HARVARD UNIVERSITY
17 Oxford Street
Cambridge, MA 02138

Mathematical Picture Language Seminar

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Zoom QR Code & Link:

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Uniqueness of BP fixed point for Ising models

Yury Polyanskiy
Massachusetts Institute of Technology

Abstract: In the study of Ising models on large locally tree-like graphs, in both rigorous and non-rigorous methods one is often led to understanding the so-called belief propagation distributional recursions and its fixed point (also known as Bethe fixed point, cavity equation etc). In this work we prove there is at most one non-trivial fixed point for Ising models with zero and certain random external fields. As a concrete example, consider a sample A of Ising model on a rooted tree (regular, Galton-Watson, etc). Let B be a noisy version of A obtained by independently perturbing each spin as follows: B_v equals to A_v with some small probability δ and otherwise taken to be a uniform +-1 (alternatively, 0). We show that the distribution of the root spin A_ρ conditioned on values B_v of all vertices v at a large distance from the root is independent of δ and coincides with $\delta=0$. Previously this was only known for sufficiently low-temperature" models. Our proof consists of constructing a metric under which the BP operator is a contraction (albeit non-multiplicative). I hope to convince you our proof is technically rather simple.

This simultaneously closes the following 5 conjectures in the literature: uselessness of global information for a labeled 2-community stochastic block model, or 2-SBM (Kanade-Mossel-Schramm'2014); optimality of local algorithms for 2-SBM under noisy side information (Mossel-Xu'2015); independence of robust reconstruction accuracy to leaf noise in broadcasting on trees (Mossel-Neeman-Sly'2016); boundary irrelevance in BOT (Abbe-Cornacchia-Gu-P.'2021); characterization of entropy of community labels given the graph in 2-SBM (ibid). Joint work with Qian Yu (Princeton).

https://mathpicture.fas.harvard.edu/seminar