Joint work with Sitan Chen, Hsin-Yuan (Robert) Huang, and Jerry Li

Quantum-enhanced Learning Using a Quantum Memory

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Introduction: what is an experiment?

- Measuring constants of nature (speed of light, charge of electron)
- Quantifying dynamical properties (rate of a chemical reaction)
- Inferring structural properties (symmetry group of a crystal)
- Learning more abstract information (the chain of chemical reactions that comprise photosynthesis, whether Yang-Mills describes the strong force)

What exactly *is* an experiment, in its full scope of generality?

Recent perspectives

Many experiments can be thought of as *quantum learning algorithms* which learn properties of (partially) uncharacterized systems via examples **[Huang, Kueng, Preskill '21]**

All experiments are examples of *quantum algorithmic measurements* (QUALMs), which are a hybrid of black box algorithms and interactive protocols **[Aharonov, Cotler, Qi '21]**

[Gutoski, Watrous '07], **[Chiribella, D'Ariano, Perinotti '08]**,...

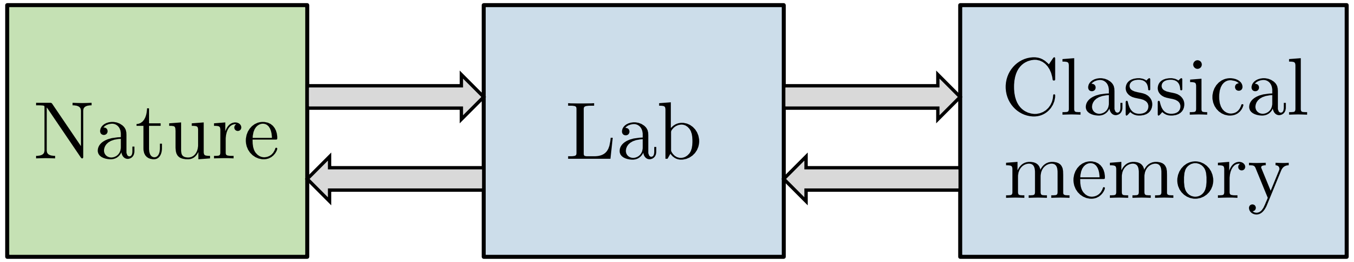
Power of quantum memory

- We would like to quantify the power of quantum memory in helping us learn about Nature via experiments

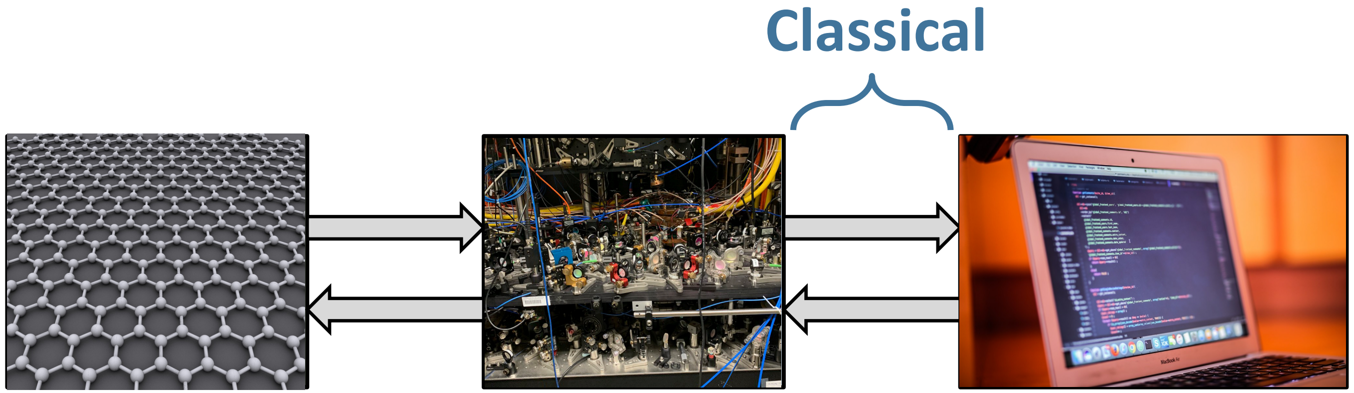
- Consider two kinds of experimental protocols:

1. Those with a **classical memory**
2. Those with a **quantum memory**

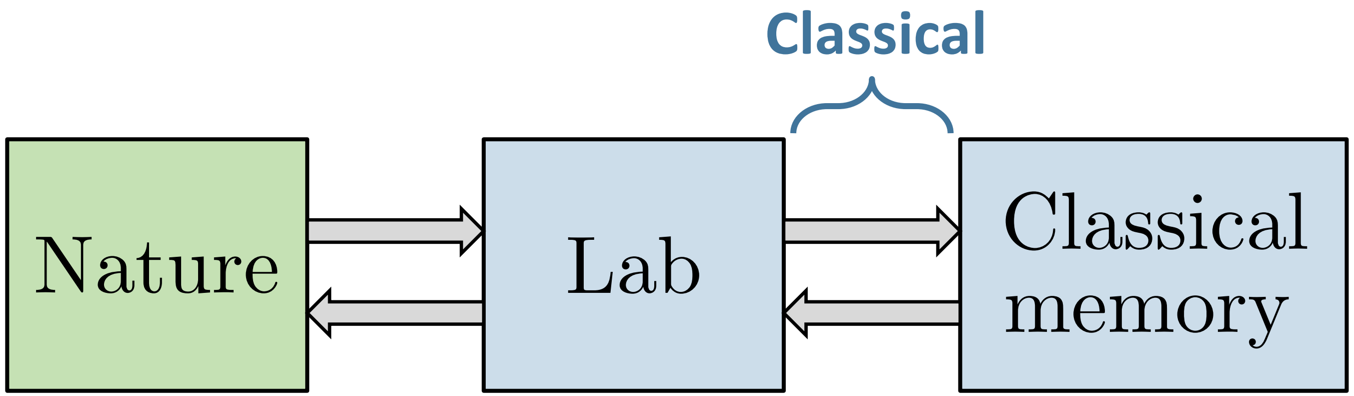
Protocols with a Classical Memory



Protocols with a Classical Memory



Protocols with a Classical Memory

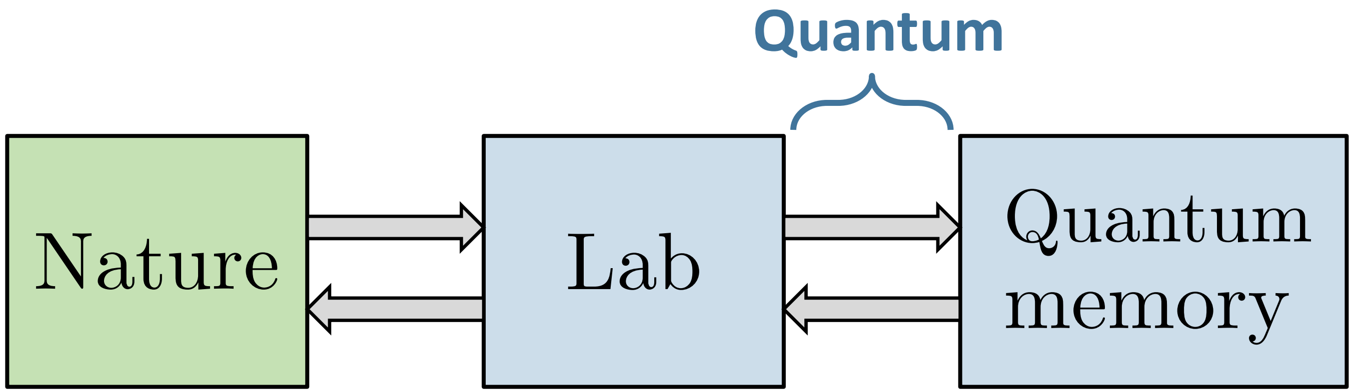


Protocol is T rounds (Nature is queried once per round)

Each round:

1. Prepare a state on the lab system (state can depend adaptively on **classical memory**)
2. Query Nature
3. Measure lab system with a POVM (can depend adaptively on **classical memory**) and store the **classical** outcome in the memory
4. Initialize lab system to all zero state

Protocols with a Quantum Memory



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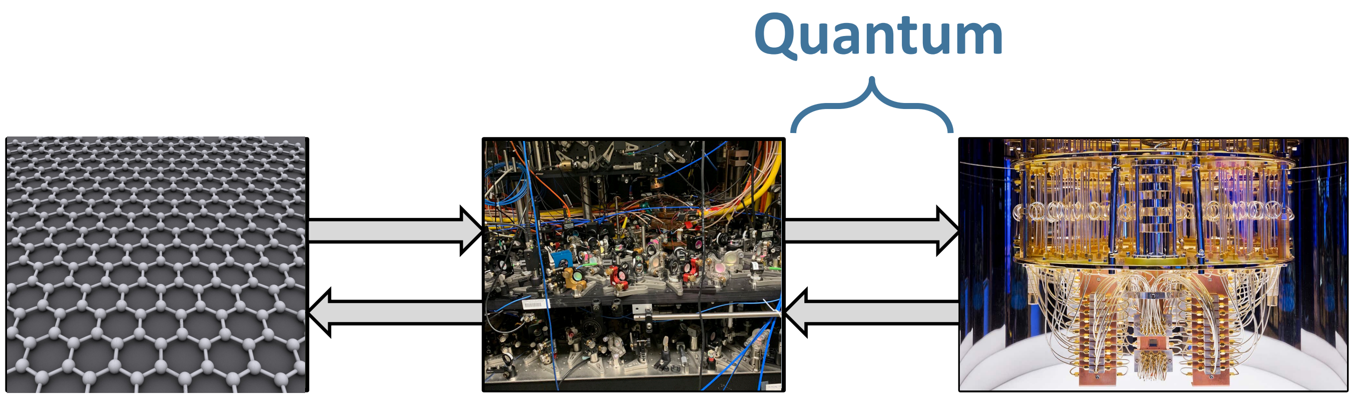
Protocols with a Quantum Memory



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Each round:

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Goal

Find tasks for which there is an ***exponential*** advantage using a **quantum memory** versus a **classical memory**

Two parts:

1. Show that a task can be done efficiently if we have access to a **quantum memory** → **Easy: exhibit one efficient protocol**
2. ★ Show that a task is exponentially hard if we only have access to a **classical memory** → **Difficult: rule out all efficient protocols**

Recent work

- Exponential separation in gate and query complexity for learning properties of classical oracles
  - Generalized Simon's problem **[Chia, Chung, Lai '20]**
  - Welded tree problem **[Coudron '20]**
- Exponential separation in query complexity for learning Pauli observables **[Huang, Kueng, Preskill '21]**
- Exponential separation in gate and query complexity for learning properties of quantum states and channels **[Aharonov, Cotler, Qi '21]**

Our work

- We synthesize and generalize the approaches of:
  [Bubeck, Chen, Li ‘20], **[Huang, Kueng, Preskill ‘21]**, **[Aharonov, Cotler, Qi ‘21]**

- We provide a new framework for proving ***exponential*** lower bounds for protocols with classical memories

- Allows us to prove new ***exponential complexity separations*** between interactive protocols with and without quantum memories

Purity Testing

Task: Distinguish between (i) the maximally mixed state, and (ii) a fixed, Haar-random pure state

Theorem [Aharonov, Cotler, Qi '21]

$$\text{Complexity}_{\text{QuantumMemory}}[\text{PurityTest}(n)] = \Theta(1)$$

$$\text{Complexity}_{\text{ClassicalMemory}}[\text{PurityTest}(n)] = \Omega(2^{n/3.5})$$

Proof of classical lower bound is around 20 pages

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Proof of classical lower bound is ½ page using learning tree framework

Learning Tree Framework (for learning states)

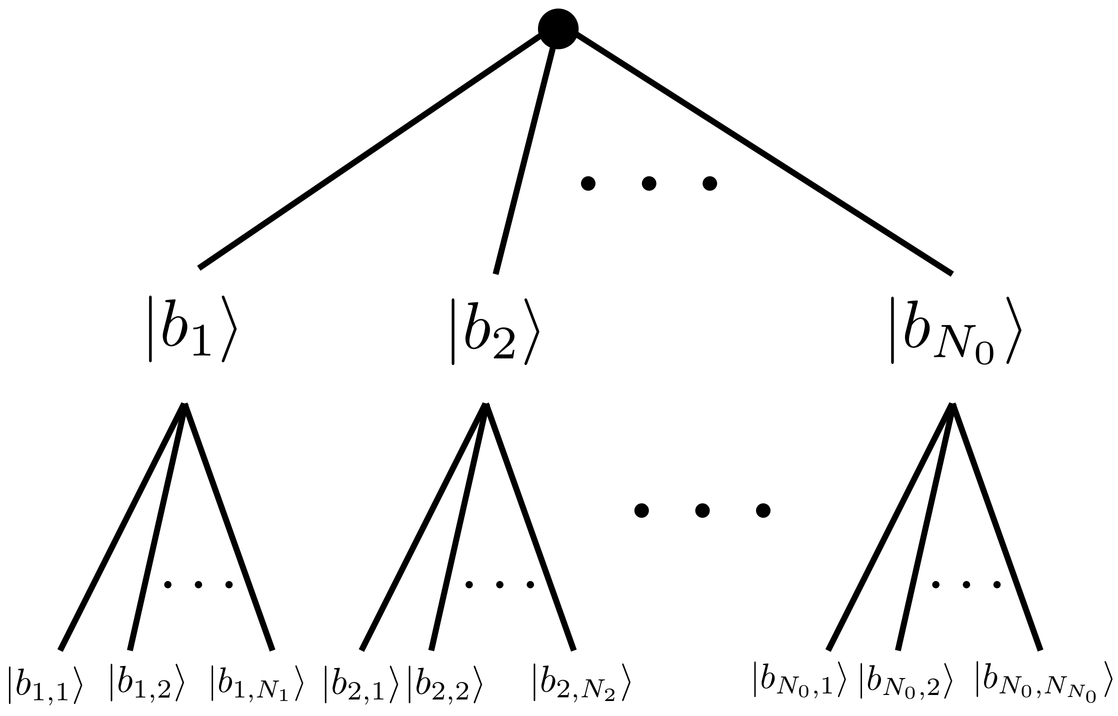
Learning Tree Framework

Suppose we have a physical source producing a state ρ, and we want to learn about it in the setting *without* a quantum memory

Obtain copy of ρ, measure using a rank-1 POVM $\{|b_i\rangle\langle b_i|\}_i$, suppose the output is q and store it in the classical memory

Obtain copy of ρ, measure using a rank-1 POVM $\{|b_{q,i}\rangle\langle b_{q,i}|\}_i$, suppose the output is r and store it in the classical memory

Obtain copy of ρ, measure using a rank-1 POVM $\{|b_{q,r,i}\rangle\langle b_{q,r,i}|\}_i$, suppose the output is s and store it in the classical memory



- A particular instantiation of the protocol is a path through the tree
- A tree of depth T corresponds to measuring T copies of the state

Learning Tree Framework

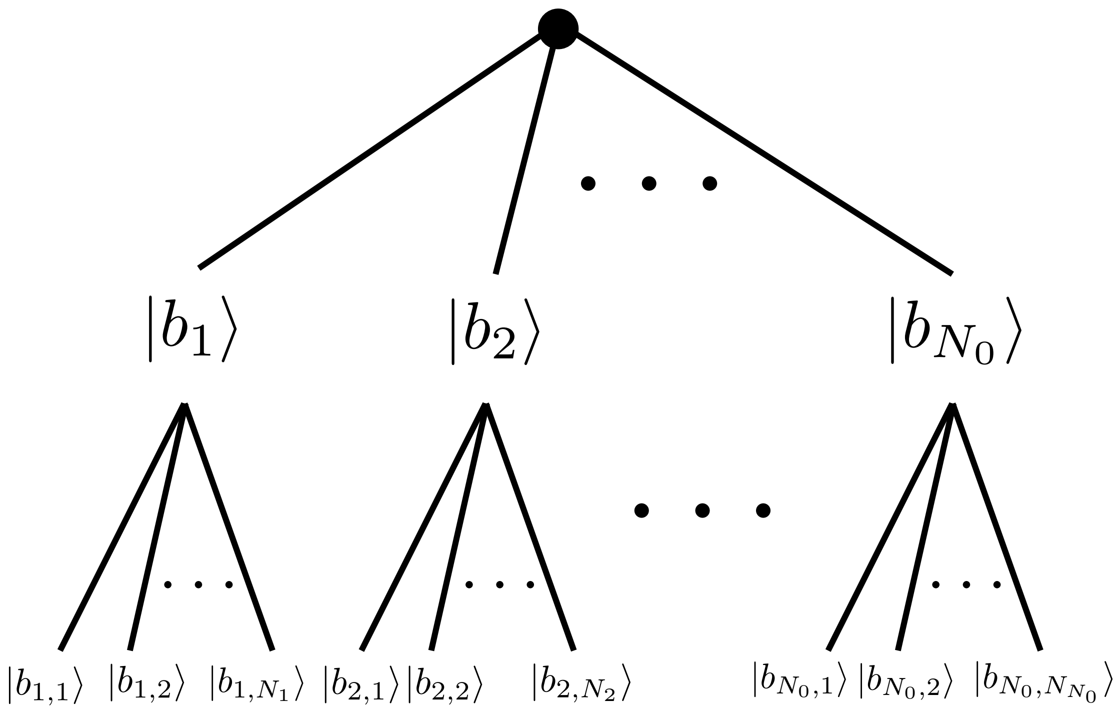
Let $v_0, v_1, \ldots, v_{T-1}$ be a root-to-leaf path through the tree

The probability of taking that path is

$$p^{\rho}(\{v_t\}) = \prod_{t=0}^{T-1} \langle b_{v_t} | \rho | b_{v_t} \rangle$$

$$p^{\sigma}(\{v_t\}) = \prod_{t=0}^{T-1} \langle b_{v_t} | \sigma | b_{v_t} \rangle$$

$$\frac{1}{2} \sum_{\text{Paths } \{v_t\}} |p^{\rho}(\{v_t\}) - p^{\sigma}(\{v_t\})|$$



Proof strategies

1. **Edge-based:** Bound the information the learner can gain from traversing any edge in the tree
2. **Path-based:** Bound the information gain for traversing an entire path.

Purity Testing

Task: Distinguish between (i) the maximally mixed state, and (ii) a fixed, Haar-random pure state

Theorem [Chen, Cotler, Huang, Li '21]

$$\text{Complexity}_{\text{QuantumMemory}}[\text{PurityTest}(n)] = \Theta(1)$$

$$\text{Complexity}_{\text{ClassicalMemory}}[\text{PurityTest}(n)] = \Theta(2^{n/2})$$

Proof of classical lower bound is ½ page using learning tree framework

Proof of Lower Bound for Classical Memory

Let $v_0, v_1, \ldots, v_{T-1}$ be a root-to-leaf path through the tree

The probability of taking that path is

$$p_{\text{mixed}}(\{v_t\}) = \prod_{t=0}^{T-1} \langle b_{v_t} | \frac{\mathbb{1}}{2^n} | b_{v_t} \rangle = \frac{1}{2^{nT}}$$

$$p_{\text{pure}}(\{v_t\}) = \mathbb{E}_{|\Psi\rangle \sim \text{Haar}} \prod_{t=0}^{T-1} \langle b_{v_t} | \Psi \rangle \langle \Psi | b_{v_t} \rangle$$

$$\frac{1}{2} \sum_{\text{Paths } \{v_t\}} |p_{\text{mixed}}(\{v_t\}) - p_{\text{pure}}(\{v_t\})|$$

$$\sum_{\substack{\text{Paths } \{v_t\} \\ p_{\text{mixed}}(\{v_t\}) > p_{\text{pure}}(\{v_t\})}} p_{\text{mixed}}(\{v_t\}) \left(1 - \underbrace{\frac{p_{\text{pure}}(\{v_t\})}{p_{\text{mixed}}(\{v_t\})}}\right)$$

$$\frac{p_{\text{pure}}(\{v_t\})}{p_{\text{mixed}}(\{v_t\})} = \mathbb{E}_{|\Psi\rangle \sim \text{Haar}} \prod_{t=0}^{T-1} 2^n \langle b_{v_t} | \Psi \rangle \langle \Psi | b_{v_t} \rangle$$

$$= \underbrace{\frac{2^{nT}}{2^n (2^n + 1) \cdots (2^n + T - 1)}}_{\geq \left(1 - \frac{T}{2^n}\right)^T} \underbrace{\sum_{\pi \in S_T} \mathrm{tr}\left(P_\pi \bigotimes_{t=0}^{T-1} |b_{v_t}\rangle\langle b_{v_t}| \right)}_{\geq 1}$$

$$T \geq \Omega(2^{n/2})$$

Exponential Separations in Learning Quantum States

Pauli State Property Testing

Task: Given a state $\frac{\mathbb{1} + \varepsilon P}{2^n}$ where P is a Pauli string, determine P

Theorem [Chen, Cotler, Huang, Li '21]

$$\text{Complexity}_{\text{QuantumMemory}}[\text{PauliStateTest}(n)] = \mathcal{O}(n)$$

$$\text{Complexity}_{\text{ClassicalMemory}}[\text{PauliStateTest}(n)] = \widetilde{\Theta}(2^n)$$

A variation of this task is gate efficient with a quantum memory

Resolves a conjecture in **[Aharonov, Cotler, Qi '21]**

Shadow tomography without a quantum memory

Task: Given a state ρ and observables $\{O_1, ..., O_M\}$, determine all $\mathrm{tr}(O_i\, \rho)$ to ε error

Theorem [Chen, Cotler, Huang, Li '21]

$$\text{Complexity}_{\text{ClassicalMemory}}[\text{Shadow}(n)] = \widetilde{\Theta}(\min(M, 2^n)/\varepsilon^2)$$

Upper bound is from **[Huang, Kueng, Preskill '21]**

Resolves an open problem from **[Aaronson '18],[Aaronson, Rothblum '19]**

Exponential Separations in Learning Quantum Channels

Fixed Unitary Problem

Intuition: It is difficult to distinguish time-independent evolution from time-dependent evolution if each is sufficiently chaotic

We will make this intuition precise by formulating a task

Fixed Unitary Problem

Task: Distinguish between (i) the maximally depolarizing channel, and (ii) a fixed, Haar-random unitary

Theorem [Aharonov, Cotler, Qi '21] [Chen, Cotler, Huang, Li '21]

$$\text{Complexity}_{\text{QuantumMemory}}[\text{FixedUnitary}(n)] = \mathcal{O}(1)$$

$$\text{Complexity}_{\text{ClassicalMemory}}[\text{FixedUnitary}(n)] = \Omega(2^{n/3})$$

Protocol with quantum memory is gate efficient [Aharonov, Cotler, Qi '21]

Generalizes the setting of [Aharonov, Cotler, Qi '21] and improves bound

Symmetry Distinction Problem

Intuition: It is difficult to determine the symmetry classes of a strongly-interacting quantum many-body system

We formulate a Task in which this can be made precise

First, recall time-reversal symmetry classes of unitaries:

Unitary

$$U \in U(d)$$

$$U^{-1} = U^{\dagger}$$

Orthogonal

$$O \in O(d)$$

$$O^{-1} = O^{T}$$

Symplectic

$$S \in \mathrm{Sp}(d/2)$$

$$S^{-1} = -\mathbf{J} S^{T} \mathbf{J}$$

$$\mathbf{J}^2 = -\mathbb{1}$$

Symmetry Distinction Problem

Task: Distinguish between (i) a fixed, Haar-random unitary, (ii) a fixed, Haar-random orthogonal matrix, and (iii) a fixed, Haar-random symplectic matrix

Theorem [Aharonov, Cotler, Qi '21] [Chen, Cotler, Huang, Li '21]

$$\text{Complexity}_{\text{QuantumMemory}}[\text{Symmetry}(n)] = \mathcal{O}(1)$$

$$\text{Complexity}_{\text{ClassicalMemory}}[\text{Symmetry}(n)] = \Omega(2^{n/3.5})$$

Protocol with quantum memory is gate efficient [Aharonov, Cotler, Qi '21]

Generalizes the setting of [Aharonov, Cotler, Qi '21]

Exponential separation with Bounded Quantum Memory

Shadow tomography with bounded quantum memory

This result is qualitatively different than the previous ones

We assume we have a quantum memory, but that it is bounded in size

Theorem [Chen, Cotler, Huang, Li '21]

Any learning algorithm with k qubits of quantum memory requires $\Omega(2^{(n-k)/3})$ copies of ρ to predict $|\mathrm{tr}(P\rho)|$ for all n-qubit Pauli observables P with probability at least $2/3$.

Combines proof techniques of all the results previously mentioned

Lower bound is polynomial when $k \sim n - \# \log(n)$

Discussion

Quantum advantage → Quantum computers better than classical computers at *classical* problems

Quantum memory advantage → **Quantum memory has advantage over classical memory for *quantum* experiments**

Discussion

- Developed a new framework for proving exponential separations between quantum learning algorithms with and without a quantum memory
- Established the first exponential tradeoff between bounded quantum memory and query complexity
- Experimental demonstration of quantum memory advantage in NISQ era
- What are effects of noise?
- Other 'physical' examples along the lines of the $\frac{\mathbb{1} + \varepsilon P}{2^n}$ task?