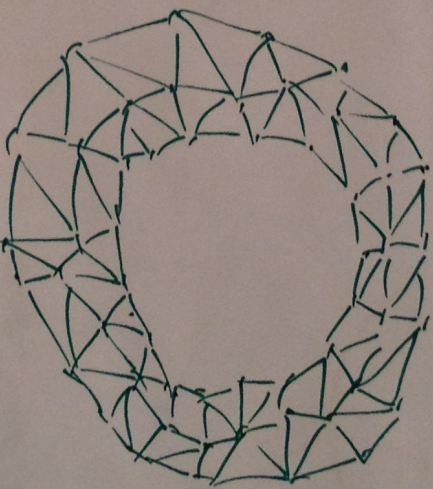
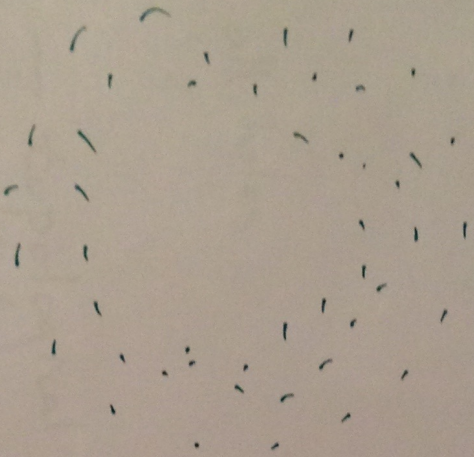


Quantum algorithm for topological analysis of data

Seth Lloyd, Silvano Carnevale, Paolo Zavaradi



Identify holes, voids, connected components via persistent homology.

Classical algorithms: $O(2^{2n})$

Quantum: $O(n^3)$

Advertisement:

On the uncomputability of the spectral gap
arXiv: 1602.05924 S.L.

(compare Cubitt et al. Nature 528, 207 (2015))

summarizes results of

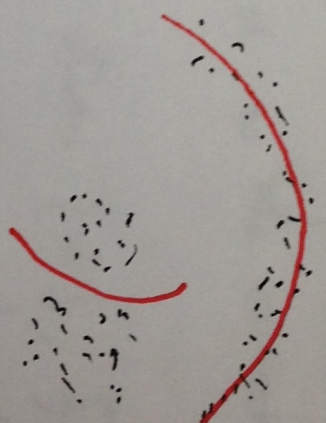
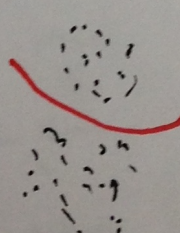
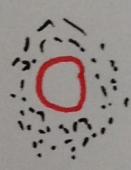
S.L. PRL 71, 903 (1993)

S. Mod. Phys. 41, 2503 (1969)

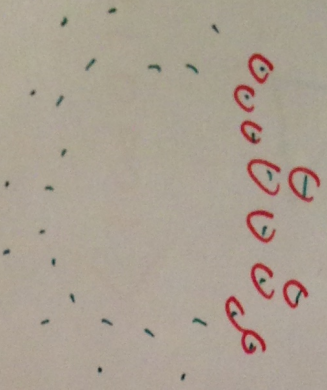
which show that

any Hamiltonian quantum system capable
of universal computation
has an uncomputable gap

Quantum machine learning algorithms

- data $\vec{X} \in \mathbb{C}^n$
 - quantum random access memory (qRAM)
 $\vec{X} \rightarrow |x\rangle$
 - Formulation:
 $\langle y | QFT | x \rangle > 1 - \epsilon$?
 - Data fitting:

 - Support vector machines:

 - Principal component analysis: eigenvectors of $\sum_i \vec{x}_i \vec{x}_i^T$
 count holes, voids, ...
 - topological invariants:
 - quantum finance
- $\Rightarrow$ all take time $O(\text{poly} \log(n))$

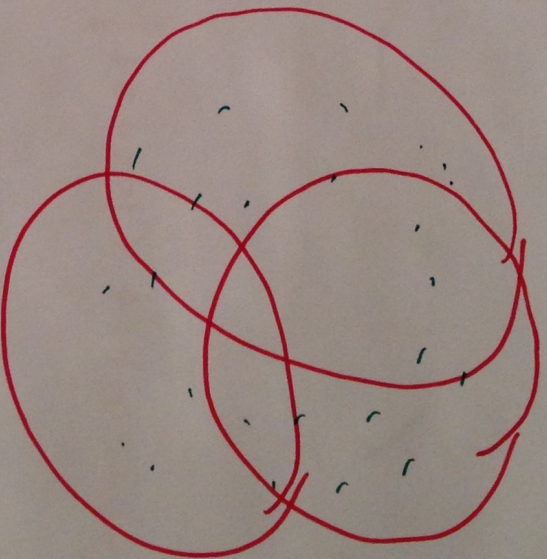
Persistent homology: group data at different scales



all holes



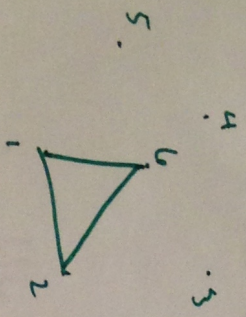
one hole



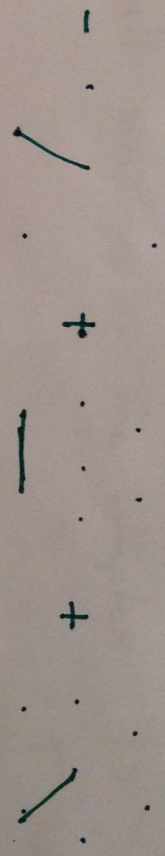
no holes

— topological features that **persist** over a wide range of grouping scales are considered to be actual features of data

Method: algebraic topology maps k -simplices to vectors $\in \mathbb{C}^2$

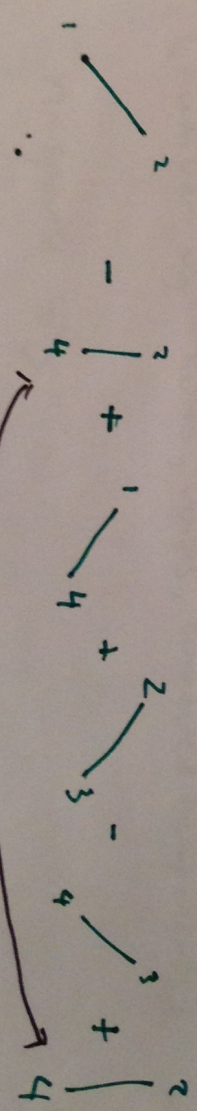
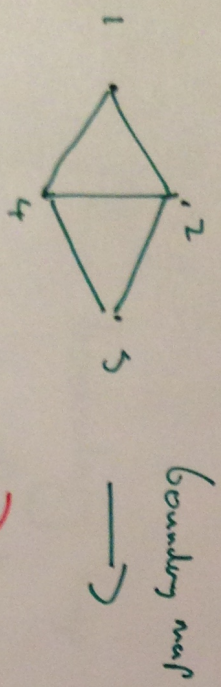


boundary map
→



$$|110001\rangle \rightarrow 2|110001\rangle = |010001\rangle - |100001\rangle + |110000\rangle$$

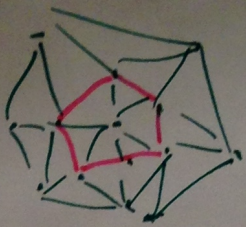
• boundary map takes vector corresponding to k -simplex to oriented sum of k -simplices on boundary



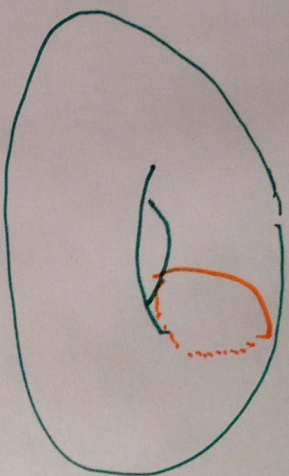
interior edges cancel

$$|1101\rangle + |0111\rangle \xrightarrow{2} |0101\rangle - |1001\rangle - |1100\rangle + |0011\rangle - |0101\rangle + |0110\rangle$$

Representatives of the homology
 correspond to sets of simplices that are **boundaryless**
 but that ~~are~~ not **boundaries**



boundaryless
 but is a boundary



boundaryless but is not a boundary

Look for chains of simplices that lie in **$\text{Ker } \partial_k$**
 but not in the **$\text{Image } \partial_{k+1}$**

Steps:

- For each grouping scale ϵ , construct quantum superposition of all simplices in the complex $|S_k\rangle = \sum |k\text{simplices}\rangle$

- Use quantum phase algorithm to decompose $|S_k\rangle$ in terms of eigenvectors of $\partial_{w1}\partial_{k+1}^\dagger + \partial_k^\dagger\partial_k$

combinatorial Laplacian

- $\text{Ker}(\partial_{k+1}\partial_{k+1}^\dagger + \partial_k^\dagger\partial_k)$

$\Leftrightarrow$ representatives of homology

dimension $(\text{Ker}) = k\text{th Betti number}$
(number of holes, voids, connected components)

- boundary map is n -sparse $\Rightarrow$ quantum algorithm takes time $O(n^3)$
(classical algorithm takes time $O(2^{2n})$)

Summary:

- Quantum algorithm for algebraic topology operates by creating quantum superposition of simplices (maps 2^n dimensional vectors onto n qubits)
- and finds eigenvectors + eigenvalues of combinatorial Laplacian
$$\sum_k \partial_k^+ \partial_k + \partial_{k+1} \partial_{k+1}^+$$
- # of zero eigenvalues (dim of kernel) gives Betti numbers + eigenstates give generators of homology
- other eigenvalues + eigenvectors give frequencies + harmonics of Laplacian — 'hear the shape of a drum'
- quantum $O(n^3)$ classical $O(2^{2n})$