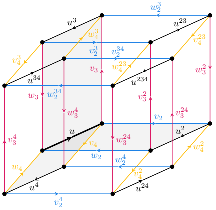

Higher dimensional digraphs from cube complexes and their spectral theory

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Outline

Buildings

- First series of buildings were introduced by J.Tits in 50s.

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- They have algebraic, analytic and number theoretical aspects.
- Buildings consist of chambers and apartments satisfying certain axioms, where each apartment consists of a set of chambers.

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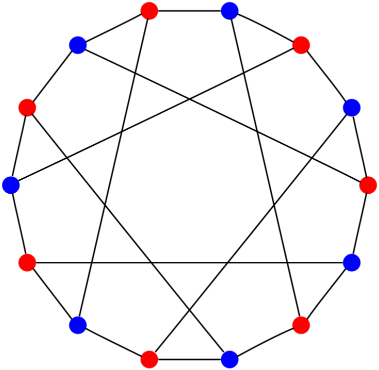

- Chambers are edges

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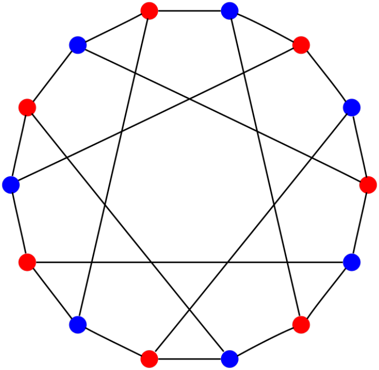

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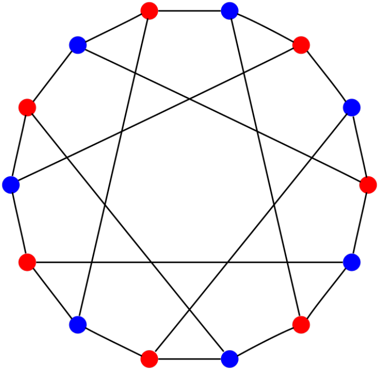

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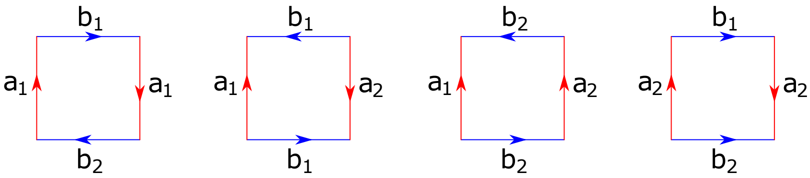
- X is a union of tessellated n-dimensional spaces called apartments, where the tiles of the tessellation are chambers.
- For any two chambers there is an apartment containing both of them.
- If two apartments F_1 and F_2 have non-trivial intersection, then there is an isomorphism from F_1 to F_2, fixing $F_1 \cap F_2$ pointwise.

Polyhedra and links

Definition

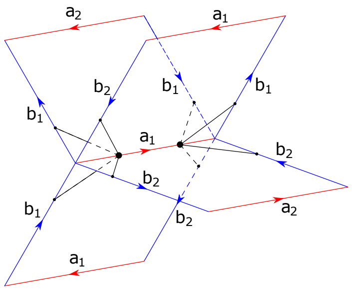
A *(generalized) polyhedron* is a two-dimensional complex which is obtained from several decorated polygons by identification of sides with the same labels respecting orientation.



Polyhedra and links

Definition

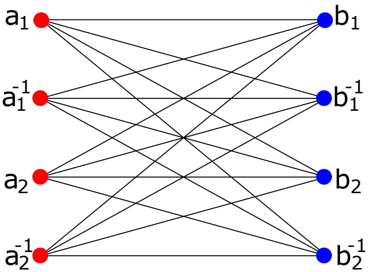
Take a sphere of a small radius at a point of the polyhedron. The intersection of the sphere with the polyhedron is a graph, which is called the *link* at this point.



Links of manifolds are spheres, but we need highly singular spaces as links to construct buildings.

Example of a link

The link of our example above is the following graph:



This graph has *diameter* (the maximal distance between two vertices) two and *girth* (the length of the shortest cycle) four.

Polyhedra and links

The following theorem connects polyhedra with buildings (the result below deals with the 2-dimensional case, but I generalised it to arbitrary dimensions).

Theorem (Ballmann, Brin 1994)

Let X be a compact two-dimensional polyhedron. If all links are graphs of diameter m and girth $2m$, then the universal cover of the polyhedron is a two-dimensional building.

Dimensions 3 and higher: joint with Ragunatapirom and Stix (2018) involving quaternion algebras. Buildings with chambers as nD cubes are constructed.

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Theorem (Vdovina 2002)

A polyhedron with given links can be constructed explicitly. Any connected bipartite graph can be realized as a link of a 2-dimensional polyhedron with $2k$-gonal faces.

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Arithmetic lattices acting simply transitively on products of trees

Let q be a prime power. Let

$$\delta \in \mathbb{F}_{q^2}^{\times}$$

be a generator of the multiplicative group of the field with q^2 elements. If $i, j \in \mathbb{Z}/(q^2-1)\mathbb{Z}$ are

$$i \not\equiv j \pmod{q-1},$$

then $1 + \delta^{j-i} \neq 0$, since otherwise

$$1 = (-1)^{q+1} = \delta^{(j-i)(q+1)} \neq 1,$$

a contradiction. Then there is a unique $x_{i,j} \in \mathbb{Z}/(q^2-1)\mathbb{Z}$ with

$$\delta^{x_{i,j}} = 1 + \delta^{j-i}.$$

With these $x_{i,j}$ we set $y_{i,j} := x_{i,j} + i - j$, so that

$$\delta^{y_{i,j}} = \delta^{x_{i,j}+i-j} = (1+\delta^{j-i}) \cdot \delta^{i-j} = 1 + \delta^{i-j}.$$

We set

$$l(i,j) := i - x_{i,j}(q-1),$$

$$k(i,j) := j - y_{i,j}(q-1).$$

Let $M \subseteq \mathbb{Z}/(q^2-1)\mathbb{Z}$ be a union of cosets stable under multiplication by q, and by addition of $q-1$.

Theorem (RSV 2018)

Each group $\Gamma_{M,\delta}$ acts simply transitively on a product of $d = |M|$ trees.

$$\Gamma_{M,\delta} = \left\langle a_i \text{ for all } i \in M \;\middle|\; \begin{array}{c} a_{i+(q^2-1)/2}a_i = 1 \text{ for all } i \in M, \\ a_i a_j = a_{k(i,j)} a_{l(i,j)} \text{ for all } i,j \in M \text{ with } i \not\equiv j \pmod{q-1} \end{array} \right\rangle$$

if q is odd, and if q is even:

$$\Gamma_{M,\delta} = \left\langle a_i \text{ for all } i \in M \;\middle|\; \begin{array}{c} a_i^2 = 1 \text{ for all } i \in M, \\ a_i a_j = a_{k(i,j)} a_{l(i,j)} \text{ for all } i,j \in M \text{ with } i \not\equiv j \pmod{q-1} \end{array} \right\rangle.$$

3D example

$$\Gamma = \left\langle \begin{array}{c} a_1, a_5, a_9, a_{13}, a_{17}, a_{21}, \\ b_2, b_6, b_{10}, b_{14}, b_{18}, b_{22}, \\ c_3, c_7, c_{11}, c_{15}, c_{19}, c_{23} \end{array} \middle| \begin{array}{c} a_i a_{i+12} = b_i b_{i+12} = c_i c_{i+12} = 1 \text{ for all } i\,, \\ a_1 b_2 a_{17} b_{22},\ a_1 b_6 a_9 b_{10},\ a_1 b_{10} a_9 b_6, \\ a_1 b_{14} a_{21} b_{14},\ a_1 b_{18} a_5 b_{18},\ a_1 b_{22} a_{17} b_2, \\ a_5 b_2 a_{21} b_6,\ a_5 b_6 a_{21} b_2,\ a_5 b_{22} a_9 b_{22}, \\ a_1 c_3 a_{17} c_3,\ a_1 c_7 a_{13} c_{19},\ a_1 c_{11} a_9 c_{11}, \\ a_1 c_{15} a_1 c_{23},\ a_5 c_3 a_5 c_{19},\ a_5 c_7 a_{21} c_7, \\ a_5 c_{11} a_{17} c_{23},\ a_9 c_3 a_{21} c_{15},\ a_9 c_7 a_9 c_{23}, \\ b_2 c_3 b_{18} c_{23},\ b_2 c_7 b_{10} c_{11},\ b_2 c_{11} b_{10} c_7, \\ b_2 c_{15} b_{22} c_{15},\ b_2 c_{19} b_6 c_{19},\ b_2 c_{23} b_{18} c_3, \\ b_6 c_3 b_{22} c_7,\ b_6 c_7 b_{22} c_3,\ b_6 c_{23} b_{10} c_{23}. \end{array} \right\rangle .$$

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- 2D examples: Wise (1996), Burger-Mozes (2000);
- Arithmetic lattices + generalized doubling construction;
- Why difficult? Each k-D cube group gives k 2D groups, which need to be compatible, and remain compatible after doubling.

C*-algebras

We begin with the abstract characterization of C*-algebras given in the 1943 paper by Gelfand and Naimark.

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- For every complex number λ and every $x \in B$: $(\lambda x)^* = \overline{\lambda} x^*$.
- For all $x \in B$: $\|x^*x\| = \|x\|\|x^*\|$.

C*-algebras and von Neumann algebras of k-graphs

One of the bridges between the cube complexes and C*-algebras are so-called k-graphs (another one is via crossed products).
Moreover, in a recent work with Nadia Larsen we suggest to look at the spectra of the k-graphs.

Definition

A countable category C is said to be a *higher rank graph* or a *k-graph* if there is a functor $d\colon C \to \mathbb{N}^k$, called the *degree map*, satisfying the *unique factorization property* (UFP): if $d(a) = \mathbf{m} + \mathbf{n}$ then there are unique elements a_1 and a_2 in C such that $a = a_1 a_2$ where $d(a_1) = \mathbf{m}$ and $d(a_2) = \mathbf{n}$. We call $d(x)$ the of x. A *morphism* of k-graphs is a degree-preserving functor.

C*-algebras and von Neumann algebras of k-graphs

Theorem (Joint work with Nadia Larsen)

There exists a strongly connected k-rank graph Δ *with* $\rho(\Delta) = (2l_1, ..., 2l_k)$ *for any integers* $l_1, ..., l_k$, *such that for any cycle* $\mu \in \Delta$, $\sum_{i=1}^{k} d(\mu)_i \in 2\mathbb{Z}$.

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Corollary

By varying $l_1, ..., l_k$ we are getting an infinite family of distinct values of λ for III_λ factors. In particular, if $l_1 = ... = l_k = l$, then $\lambda = (2l)^{-2}$.

Definition

A k-dimensional digraph DG is a directed graph with V a finite set of vertices, E finite set of edges, and the edge set decomposes as a disjoint union $E = E_1 \sqcup E_2 \sqcup \cdots \sqcup E_k$ with E_i for $i = 1, \ldots, k$ regarded as edges of colour i, such that there is a bijection of all directed paths of length two formed of edges of colours given by ordered pairs (i, j) with $i \neq j$ in $\{1, 2, \ldots, k\}$, and:

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(F1) If xy is a path of length two with x of colour i and y of colour j, then $\phi(xy) = y'x'$ for a unique pair (y', x') where y' has colour j, x' has colour i and the origin and terminus vertices of the paths xy and $y'x'$ coincide. We write this as $xy \sim y'x'$.

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(F2) For all $x \in E_i, y \in E_j$ and $z \in E_l$ so that xyz is a path on E, where i, j, l are distinct colours, if $x_1, x_2, x^2 \in E_i, y_1, y_2, y^2 \in E_j$ and $z_1, z_2, z^2 \in E_l$ satisfy

$$xy \sim y^1x^1, x^1z \sim z^1x^2, y^1z^1 \sim z^2y^2$$

and

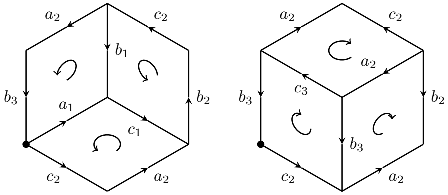
$$yz \sim z_1y_1, xz_1 \sim z_2x_1, x_1y_1 \sim y_2x_2,$$

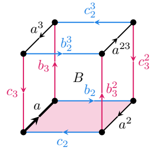
it follows that $x_2 = x^2, y_2 = y^2$ and $z_2 = z^2$.

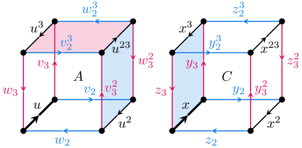
Definition (BGV)

Let G be a k-dimensional digraph on n disjoint alphabets $X_i, i = 1, ..., n$ such that any two alphabets generate a bi-reversible automaton with an infinite group generated by this automaton. We will call it nD automaton.

Pictures behind the proofs







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- Let $\Gamma = \mathbb{Z} * \mathbb{Z}$, the free group on two generators a and b.
- The Cayley graph of Γ with respect to the generating set $\{a, b\}$, $Cay(\Gamma, \{a, b\})$, is a homogeneous tree of degree 4.
- The vertices of the tree are elements of Γ *i.e.* reduced words in $S = \{a, b, a^{-1}, b^{-1}\}$.

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- Ω has a natural compact (totally disconnected) topology :
- if $x \in \Gamma$ then let $\Omega(x)$ be all semi-infinite words with the prefix x
- $\Omega(x)$ is open and closed in Ω and the sets $g\Omega(x)$ and $g(\Omega \setminus \Omega(x))$, where $g \in \Gamma$ and $x \in S$, form a base for the topology of Ω.

Graph C*-algebras

Left multiplication by $x \in \Gamma$ induces an action α of Γ on $C(\Omega)$ by

$$\alpha(x)f(w) = f(x^{-1}w).$$

$C(\Omega) \rtimes \Gamma$ is generated by $C(\Omega)$ and the image of a unitary representation π of Γ
such that $\alpha(g)f = \pi(g)f\pi^*(g)$ for $f \in C(\Omega)$ and $g \in \Gamma$ and every such C^*-algebra is a quotient of $C(\Omega) \rtimes \Gamma$.

Graph C*-algebras

For $x \in \Gamma$, let p_x denote the projection defined by the characteristic function $\mathbf{1}_{\Omega(x)} \in C(\Omega)$.
For $g \in \Gamma$, we have

$$g p_x g^{-1} = \alpha(g)\mathbf{1}_{\Omega(x)} = \mathbf{1}_{g\Omega(x)}$$

and therefore for each $x \in S$,

$$p_x + x p_{x^{-1}} x^{-1} = \mathbf{1}.$$

$$p_a + p_{a^{-1}} + p_b + p_{b^{-1}} = \mathbf{1}$$

Partial isometries

For $x \in S$ we define a *partial isometry* $s_x \in C(\Omega) \rtimes \Gamma$ by

$$s_x = x(\mathbf{1} - p_{x^{-1}}).$$

Then,

$$s_x s_x^* = x(\mathbf{1} - p_x)x^{-1} = p_x$$

and

$$s_x^* s_x = \mathbf{1} - p_{x^{-1}} = \sum_{y \neq x^{-1}} s_y s_y^*.$$

These relations show that the partial isometries s_x, for $x \in S$, generate a C^*-algebra $\mathcal{O}_A$.
The K-theory of this C^*-algebra is $\mathbb{Z} \times \mathbb{Z}$.

Transition matrix

Where

$$A = \begin{pmatrix} \mathbf{1} & 0 & \mathbf{1} & \mathbf{1} \\ 0 & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & 0 \\ \mathbf{1} & \mathbf{1} & 0 & \mathbf{1} \end{pmatrix}$$

relative to $\{a, a^{-1}, b, b^{-1}\} \times \{a, a^{-1}, b, b^{-1}\}$.

Related projects and further directions of research

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- Applications of harmonic maps to study of buildings and higher-dimensional complexes (with G.Daskalopoulus and C.Mese).

Related projects and further directions of research

- Higher-dimensional Thompson groups and their C*-algebraic invariants (with M.Lawson and A.Sims).
- Unitary representations of higher-dimensional Thompson groups.
- Multi-dimensional picture languages coming from buildings.
- K-theory of n-dimensional polyhedral algebras (with PhD students S.Matter and C.Radu).
- Non-residually finite higher-dimensional CAT(0) groups (with N. Ragunatapirom and J.Stix).
- Systematic study of aperiodic tilings using higher-rank graph C*-algebras.
- Higher-dimensional expanders.
- Groups acting on higher-dimensional hyperbolic buildings.
- Low complexity algorithms on knot recognition and higher-dimensional words recognition (with O.Kharlampovich).
- Applications of harmonic maps to study of buildings and higher-dimensional complexes (with G.Daskalopoulus and C.Mese).
- Applications to algebraic geometry: Beauville surfaces and fake quadrics (with N.Boston, N.Peyerimhoff, J.Stix).

Relevant references

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3. J. Stix, A. Vdovina, *Simply transitive quaternionic lattices of rank 2 over Fq(t) and a non-classical fake quadric*, Mathematical Proceedings of the Cambridge Philosophical Society (2017) 163(3), 453-498.
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6. G. Daskolopoulos, C. Mese, A. Vdovina, *Superrigidity of hyperbolic buildings*, Geometric and Functional Analysis (2011) 21, 905-919.
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