A NATURAL LIMITATION FOR PROPERLY HUMAN SCIENTIFIC PROGRESS.

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This is a philosophical problem

Claim: there is a lot of trash in published philosophical literature, but it is natural for scientists to ask 'philosophical' questions on their professional work, without obligation to follow 'official' philosophy, even if published in refereed scientific-looking journals.

A. Jaffe and F. Quinn " "Theoretical mathematics": towards a cultural synthesis of mathematics and theoretical physics". Bull. Amer. Math. Soc. **29**,1-13(1993).

D. Ruelle. "Post-human mathematics" arXiv:1308.4678 [math.HO] [ESI conference in Vienna (April 2013)].

The specific problem of progress for mathematics

It follows rather directly from Gödel that mathematical progress per mathematician should diminish with time. (The proof of a statement of length L has minimal proof length $>> L$.)

But there remains the real problem is what relation there is between human mathematics and the set of consequences of the ZFC axioms.

Scientific and philosophical ideas

Scientific concepts are of mathematical or experimental-observational nature.

Philosophical concepts refers to the functioning of the human brain in an essential manner using intuition, but without analysis of the origin of the concepts.

(Examples: intuition, understanding, existence, God, cogito ergo sum)

Opinions of mathematicians on philosophers

C.F. Gauss: "When a philosopher says something that is true then it is trivial. When he says something that is not trivial then it is false".

A.N. Whitehead: "The safest general characterization of the European philosophical tradition is that it consists of a series of footnotes to Plato".

[I. Niiniluoto "Scientific Progress" in *The Stanford Encyclopedia of Philosophy*, 2019 (E. N. Zalta ed.).]

[J. Avigad "Varieties of mathematical understanding". Bull. Amer. Math. Soc. **59**,99-117(2022).]

Natural limitations to human scientific progress.

Our discussion:

- estimating human intellectual capabilities:
  properly human intelligence
- estimating complexity of things investigated
  we restrict at first to mathematics

Our conclusion:

- the rate of mathematical progress per mathematician
  should be decreasing with time

Human mathematics.

Mathematics is based on a system of axioms
(currently: Zermelo-Fraenkel-Choice)
proofs translatable in formal language

Some theorems have very long proofs with respect to
the length of their statement
[this is related to Gödel's incompleteness theorems, e.g. :
undecidability of halting problem for Turing machine]

Human mathematical understanding based on structures
(groups, compact spaces, etc.)
[structures are human constructs depending on time:
Bourbaki, Grothendieck, categories and functors, Voevodsky]

Compare

$3^2 + 4^2 = 5^2$ $\qquad$ $3^3 + 4^3 + 5^3 = 6^3$

How natural is human mathematics?

Great diversity of intellectual profiles of best mathematicians

(Early 20-th century: Poincaré, Hilbert, Gödel, von Neumann)

$\Rightarrow$ diverse directions of human mathematical progress

$\neq$ unique human racing speed limit (Usain Bolt: 44.72 km/h)

explained by human evolutionary argument

Proofs understandable by humans

$\neq$ computer-verified formal proofs

Compared with computer, human brain is

slow, prone to error, has limited memory, and is highly parallel

uses human understanding (structures)

$\Rightarrow$ creativity

Reflections on human mathematical thinking

Reflections of various mathematicians
    (Gauss, Poincaré, Hadamard, Hardy, etc.)
    on mathematical thinking

Role of
    unconscious thinking
    short term memory (working memory)
    combinatorial nature of mathematical thinking

These reflections do not amount to a theory of
    mathematical creativity

Progress of human mathematics.

New domains [computational complexity: NP completeness]

Short-term memory $\Rightarrow$ definition of structures (how?)

Human mathematics understandable by humans
is in human (not formal) language
$\neq$ theorems provable from axioms
$\neq$ computer-verified formal proofs

The rate of mathematical progress per mathematician
should be decreasing with time

Creative human mathematical thinking.

Human-understandable jargon based on natural language
(often in written form)
(translation to axiom-based formalization usually non explicit).

Understandable: manipulates short formulations (sentences)
with concepts which are defined and named.

Properly human mathematics falls short of statements
following from ZFC (e.g., Hale's proof of Kepler's
conjecture on packing of spheres).

Possibly non-natural nature of the structural mathematics
(of Bourbaki, Grothendieck, etc.)

Summary of human mathematics

Human mathematics is based on axioms

Human mathematics uses natural language+jargon, it can be translated into formal language

Because of short-term working memory and trying to avoid errors, we use short formulations (sequence of sentences) with named concepts

Mathematical thinking is combinatorial and partly unconscious it is creative but we don't really know how it works

Therefore purely human mathematics is different from a set of consequences of the axioms

Nonmathematical sciences.

Observational or experimental protocols

making contact with reality at a certain approximation

Scientific theory:

piece of reality identified with mathematical structure

Various ingredients to the rate of scientific progress:

political decisions (Superconducting Super Collider 1993)

Moore's law: number of transistors in dense integrated circuit doubles every two years (1970 - 2020)

Lack of general statement