# Holographic Scattering from Quantum error-correction

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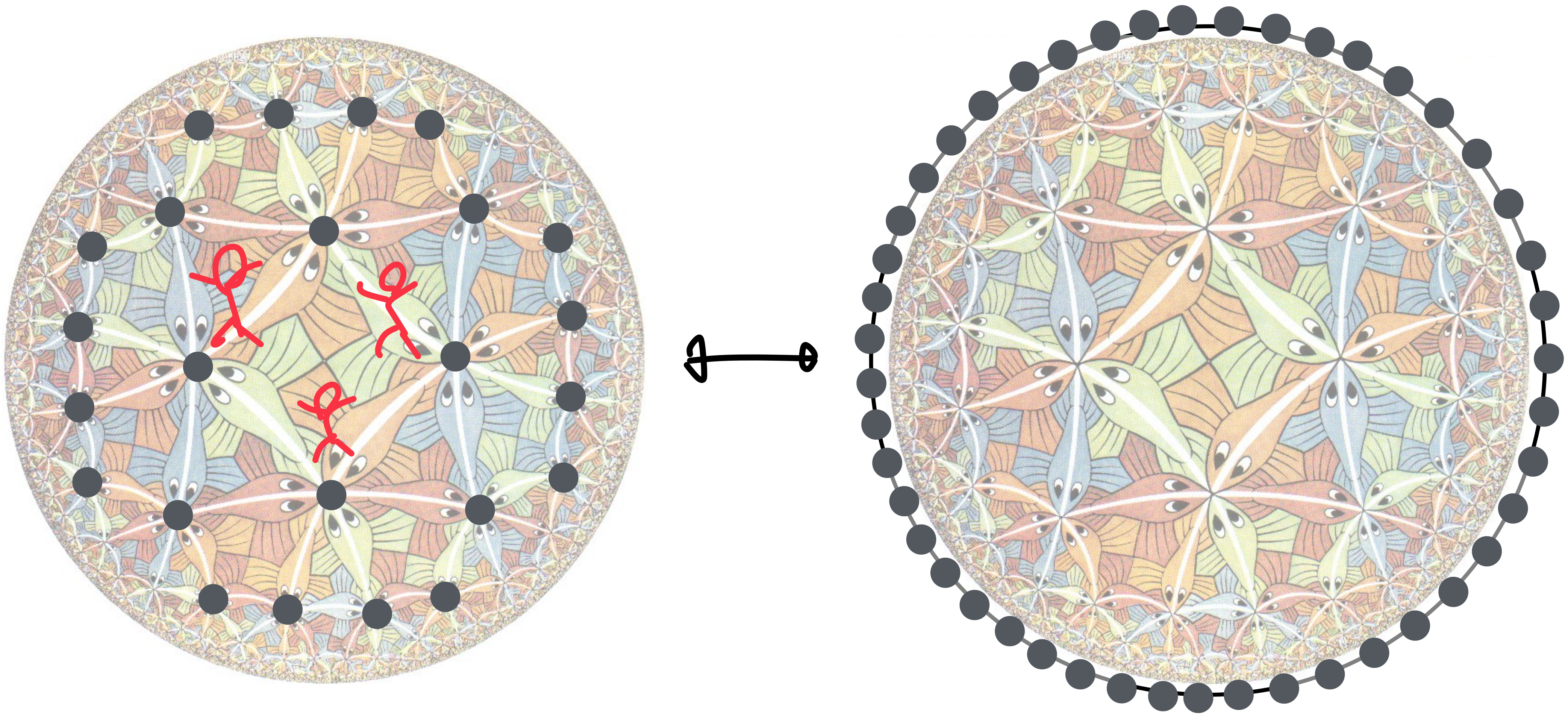
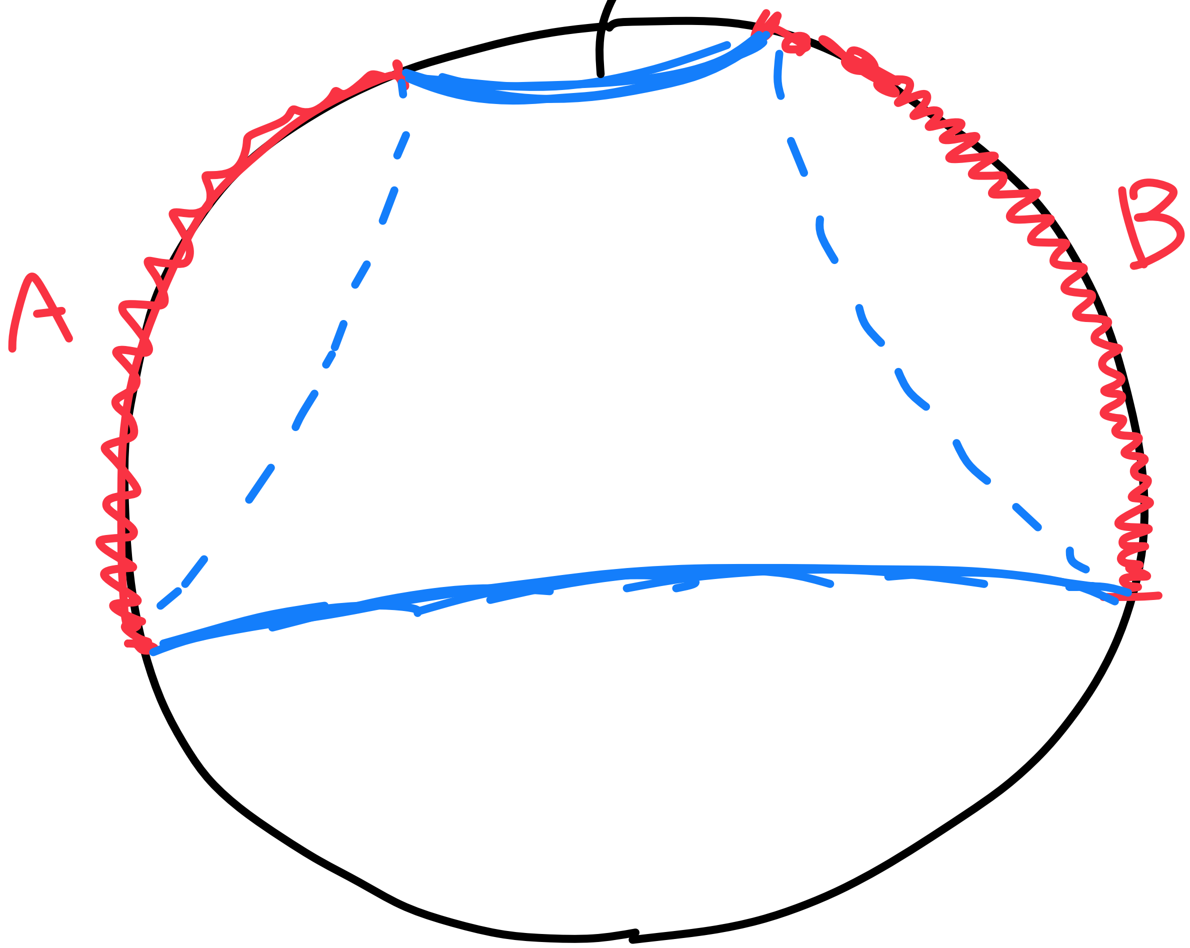
# AdS/CFT correspondence

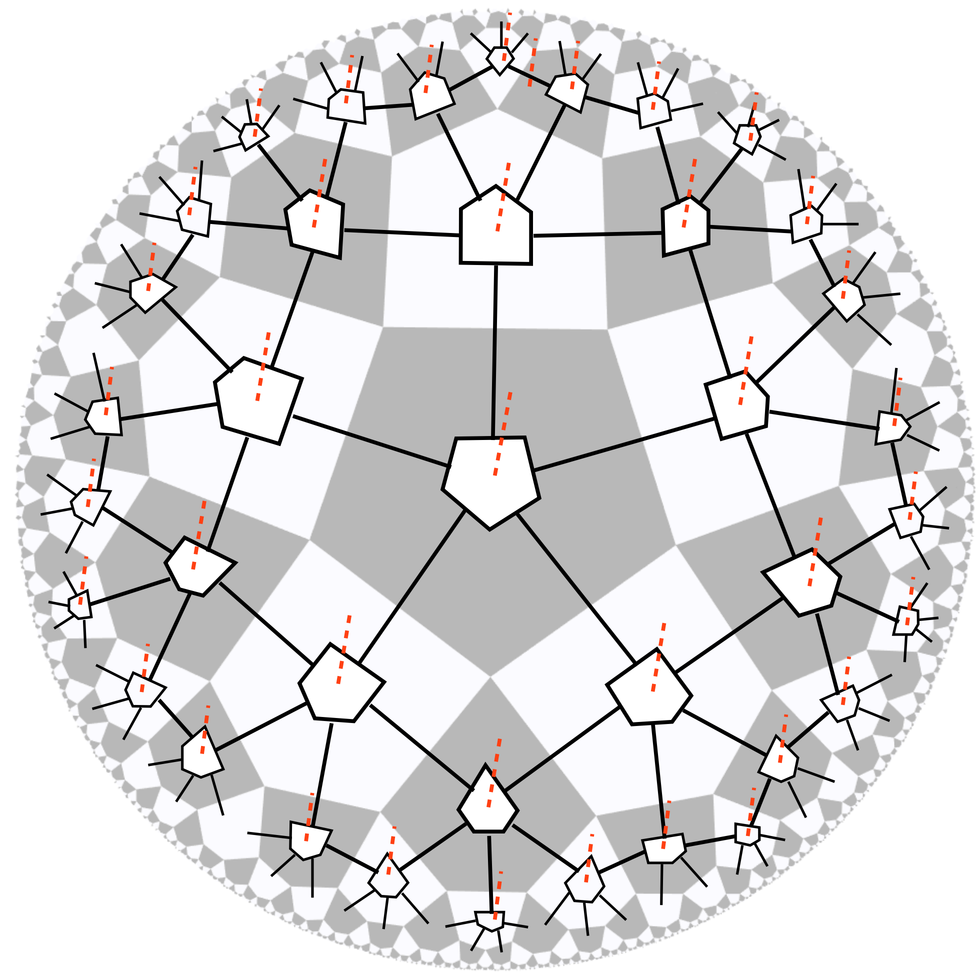


bulk physics $\longleftrightarrow$ boundary physics

# Spacetime from entanglement

minimal surface



Entanglement ↔ connected wedge

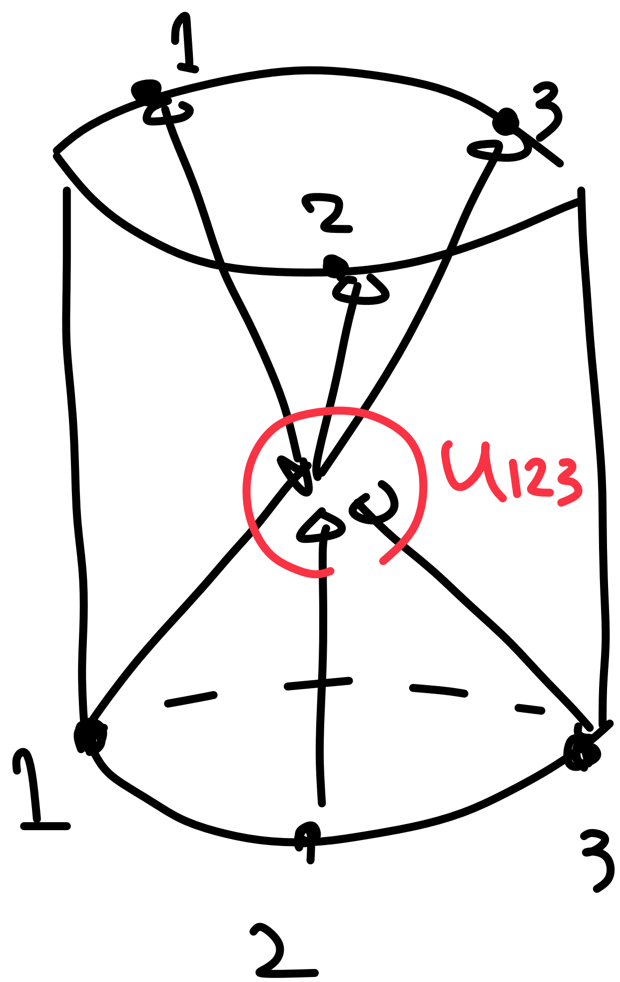
(like wormhole)

Ryu-Takayanagi formula

Tensor-network

@ quantum error-correction

# Holographic Scattering?




$U_{123}$ : Arbitrary unitary op.

How is $U_{123}$ realized on boundary QM?

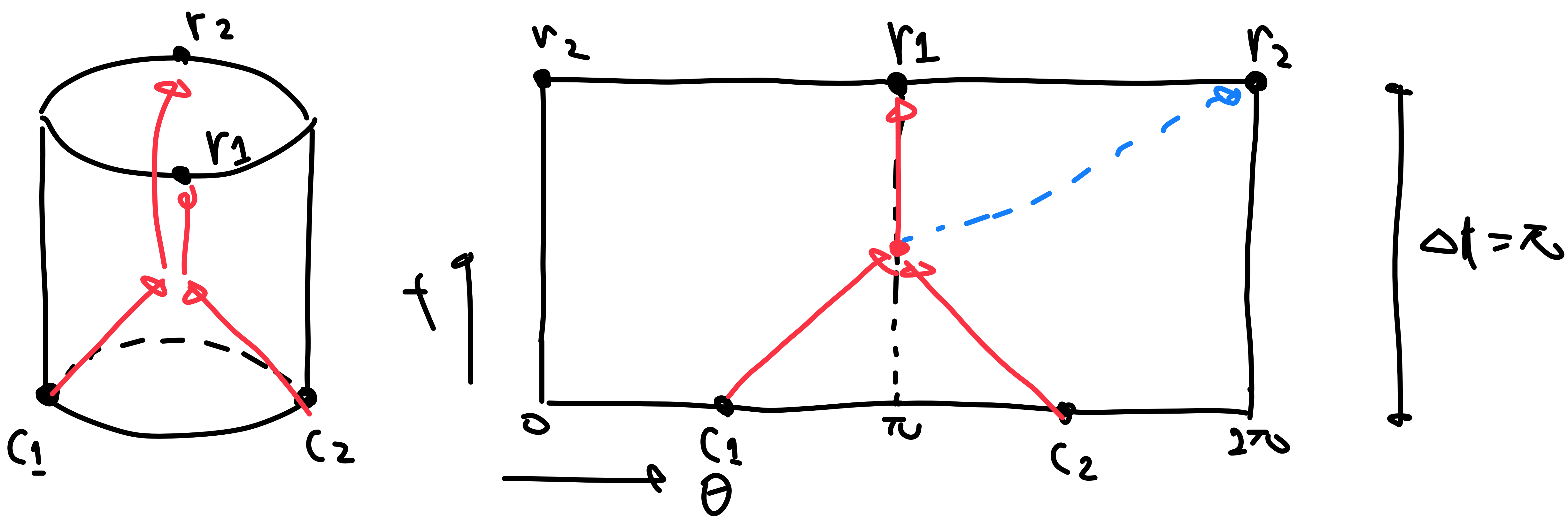
Important work by Alex May (2019)

Holographic Scattering

$\Updownarrow$

Entanglement

# 2 particle Scattering Puzzle

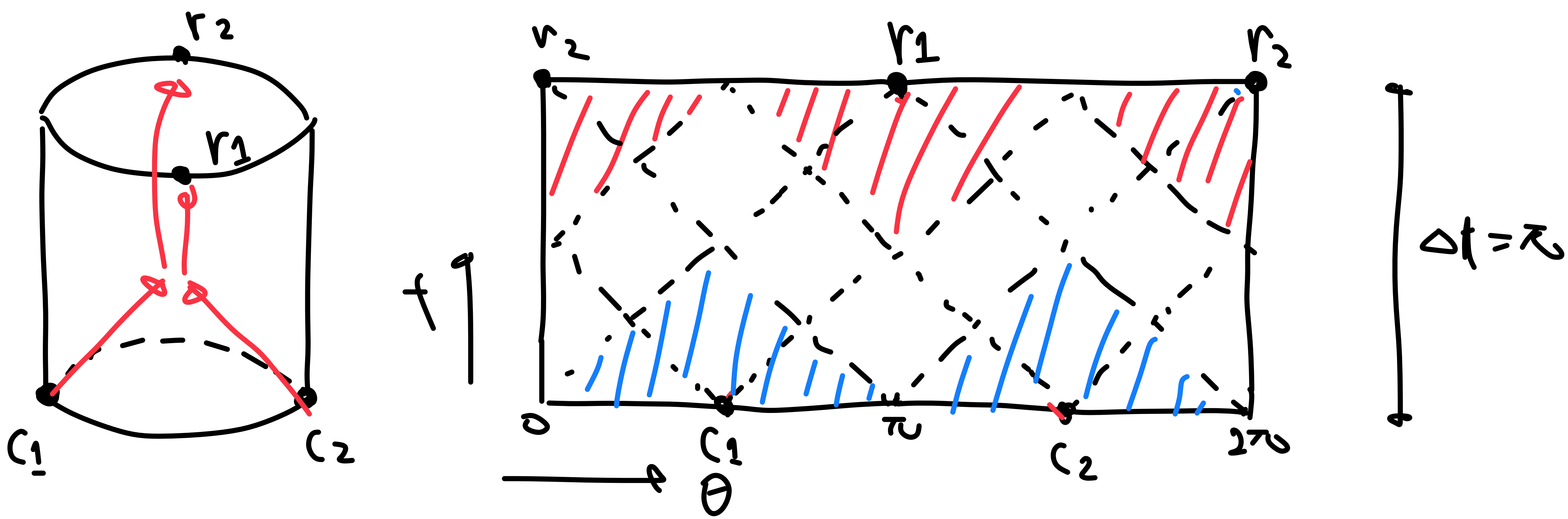



bulk $\Delta t = \pi$ boundary $\Delta t > \pi$

Not enough time for boundary scattering!!

(Alex May, 2019)

# Future and Past light cones

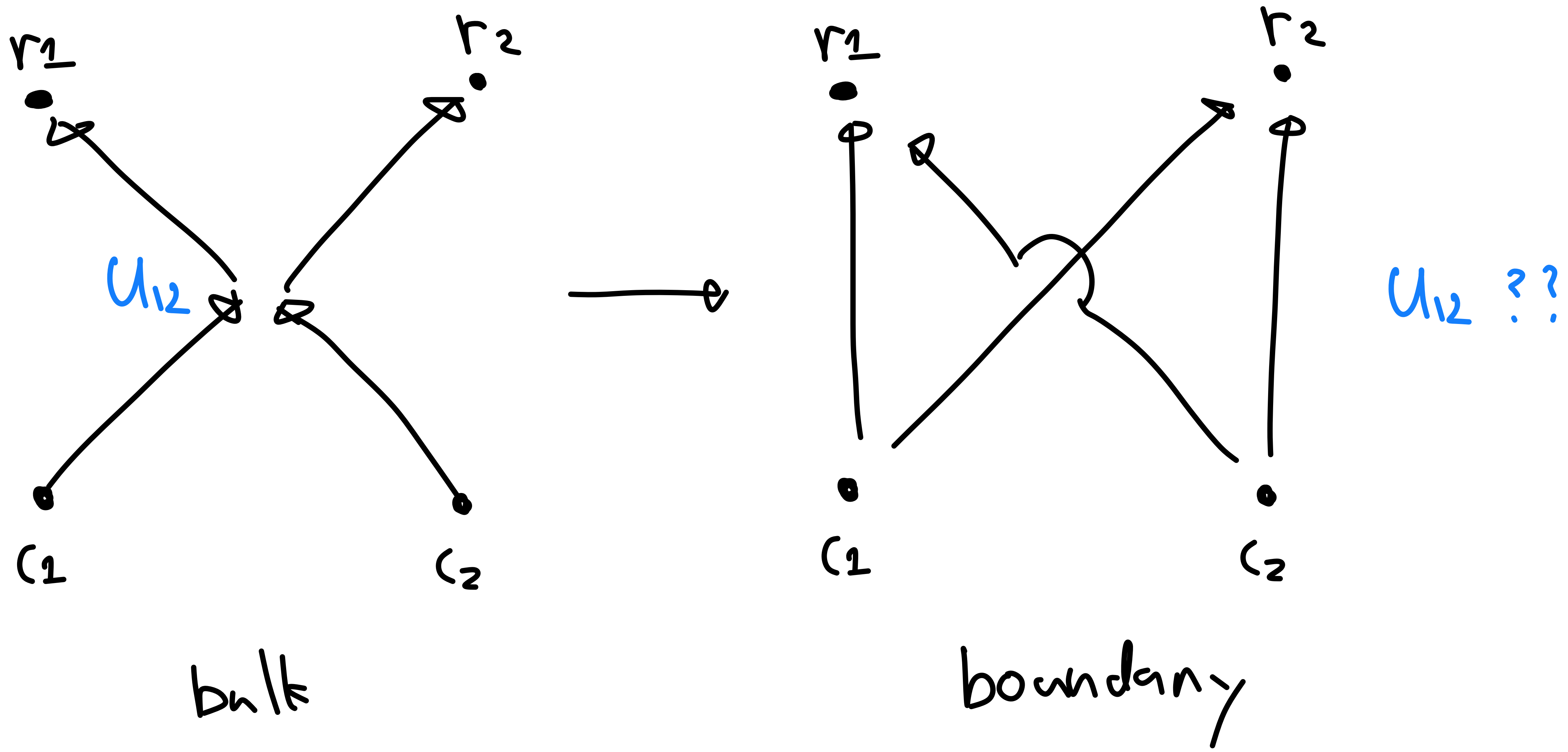



$= J_+(c_1) \cap J_+(c_2)$ Future of $c_1, c_2$.

$= J_-(r_1) \cap J_-(r_2)$ Past of $r_1, r_2$.

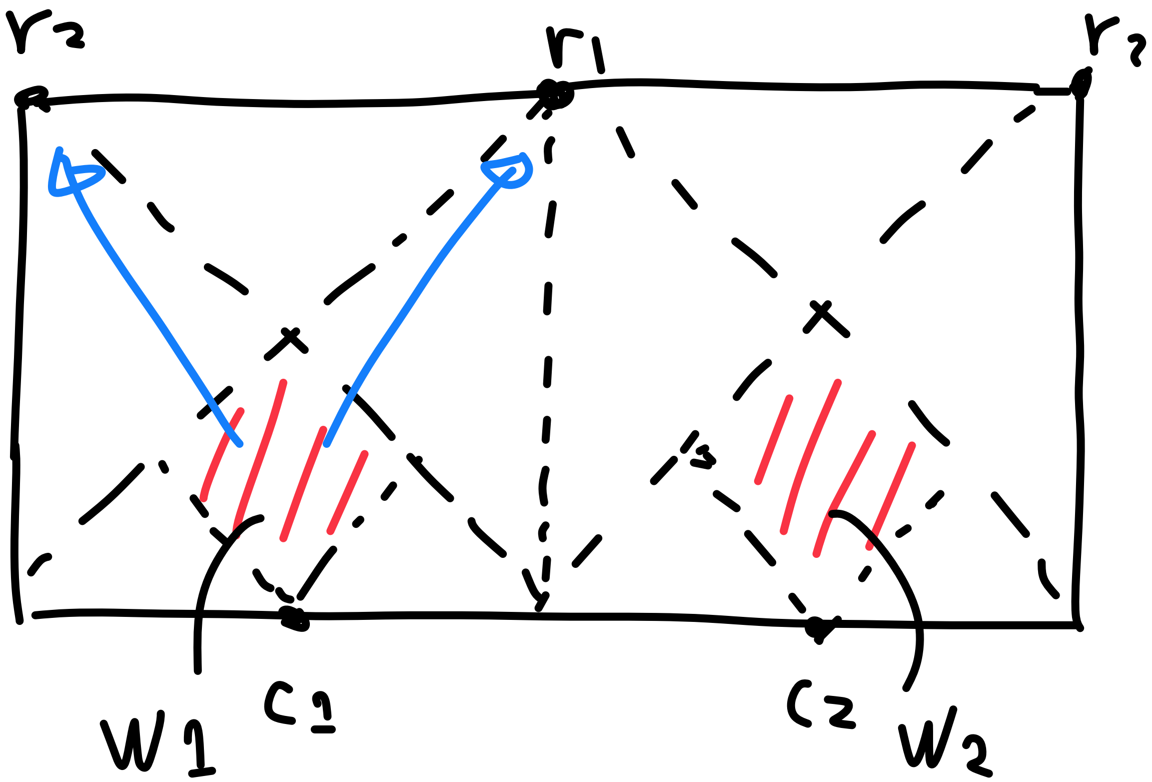
No overlap !! (No direct boundary scattering)

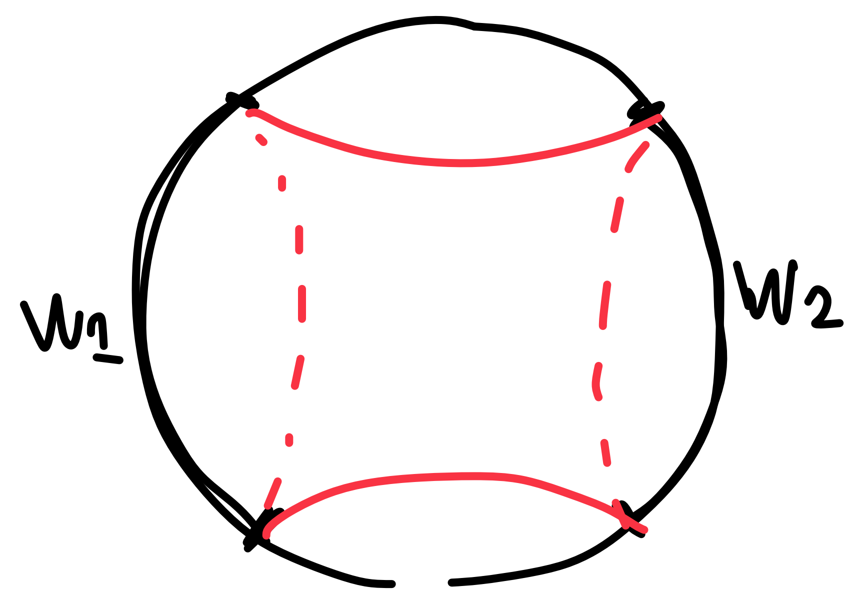
# Boundary causal graph



# Resolution: Entanglement as resource

[Alex May, 2019]




$$W_1 = J_+(c_1) \cap J_-(r_1) \cap J_-(r_2)$$

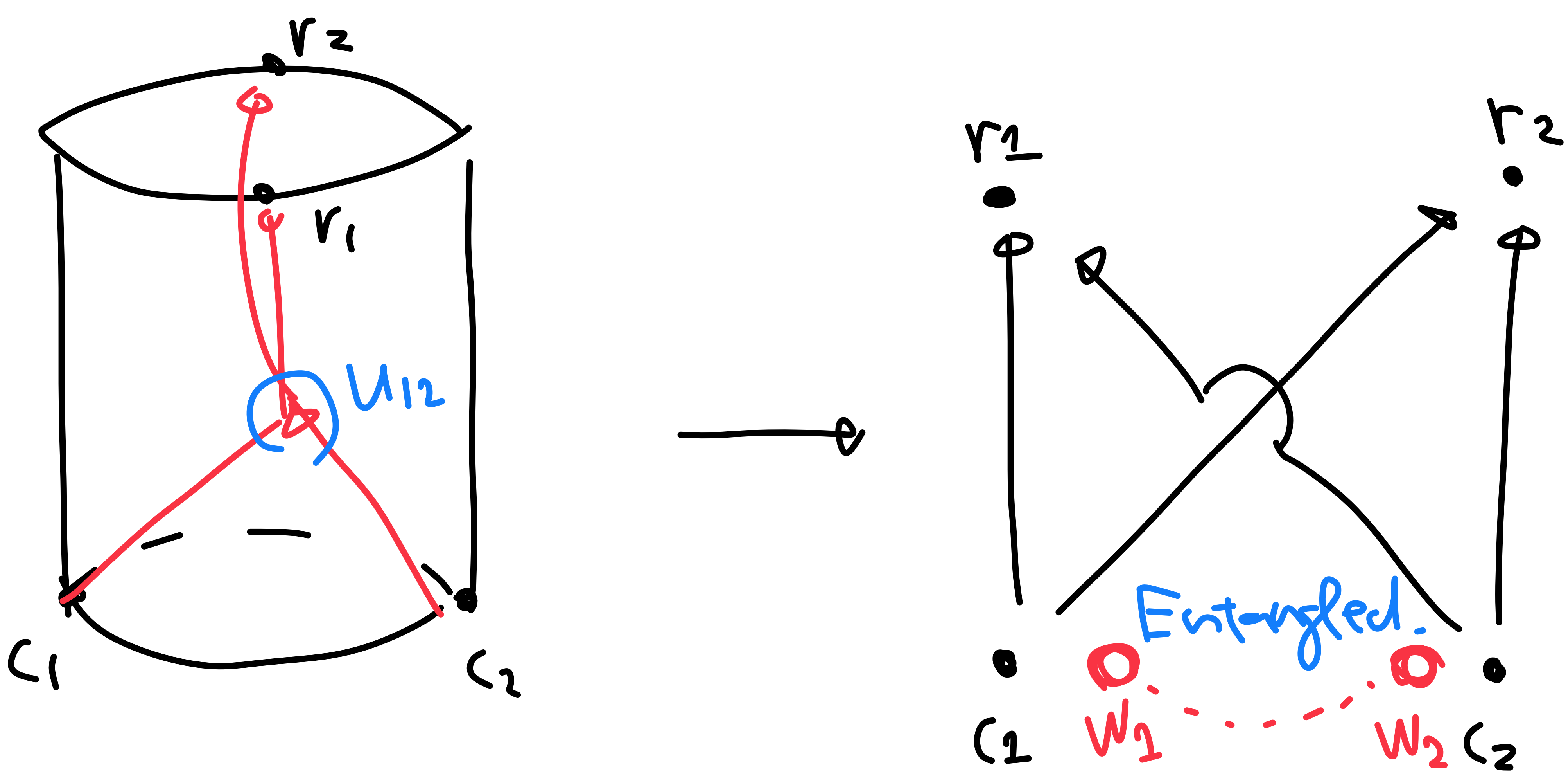
$$I(W_1, W_2) = O\left(\frac{1}{G_N}\right)$$

$W_1$ can signal to $r_1, r_2$

$W_1$ & $W_2$ entangled!

(via Ryu-Takayanagi formula)

## With entanglement ...



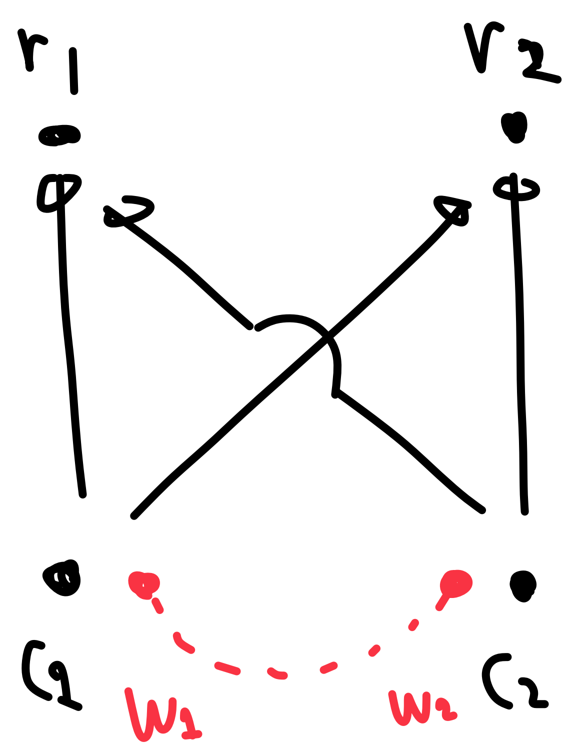

bulk　　　　　　boundary

### Idea

Somehow use $W_1 - W_2$ entanglement to implement $U_{12}$

# Entanglement-Assisted Quantum Computation

Some $U_{12}$ can be implemented efficiently!! (@I)




- Teleportation "B84 protocol" (1984)

  Quantum cryptography

- Arbitrary $U_{12}$: Port-based teleportation

  (Hiroshima-Ishizaka 2008)

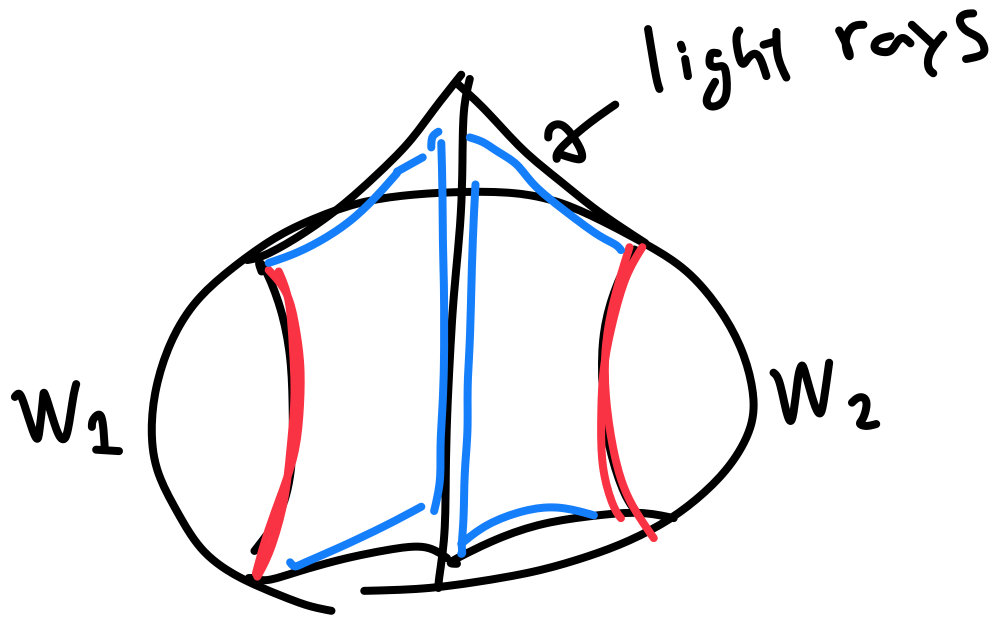
  (Beigi-König 2012)

- But.... For $n$ qubit, $\exp(n)$ EPRs needed in general.

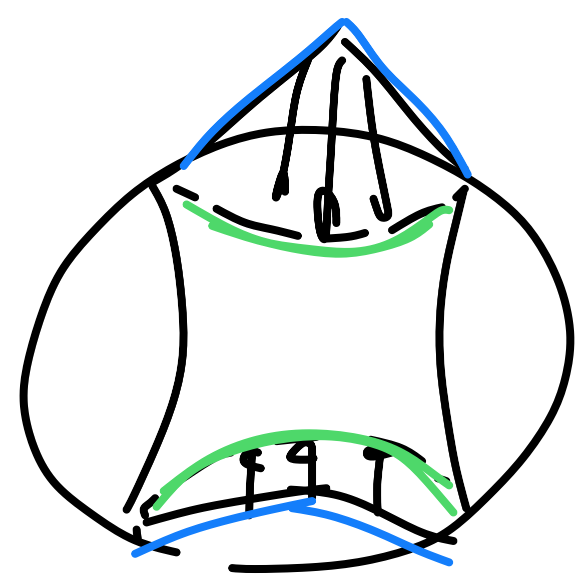
# Scattering implies entanglement

- [Thm] If $c_1, c_2 \to r_1, r_2$ scattering is possible on bulk, then $W_1$ & $W_2$'s wedges are connected.
- [idea] Use the focusing conjecture.
- [May, Pennington, Sorce 2020]

[New result] more precise proof. & works for $N \geq 3$ particles




Future evolution



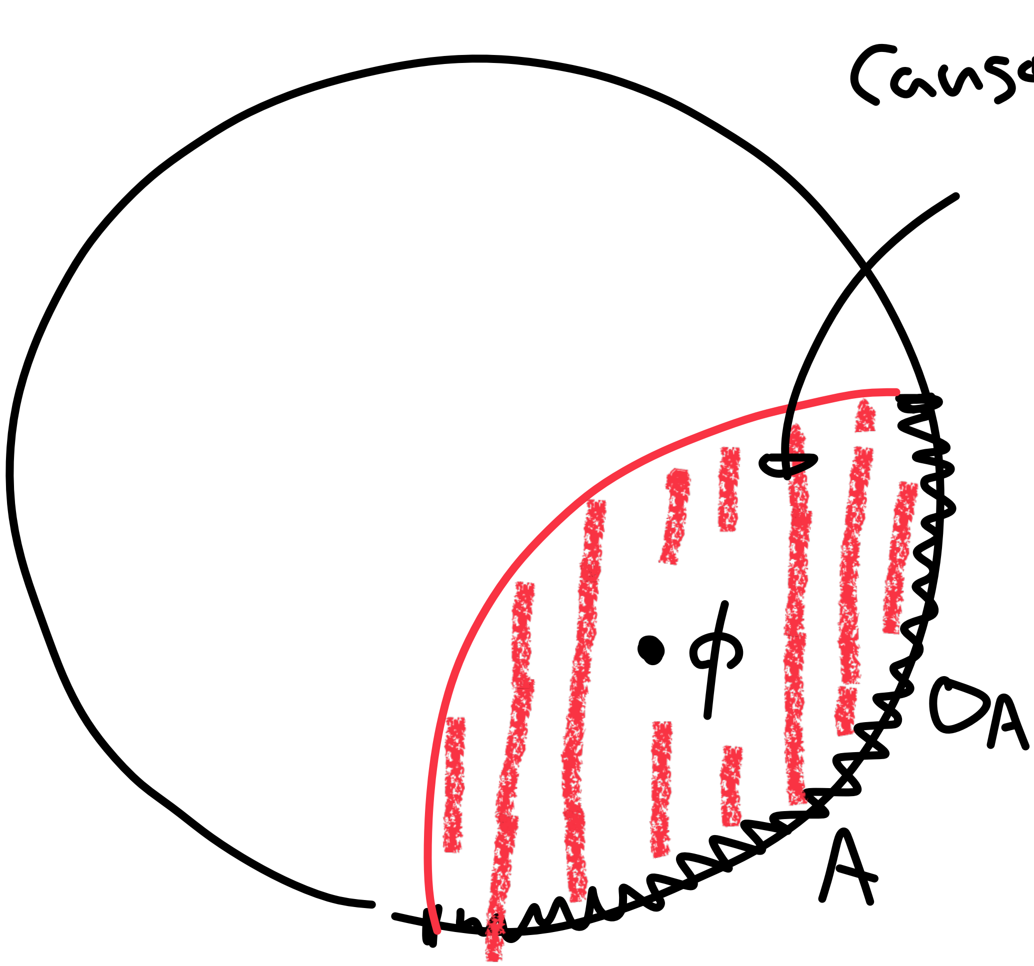
Backward evolution.

"Area" decreasing

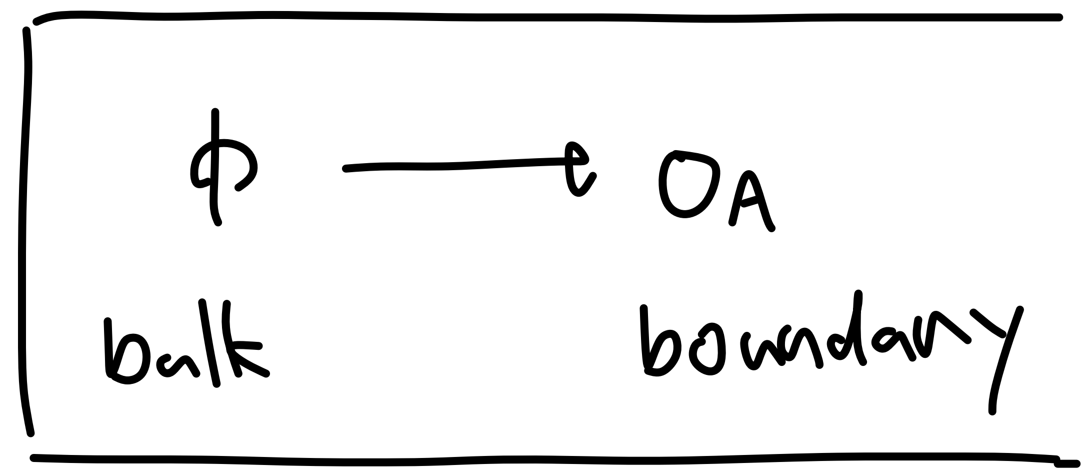
Scattering from

quantum error-correction?

# Bulk operator from boundary




## Causal wedge reconstruction

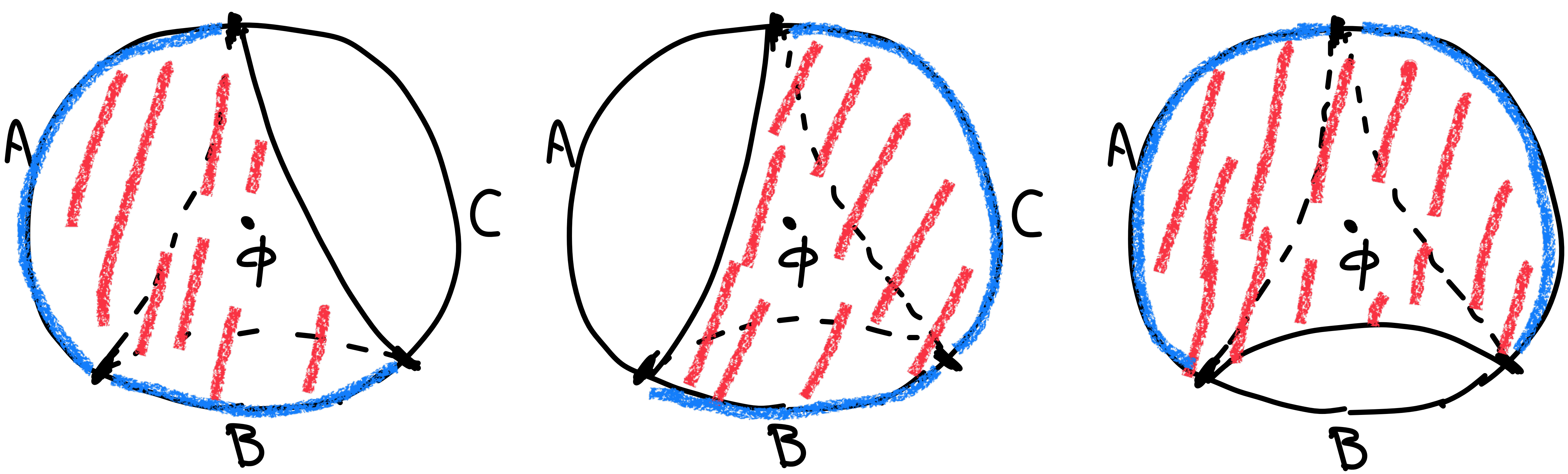


# Holographic Quantum error-correction

$\phi \longrightarrow O_{AB}, O_{BC}, O_{CA}$ (different, but equivalent)

Quantum error-correction:

Losing one subsystem is fine

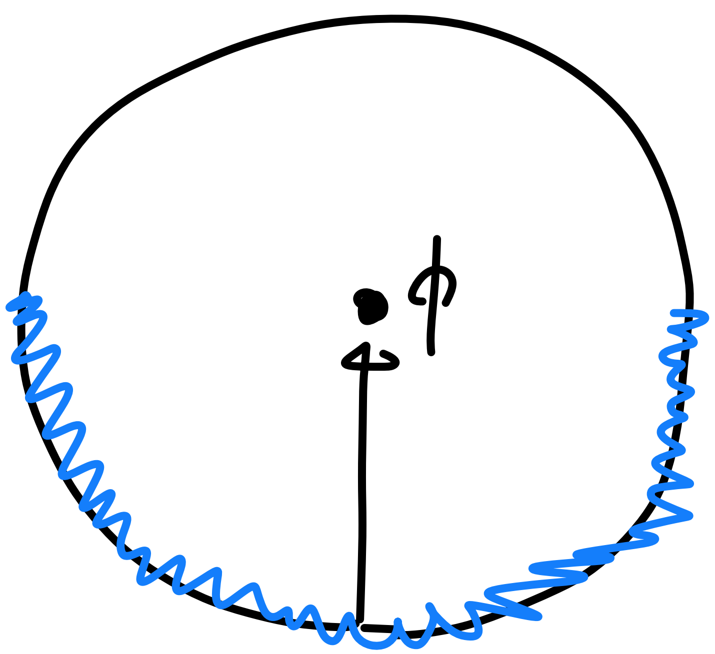



$O_{AB}$ $\qquad$ $O_{BC}$ $\qquad$ $O_{CA}$

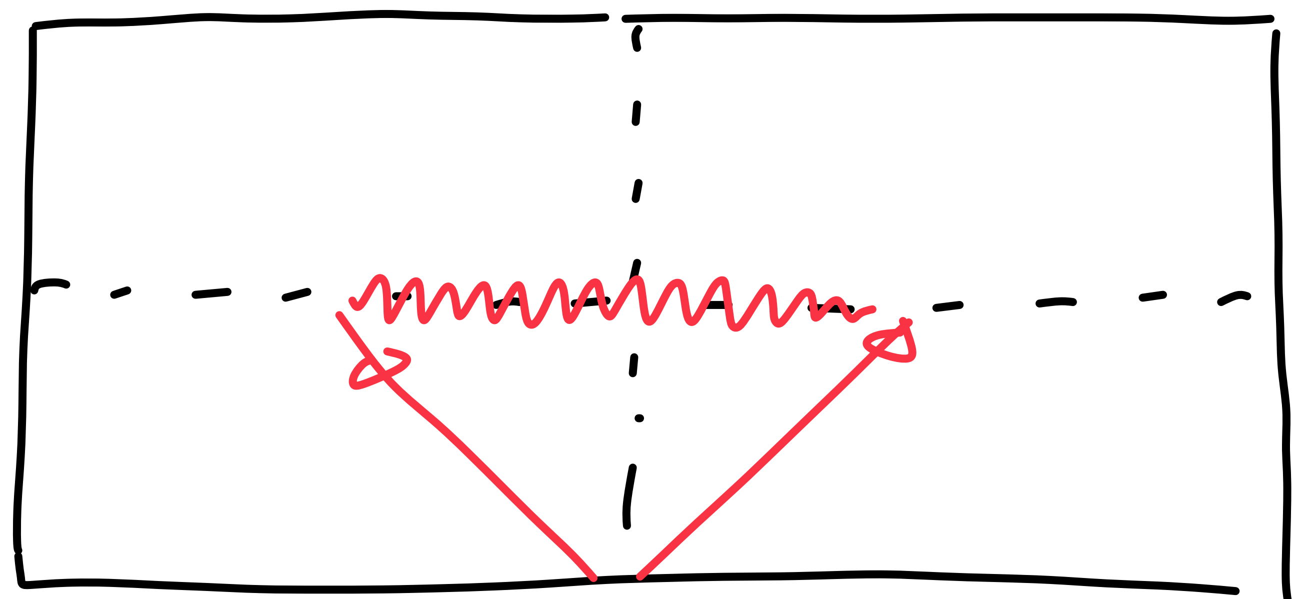
# Quantum error-correction and entanglement

Not enough time to delocalize quantum information

→ Entanglement is needed for Q error-correction.

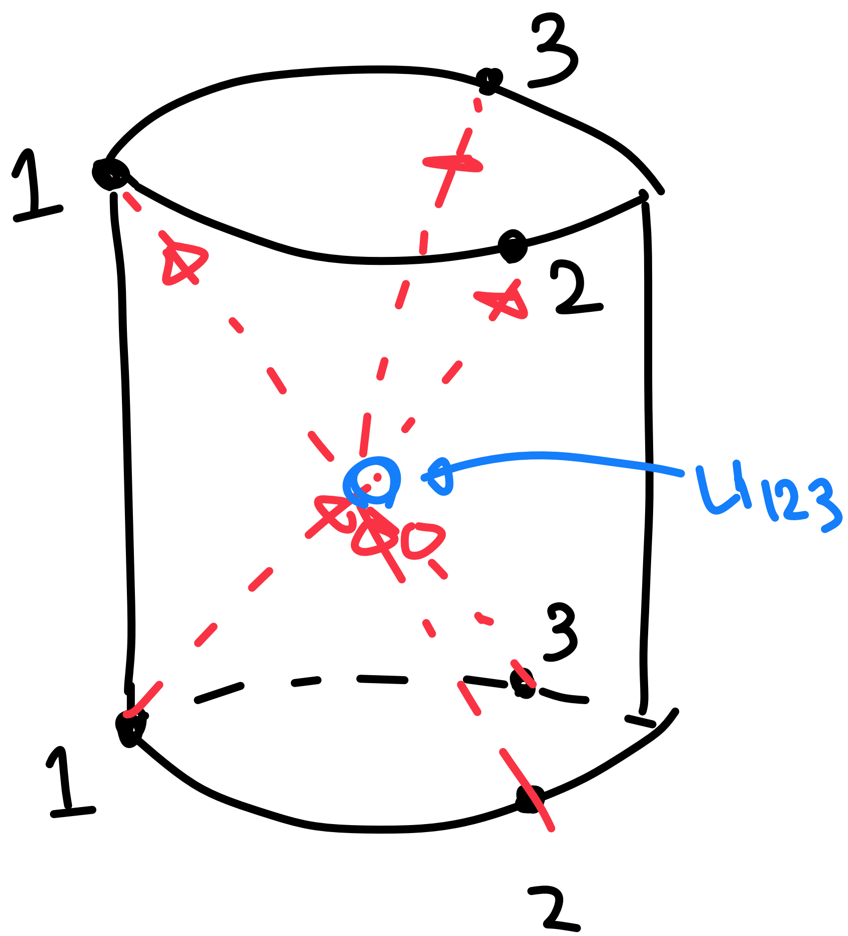


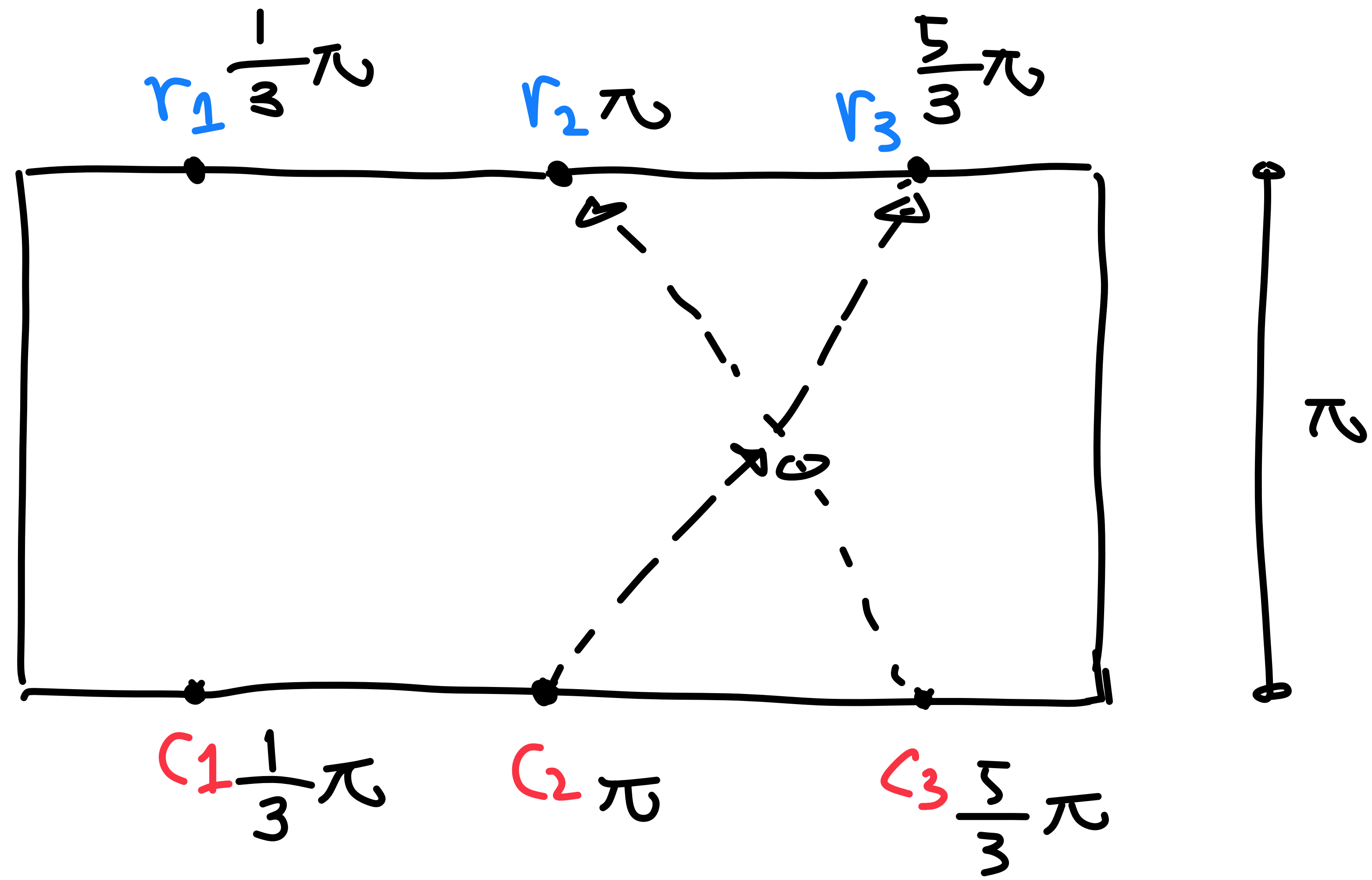

bulk



boundary

# 3 particle scattering (or $N \geq 3$)

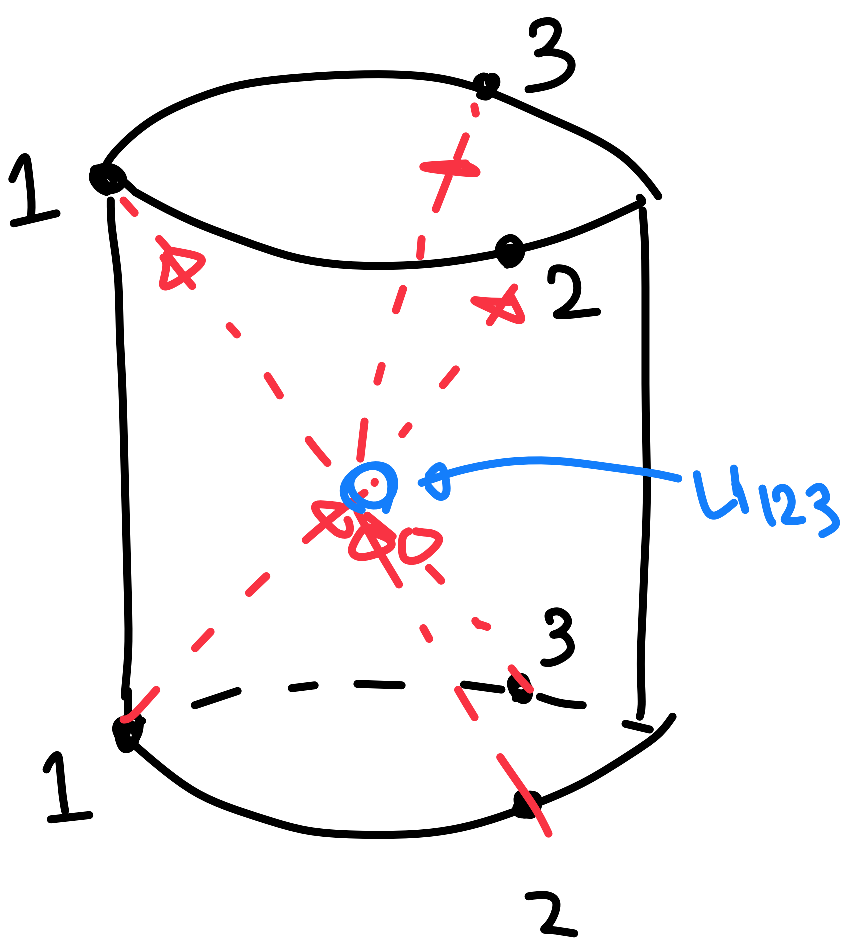


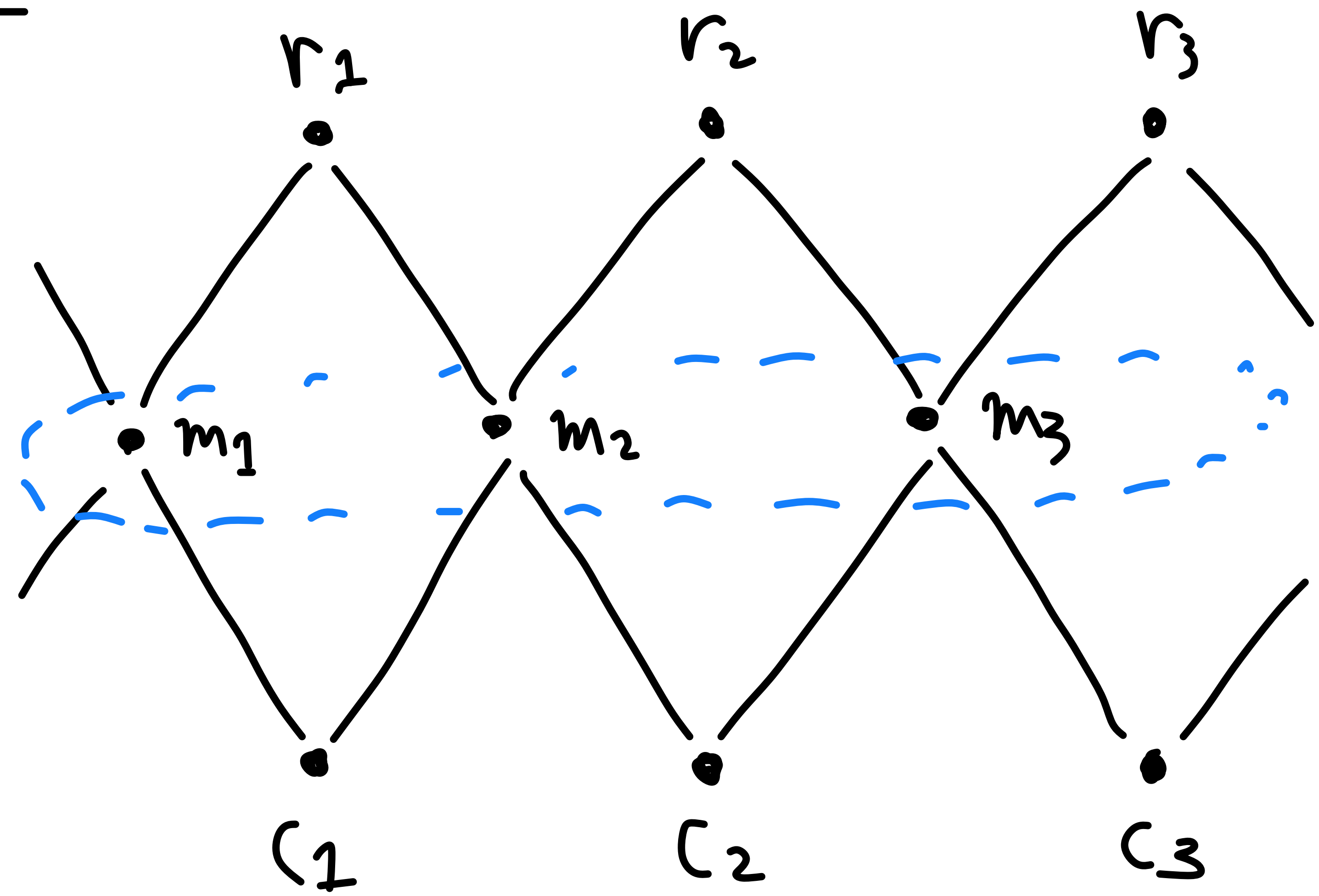

- 2-to-2 scattering is possible

  3-to-3, not possible ...

# Boundary Causality




bulk



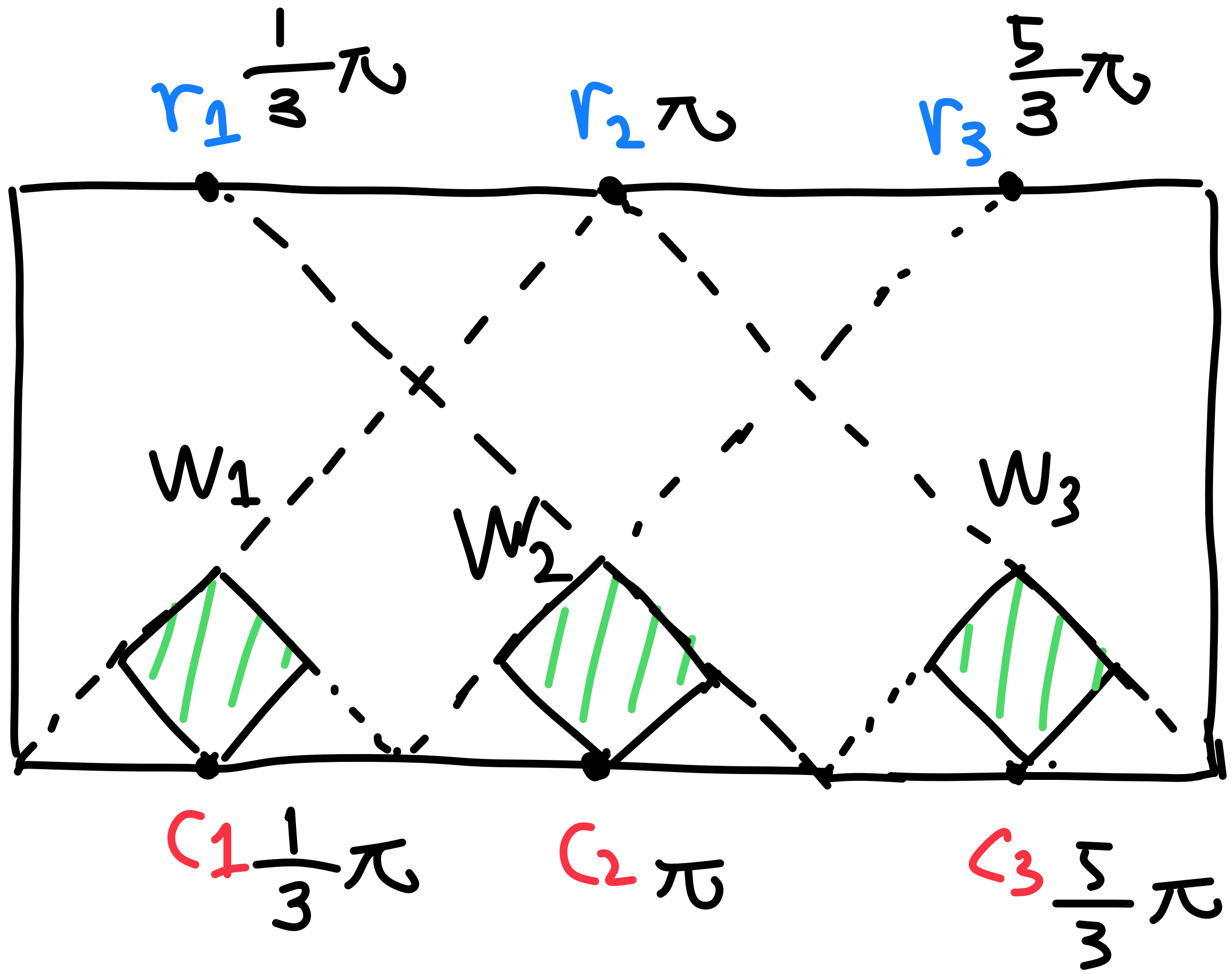

boundary

QECCs ?

| Input | | Intermediate | | Output |
|---|---|---|---|---|
| $c_j$ | $\longrightarrow$ | $m_j$ | $\longrightarrow$ | $r_j$ |

Interaction happens here ?

# Entanglement for 3 wedges ?



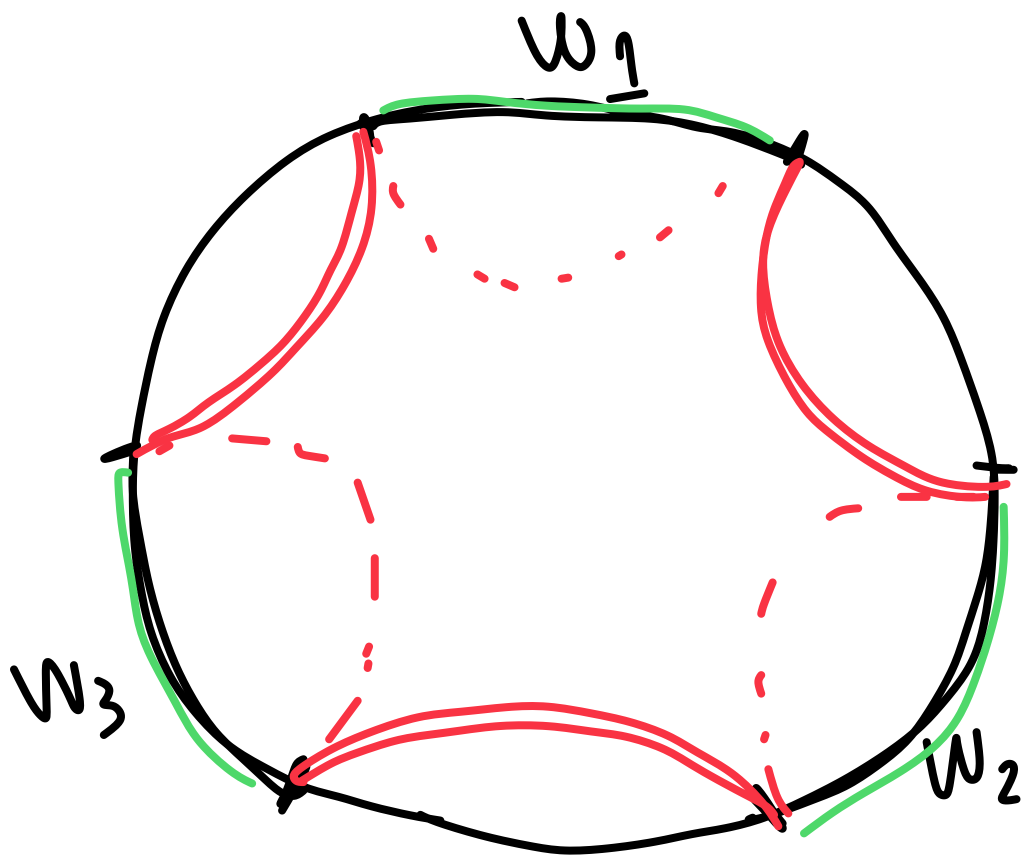

$W_j$'s are entangled

How ??

$$W_2 \equiv J_-(r_1) \cap J_-(r_2) \cap J_-(r_3) \cap J_+(c_2)$$

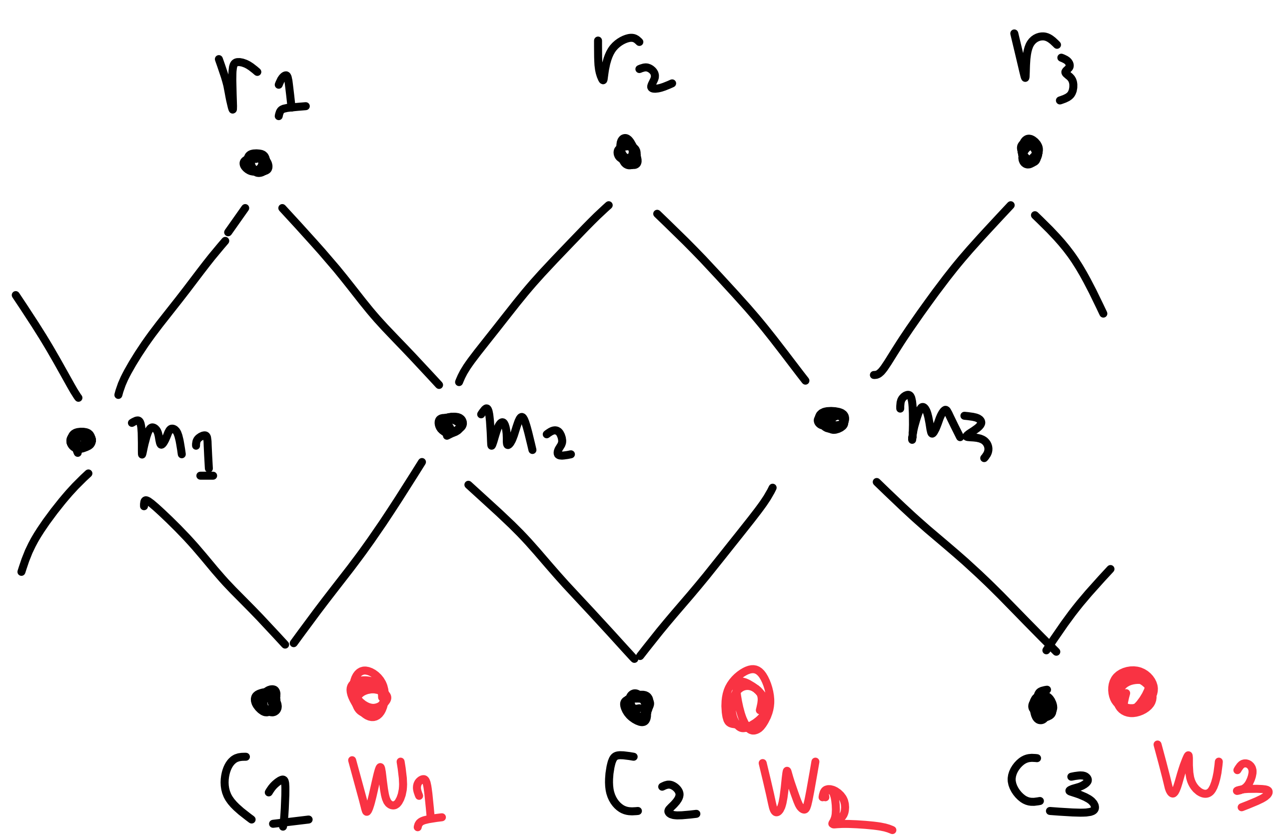
can signal to $r_1, r_2, r_3$

## Connected wedge (Multi-party entanglement)




- $I(W_1, W_2W_3) = O\left(\frac{1}{G_N}\right)$
- $I(W_1, W_2) \simeq 0$

No EPR pairs!!

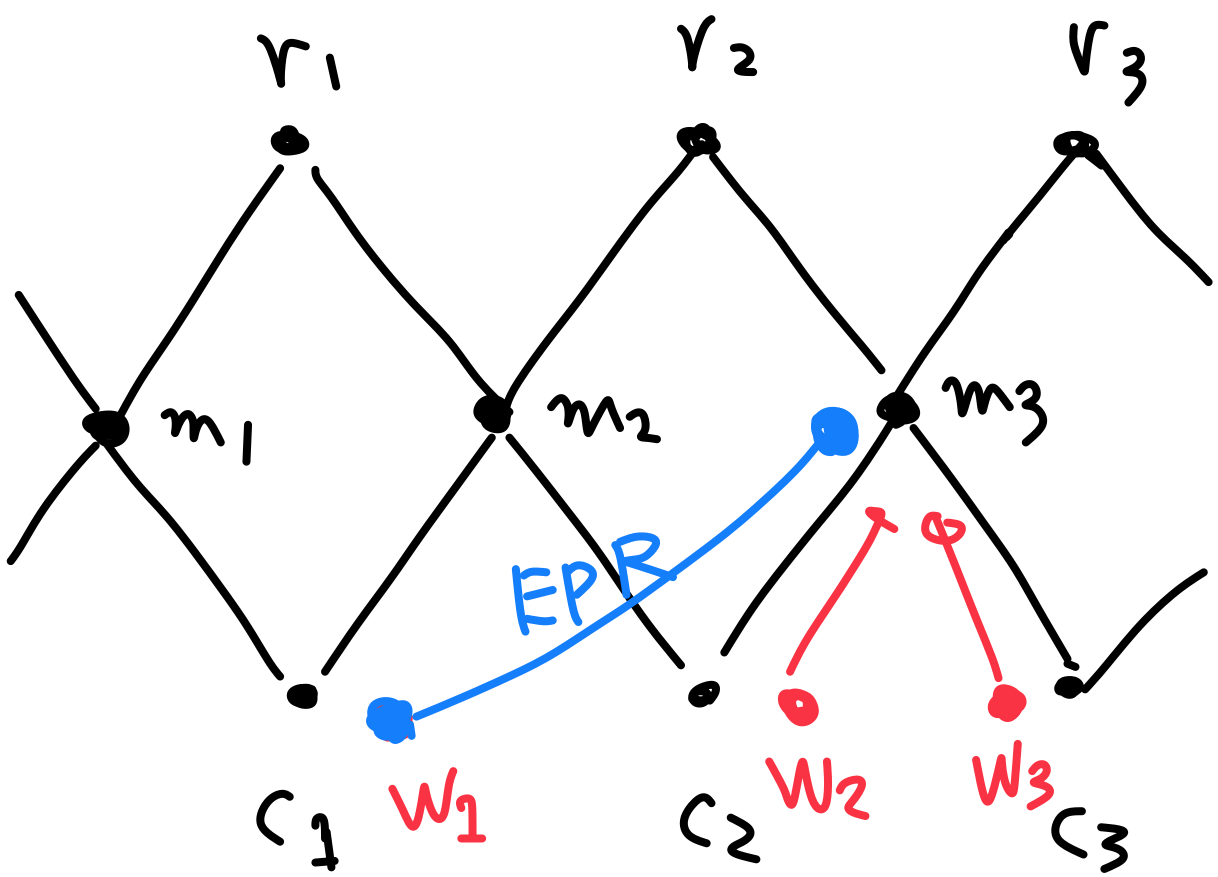



- "Teleportation" does not work ⋯.

# Entanglement in spacetime

EPR pair between different time slices

$c_1 \to m_1, m_2, m_3$ encoding is possible!!

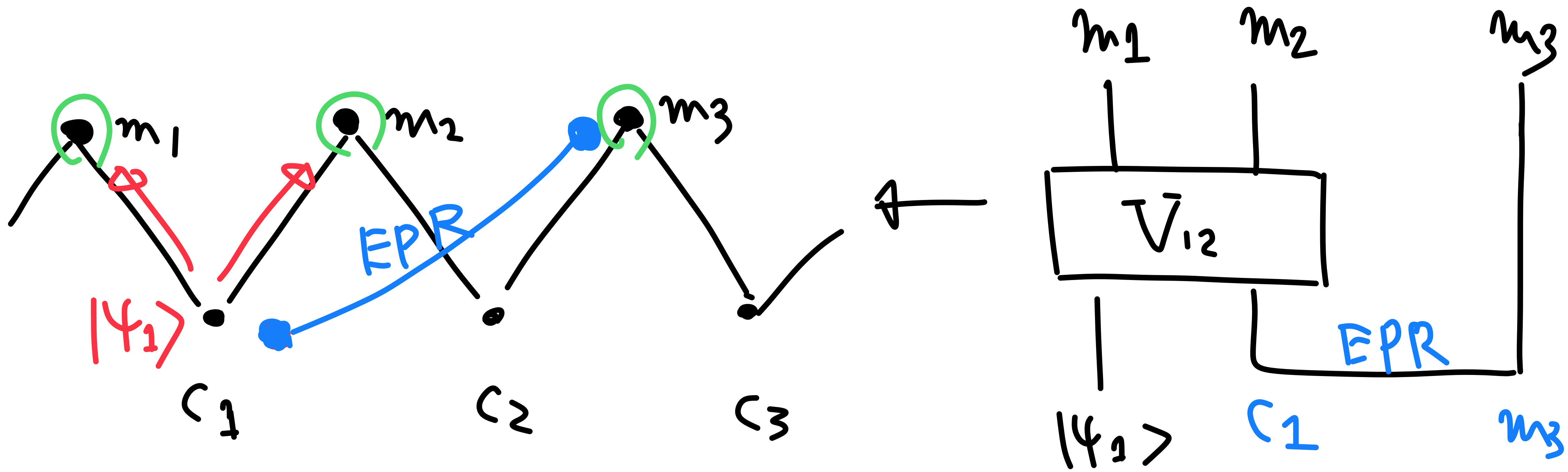



- $c_1$ and $m_3$ can share EPR pair

$$I(W_1, W_2 W_3) = O\left(\frac{1}{G_N}\right)$$

# Entanglement-assisted QECCs

- Non-local encoding

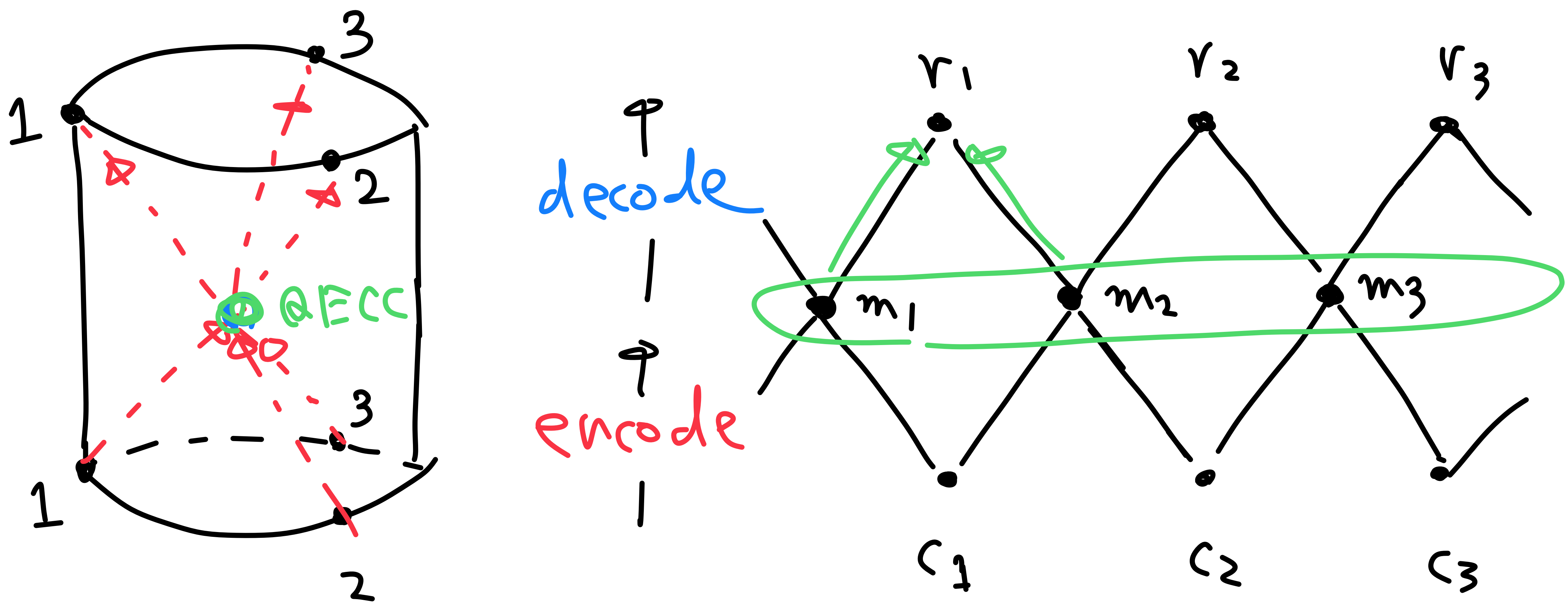



- $V_{12}$ does not need to touch $m_3$

- Same mechanism as Hayden-Preskill protocol

  (BH – Early radiation entanglement)

# Encoding and Decoding

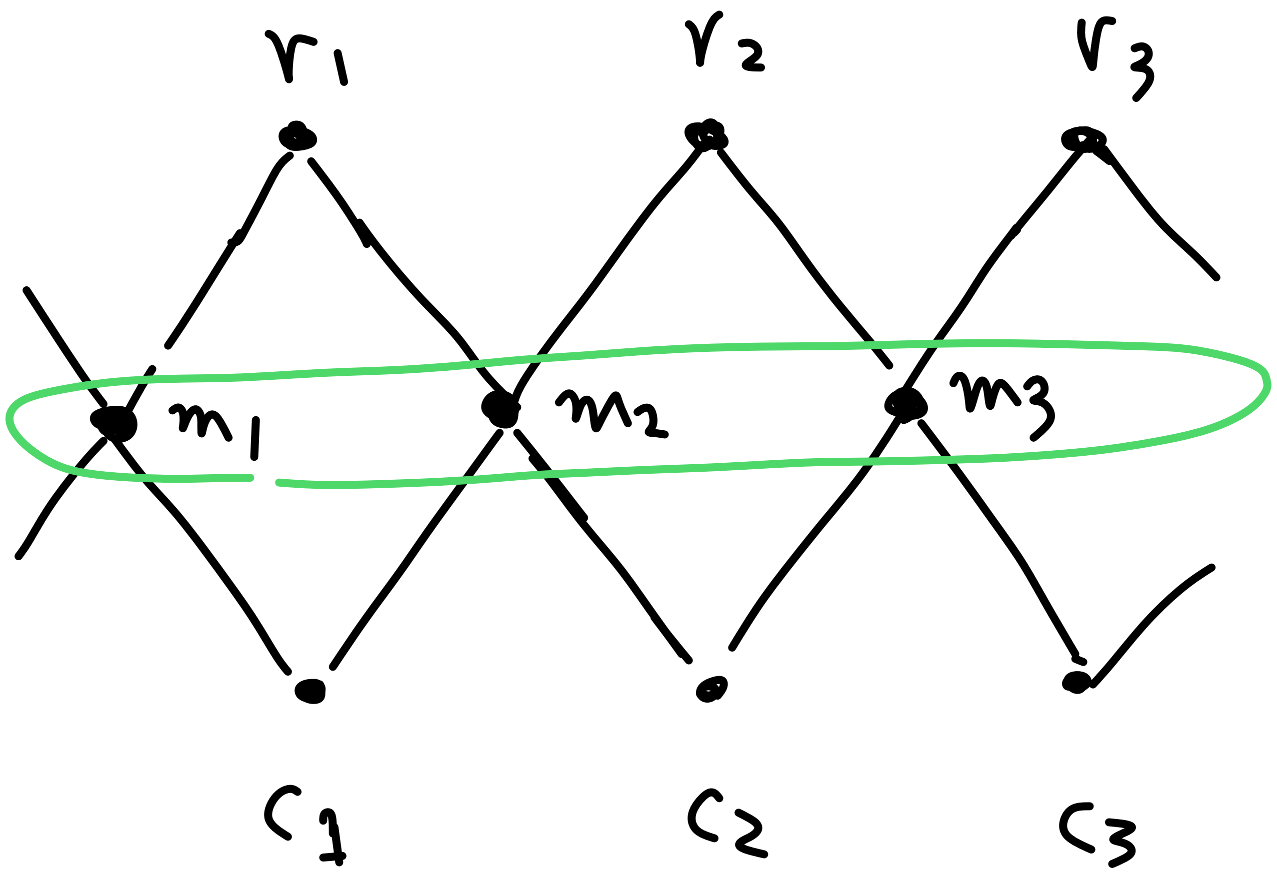



- $r_2$ has access to $m_2$ and $m_3$

  $\longrightarrow$ $|\psi_1\rangle$ can be reconstructed on $r_1$, $r_2$, $r_3$.

# Interaction in QECCs

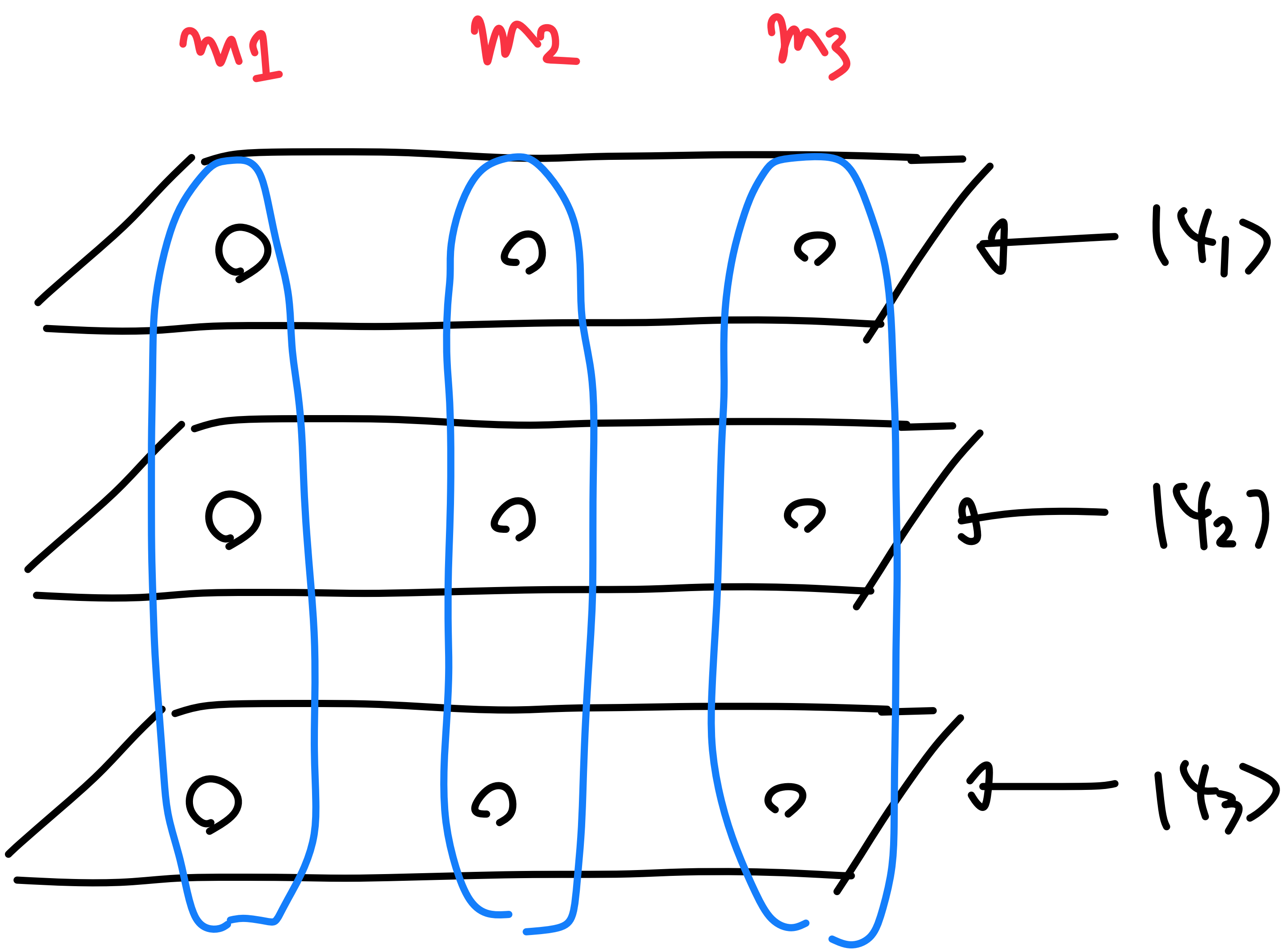
# Interaction in QECCs




"Logical operators" act on $m_1, m_2, m_3$

# Transversal logical operators

- Transversal (factorized) logical op only




$$V_1 \otimes V_2 \otimes V_3 = V$$

$$e^{i\theta(X_1X_2X_3 + Z_1Z_2Z_3)}$$

is not possible

# Implementable Logical gates

- Arbitrary Clifford gate (maps Pauli to Pauli)

  n-qubit $O(n)$ EPR pairs

  Reed - Solomon code (generalization of Qutrit code)

  Random Clifford encoding

- Phase gate

  quadratic

  $$|x_1, x_2, x_3\rangle \longrightarrow e^{i\theta(x_1, x_2, x_3)} |x_1, x_2, x_3\rangle$$

  need $O\left(\log\left(\frac{1}{\theta_{min}}\right)\right)$ EPR pairs

# Eastin - Knill theorem

- Transversal gates form discrete groups

  → Not all $U_{123}$ are transversally implementable

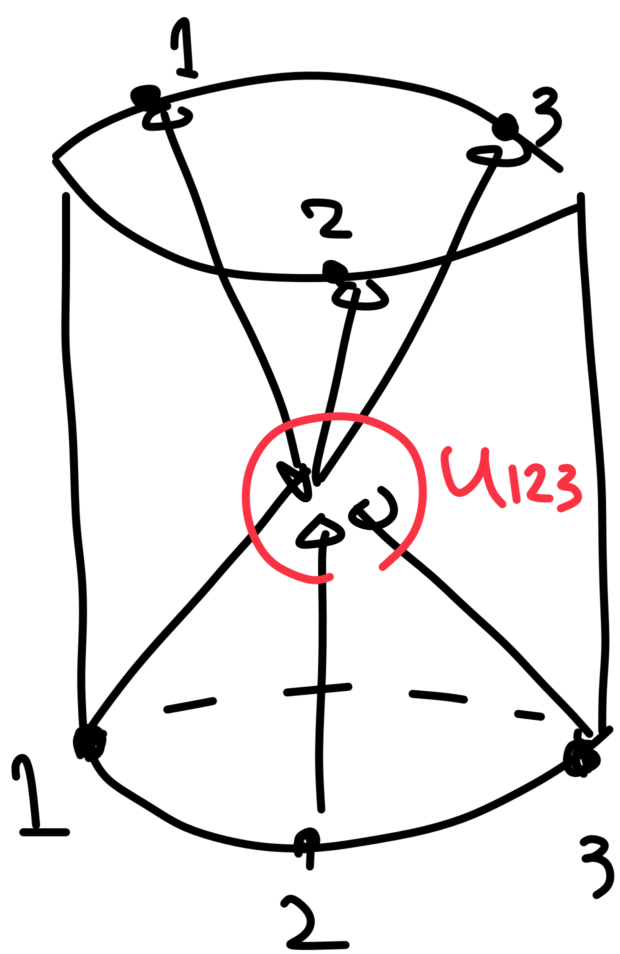
- Approximate transversal gate avoids this no-go theorem

  → AdS/CFT avoids Eastin-Knill in a clever way ?

# Summary and Outlook

## Take home message

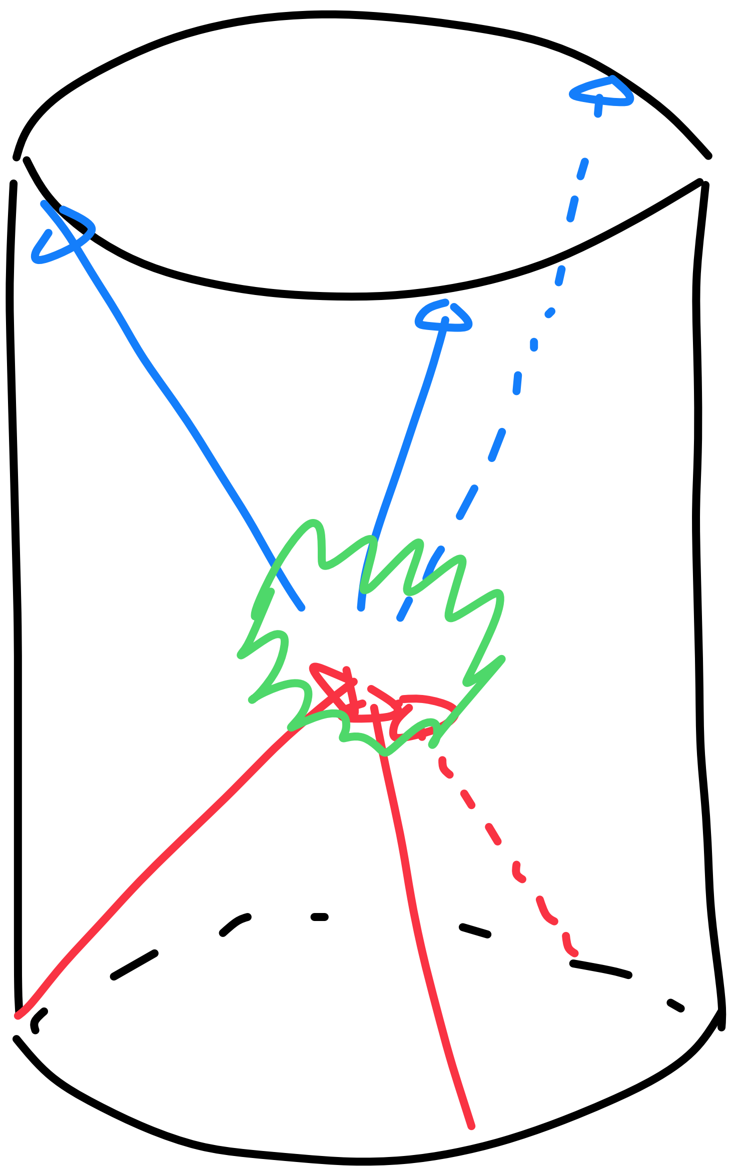
- Interaction in bulk AdS emerges from logical op in QECCs.
- QECC encoding is possible due to pre-existing entanglement among wedges $W_i$'s.




- Some $U_{123}$ can be implemented.

Arbitrary gate ??

# Analogy to fault-tolerant quantum computing



output

↑ decoding from QECC

interaction Transversal logical ops

↑ encoding into QECC

input

# Future Problems

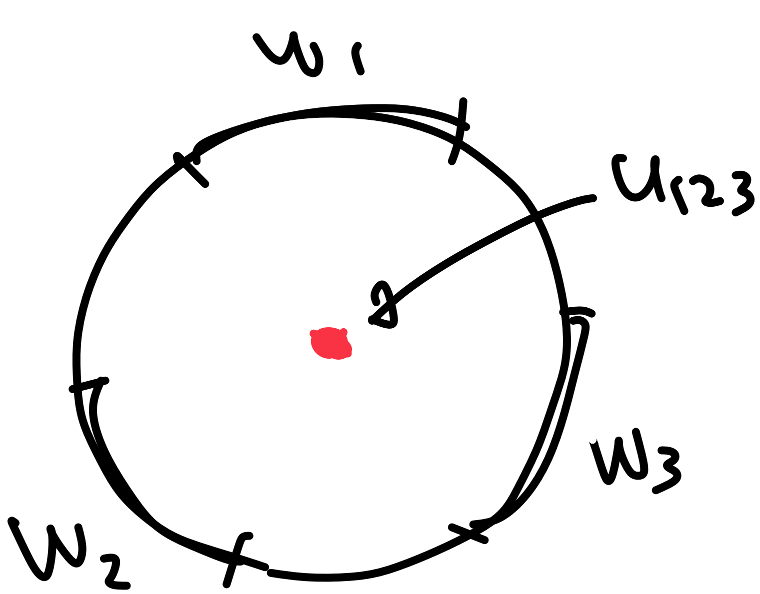
- $L = (\nabla\phi)^2 + m_2\phi^2 + \underbrace{m_3\phi^3 + m_4\phi^4 + \cdots}_{\text{interaction}}$

  How does this affect $L_{boundary}$ ?

- Complexity of holographic scattering ?

  Restriction on $U_{123\cdots n}$ ?

- Entanglement wedge reconstruction




Explicit form of bulk op ?

- HKLL reconstruction

  $\phi = \int O(x,t)$

Thank you !!