$\Longrightarrow \qquad \boxed{(-\underline{\underline{100}}, 0]} \times \mathbb{R}^2$$

~~A~~BSURD!

# Enter QCA

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{ circuits + shifts }

Enter QCA ~~?~~

$\exists\, \alpha$, an ~~circuit~~ automorphism of quasi-local alg on $\mathbb{R}^3$ $(\mathbb{Z}^3)$

s.t. $\alpha(H_{3FWW}) \underset{\varphi}{=} H_{trivial} = \sum_i \sigma_i^z - \sigma_1^z \sigma_{15}^z$

up to rearrangement

KEY: $\alpha$ does NOT BLEND into Id

# Invertible subalgebras

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**Def** **Invertible** subalgebras :

$$\Lambda = (\mathbb{Z}/L)^{D-1} \qquad \mathbb{C}^d \text{ per lattice point.}$$

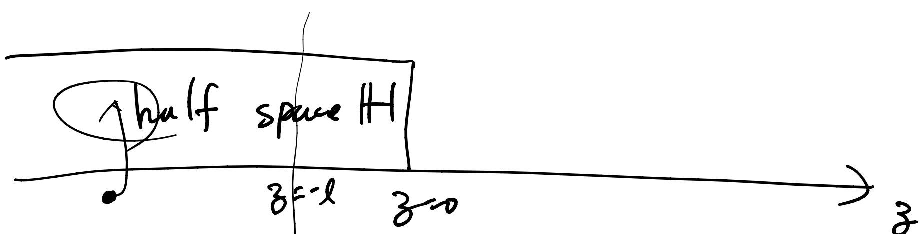
$$\mathcal{A} \subseteq \mathrm{Mat}(\Lambda)$$

$$O = \sum_j A_j B_j \qquad A_j \in \mathcal{A}, \quad B_j \in \mathcal{A}'$$

s.t.

$$\begin{cases} \mathrm{diam} \bigcup_j \mathrm{supp}(A_j) \le \ell = O(L^0) \\ \mathrm{diam} \bigcup_j \mathrm{supp}(B_j) \le \ell \quad \text{"} \end{cases}$$

QCA : $\alpha$




$$\alpha(\mathrm{Mat}(\mathbb{H})) = \mathrm{Mat}(\mathbb{H}^{\text{interior}}) \otimes \mathcal{B}$$

Bd alg of $\alpha$

**Thm** $\mathcal{A}$ sub alg on $(D-1)$-spaces is invertible iff it is Bd alg of some QCA in $D$-dim.

**Thm** A <u>sub alg</u> on $(D-1)$-spaces is <u>invertible</u> iff it is Bd alg of some QCA in $D$-dim.

Classification results

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— Inv. alg on $D=0$ is trivial. (GNVW)

$D=1$ QCA circuit + shift

— Inv. alg " $D=1$ trivial. (Freedman – Hastings)

($\exists$ circuit transform $A \Rightarrow$ Mat)

— Inv. alg. (gen. by Pauli) $D=1$, trivial by Clif. circuits

$\Rightarrow$ $D=2$ Every QCA is circuit + shifts

(cliff.) cliff.

— $D=2$ $\boxed{\text{Inv. alg.}}$ may be nontrivial.

generated by hopping op. for 3F.

Any D

— $(\text{Pauli inv. alg})^{\otimes 2}$ over $\mathbb{C}^2$ is trivial.

— $(\text{Weyl inv.})^{\otimes 2}$ over $\mathbb{C}^p$ is trivial if prime $p \equiv 1$ mod 4

$\left\{ \begin{pmatrix} 1 & & & \\ & e^{2\pi i/p} & & \\ & & e^{4\pi i/p} & \\ & & & \ddots \end{pmatrix}, \begin{pmatrix} 0 & & & 1 \\ 1 & 0 & & \\ & 1 & 0 & \\ & & 1 & 0 \end{pmatrix} \right\}$

— $(\text{Weyl inv. alg})^{\otimes 4}$ over $\mathbb{C}^p$ ($D=2$) is trivial

if prime $p \equiv 3$ mod 4.

.... There are exactly two inv. alg over $\mathbb{C}^2$ in $D=2$.

# References

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Gross--Nesme--Vogt--Werner https://arxiv.org/abs/0910.3675

1D QCA with strict locality is a circuit plus shift.
The notion of boundary algebras appeared.

Haah--Fidkowski--Hastings https://arxiv.org/abs/1812.01625

Our first QCA paper, that constructs a nontrivial 3D QCA. The nontriviality argument is a reduction to the nonexistence of a commuting Hamiltonian for chiral states in 2+1D.
The constructed QCA maps Pauli matrices to tensor product of Pauli matrices, and against quantum circuits of the same property (Clifford) the nontriviality is proven.

Freedman--Hastings https://arxiv.org/abs/1902.10285

Flows in higher dimensions
Triviality of 2D QCA over quantum spins against quantum circuits of strict locality.

Haah https://arxiv.org/abs/1907.02075

Specialization to translation invariant Clifford QCA
Exponents of such QCA
Nontrivial examples in D=3 over prime dimensional degrees of freedom at each site.
Characterization of boundary algebras has antihermitian forms over translation group algebra.

Freedman--Haah--Hastings https://arxiv.org/abs/1910.07998

Group structure of QCAs
Ancilla removal
Discussion on equivalence relations of QCA (blending and circuit).