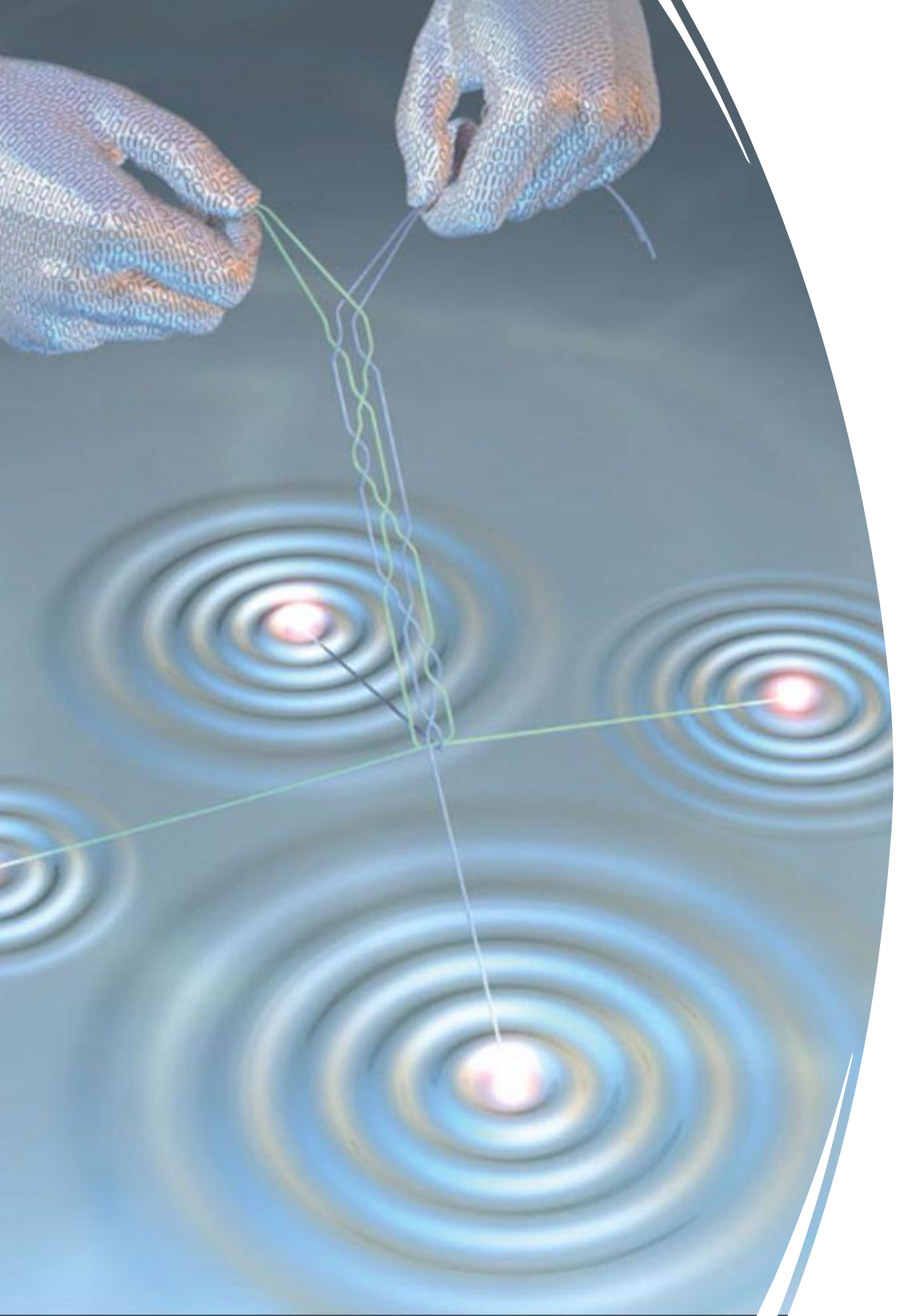
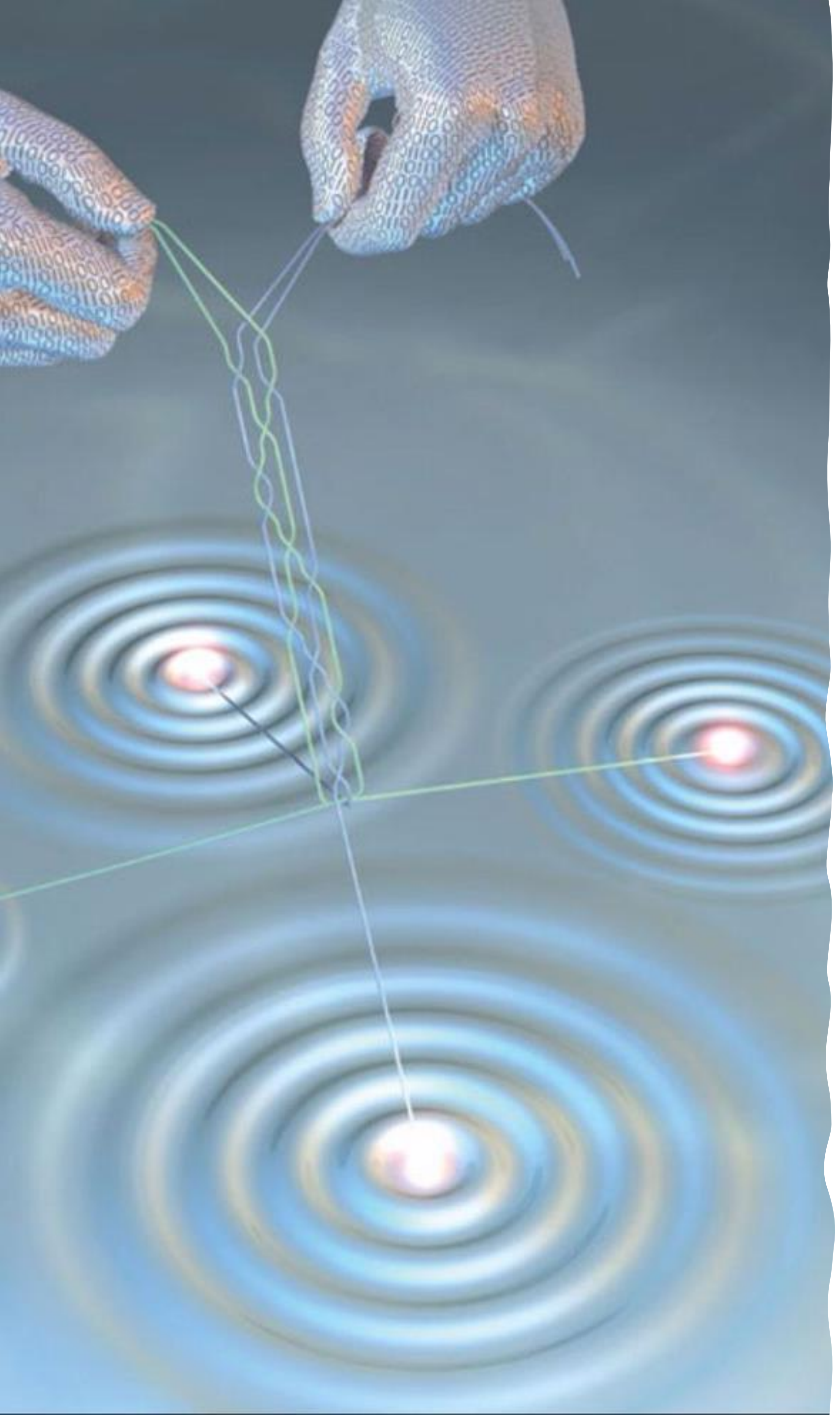


Realizing Universal Circuit Set with an S_3 Quantum Double

Liyuan Chen
Harvard University
Nov 12, 2024



Outline

- Introduction
- Universal Quantum Computation
- Circuit for $\mathcal{D}(S_3)$ anyon manipulation
- Active Error Correction
- Summary and Outlook

Collaborators

Yuanjie Ren
MIT

Ruihua Fan
UC Berkeley

Arthur Jaffe
Harvard

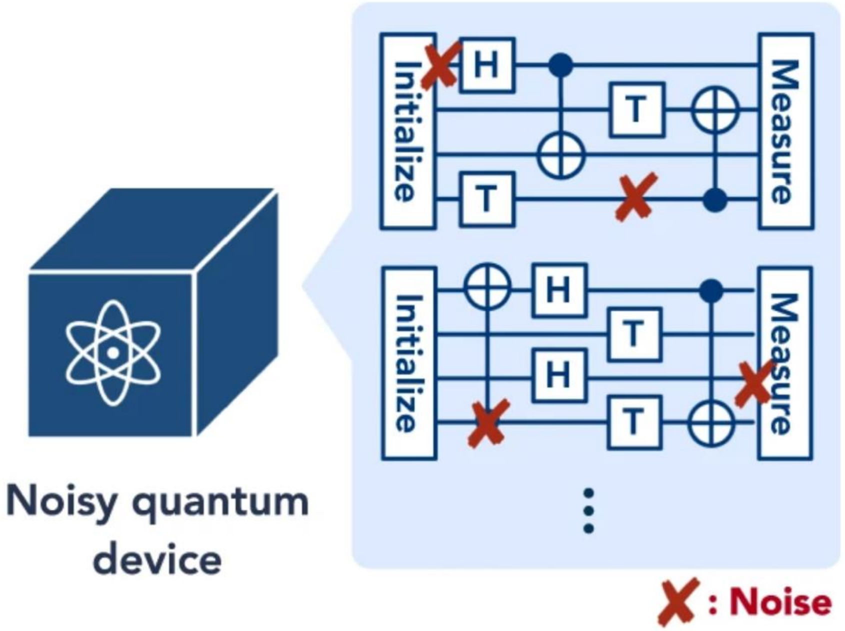
Liyuan Chen, Yuanjie Ren, Ruihua Fan, and Arthur Jaffe, arXiv:2411.xxxxx

Quantum Computers

- Qubits $\alpha|0\rangle + \beta|1\rangle$
- Overpowers Classical Computers

Figure taken from QuEra, 2023

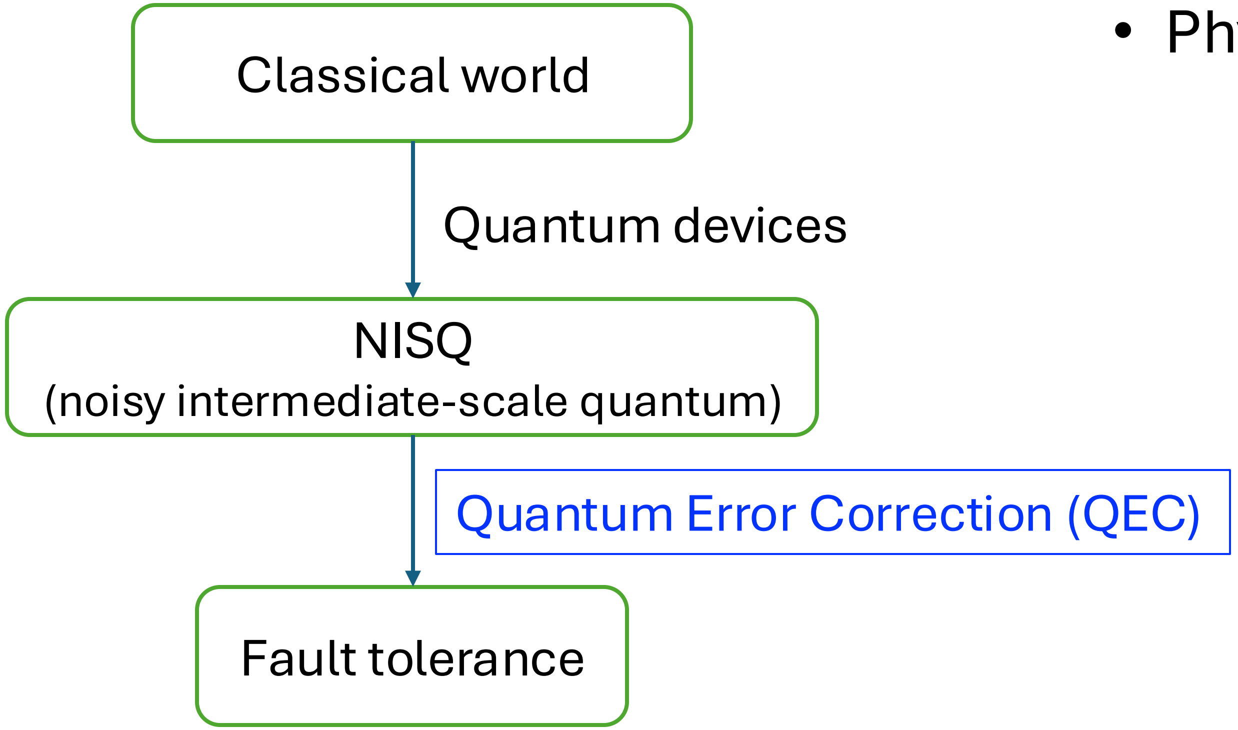
- Noisy Intermediate-Scale Quantum (NISQ)



Chen et al., 2023

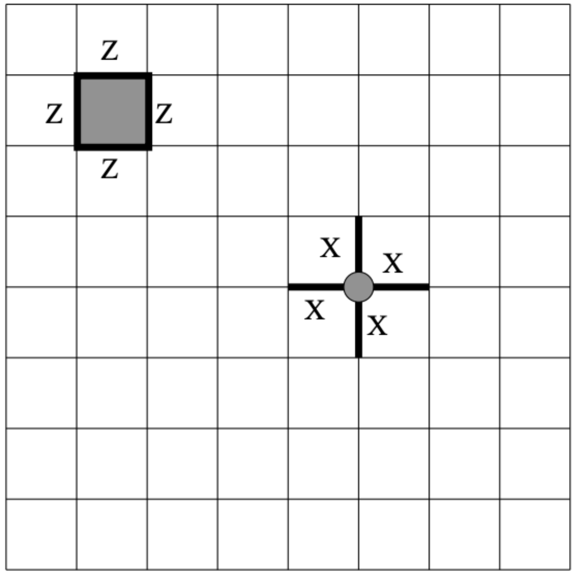
Practical Quantum Computer

- NISQ to Fault Tolerance



Noise → Errors

- Error-correcting code (ECC) Shor, 1995
- Physical → logical $|00\cdots0\rangle \rightarrow |\bar{0}\rangle$



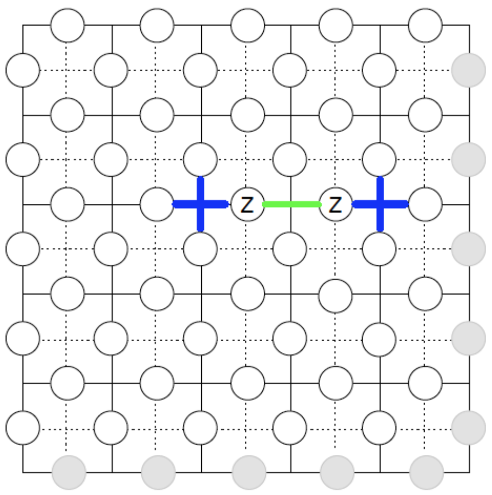
Toric Code/ Surface Code

Dennis et al., 2001

Topological Codes

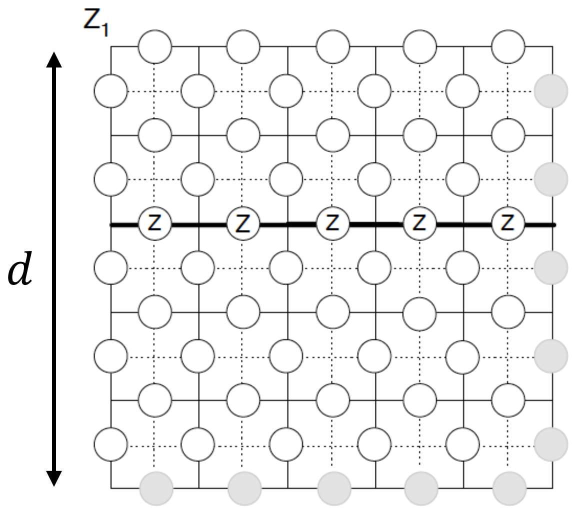
- Topological Stabilizer code

Errors: Pauli Z string



Local Z errors

Detect & correct



Z loop — logical error

Logical error rate: $p \rightarrow O(p^{d/2})$

Figures taken from Browne, 2014

Universal Quantum Computation (UQC)

- Quantum Circuit
- Qubits + Unitary Gates + Measurements

- Universal Gate Set
- Basic gates → Any large unitary
- $\{CNOT, H, S\} + T$
- Clifford gates + magic gate

$$|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}, \qquad |1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

$$T = \begin{bmatrix}1 & 0\\0 & e^{i\frac{\pi}{4}}\end{bmatrix}$$

- UQC + QEC
- Fault-Tolerant Universal Gate Set

Compatibility

- Topological stabilizer codes
- Surface Code (2D)

  No fault-tolerant T gate

  Magic-state distillation

  Bravyi and Kitaev, 2004

- 3D Color Code

  UQC by lattice surgery

  Extra dimension

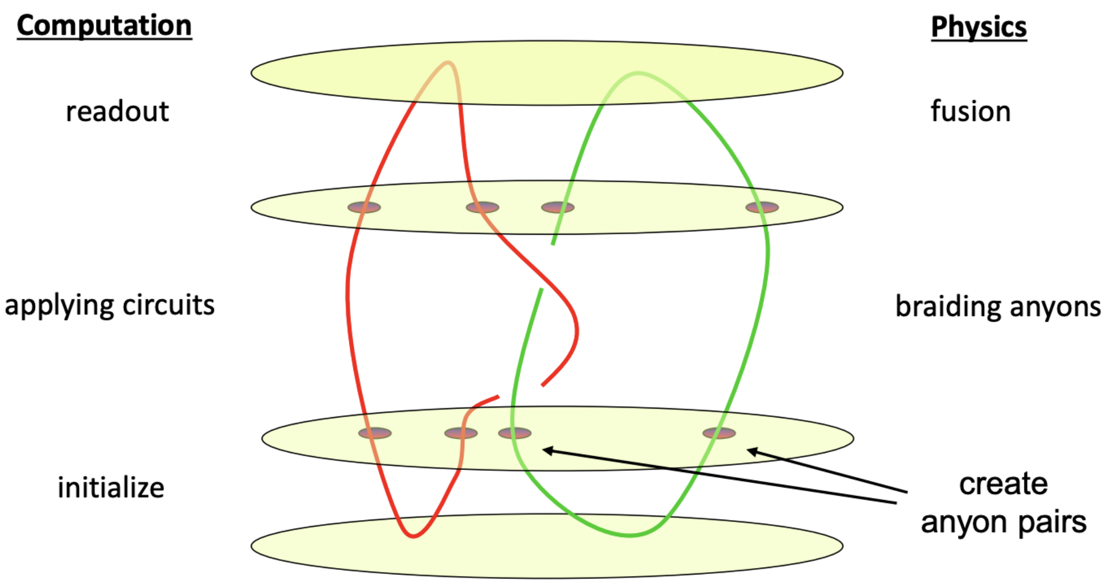
  Bombin, 2013

  Kubica et al., 2014

Extra resources are necessary!

Topological Quantum Computation (TQC)

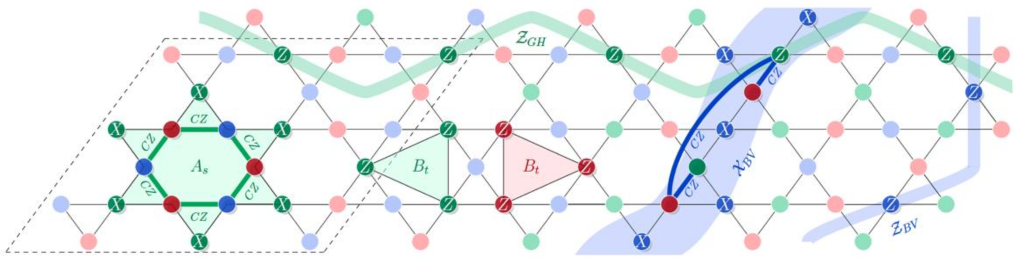
- Non-Abelian topological codes (2D)
- Excitations – non-Abelian anyons Kitaev, 2003
- Braiding & Fusion (topological)



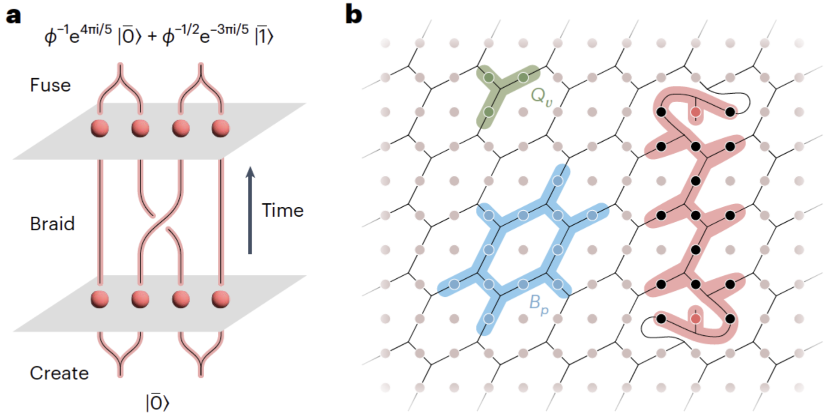
Multiple fusion outcomes $\alpha \times \beta = \gamma + \delta + \cdots$

Figure taken from Rowell and Wang, 2018

- Recent Experiments — Simulation Google, 2023 Iqbal et al., 2024
- Kitaev's Quantum Double model $\mathcal{D}(D_4)$



- Fibonacci Topological Order Xu et al., 2024



Practical TQC – A route to FTUQC

- UQC + QEC +Preparability

- Kitaev's Quantum Double model $\mathcal{D}(A_5)$

  Kitaev, 2003

  UQC by braiding

  60-dim'l local Hilbert space

- Fibonacci Topological Order

  Schotte et al., 2022
  Xu et al., 2024

  UQC by braiding

  2-dim'l local Hilbert space

  Complicated Circuit

  Weight 16 syndrome measurement

- Quantum Double Model $\mathcal{D}(S_3)$

  Verresen et al., 2021
  Liu et al., 2022
  Bravyi et al., 2022

  Preparability

  6-dim'l local Hilbert Space – qubit + qutrit

  Constant-Depth Preparation

  Computational Power

  Mochon, 2003
  Cui et al., 2015

  UQC via braiding + measurement

  Error Correction

  Wootton et al., 2014
  Wootton et al., 2016

  Error Threshold under certain conditions

$\mathcal{D}(S_3) \Rightarrow$ optimal solution?

Compatibility

Many things to do!

$\mathcal{D}(S_3)$ UQC

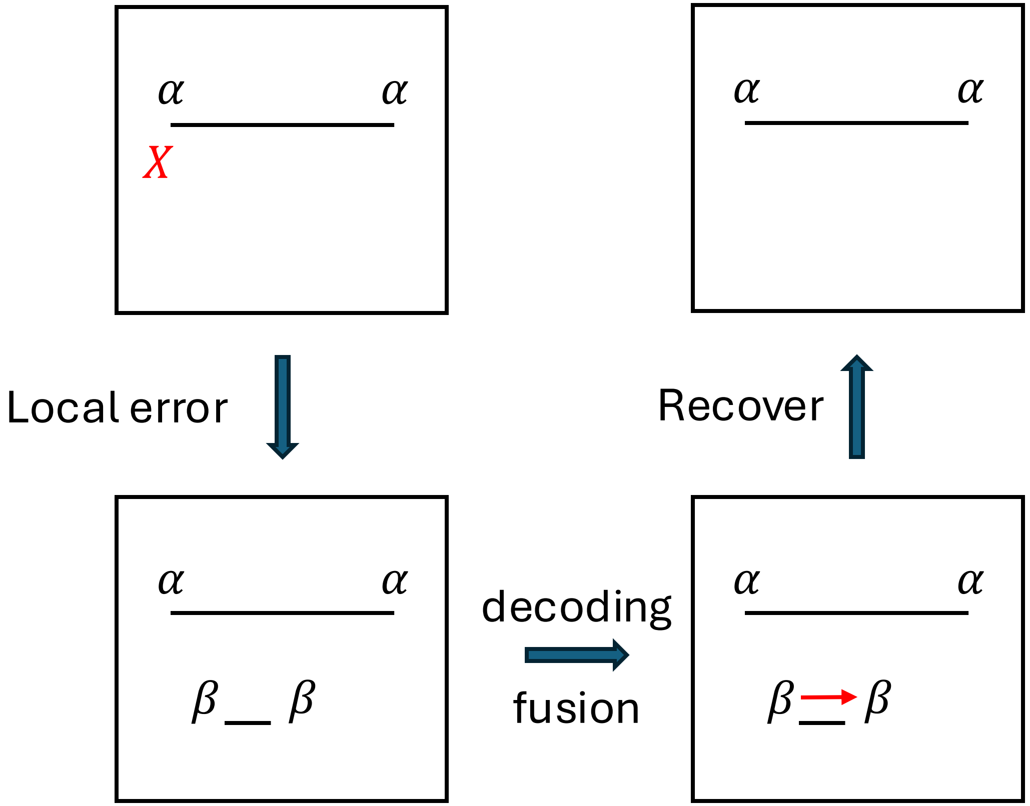
- Encoding
- Local degrees of freedom   Mochon, 2003

  Changed by single site errors

- U-model by Cui, Hong and Wang
- Anyon type   Cui, 2015

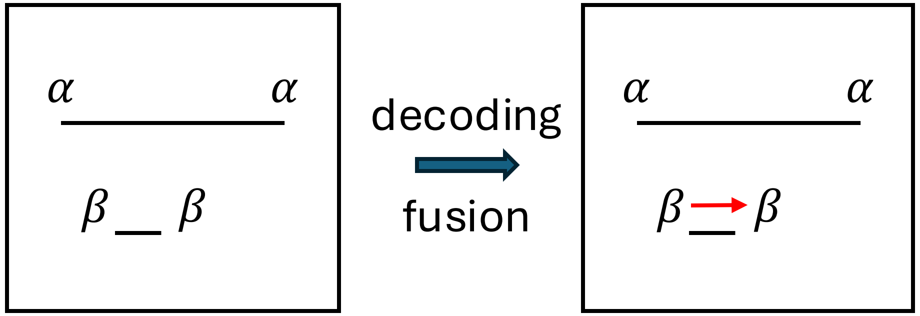
  Universal Logical Gate Set
  Braiding + Measurement

How to realize it (potentially) fault-tolerantly?



$\mathcal{D}(S_3)$ UQC (cont.)

- U-model by Cui, Hong and Wang
- Anyon type



Universal Logical Gate Set
Braiding + Measurement

Remotely

- Realize all operations in U-model remotely

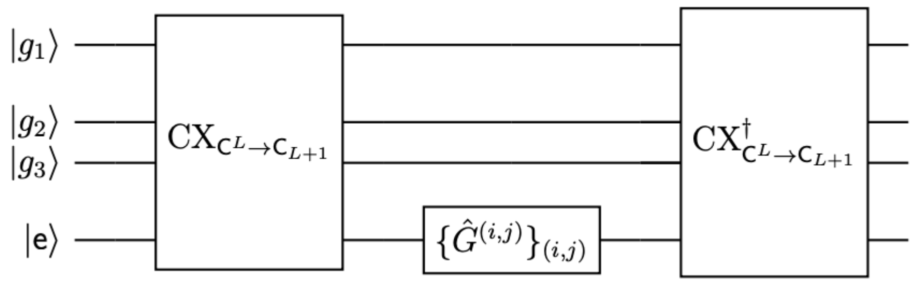
Anyon Inteferometer – Remote measurement

Multi-qutrit gate

$\mathcal{D}(S_3)$ Circuits

- Noiseless
- Ground State
- Single Anyon movement

Verresen et al., 2021
Liu et al., 2022
Bravyi et al., 2022



- Circuit-level noise?
- Fusion $\alpha \times \beta = \gamma + \delta + \cdots$
- Coherent movement with fusion?

- Realize all operations in U-model remotely

  Anyon Inteferometer – Remote measurement

  Multi-qutrit gate

- Circuits for $\mathcal{D}(S_3)$ on qutrits and qubits

  Coherent movement and error correction

$\mathcal{D}(S_3)$ TQC

- UQC + QEC +Preparability
- QEC for non-Abelian anyons

  Instantaneous error correction

  Error thresholds

  Wootton et al., 2014
  Wootton et al., 2016
  Brell et al., 2014
  Burton et al., 2017
  Schotte et al., 2022

Non-Abelion cannot be moved instantaneously !

- Continuous Error Correction
- Ising anyons

  Hutter and Wootton., 2016

$$p_c \approx 3 \times 10^{-17}$$

- Realize all operations in U-model remotely

  Anyon Inteferometer – Remote measurement

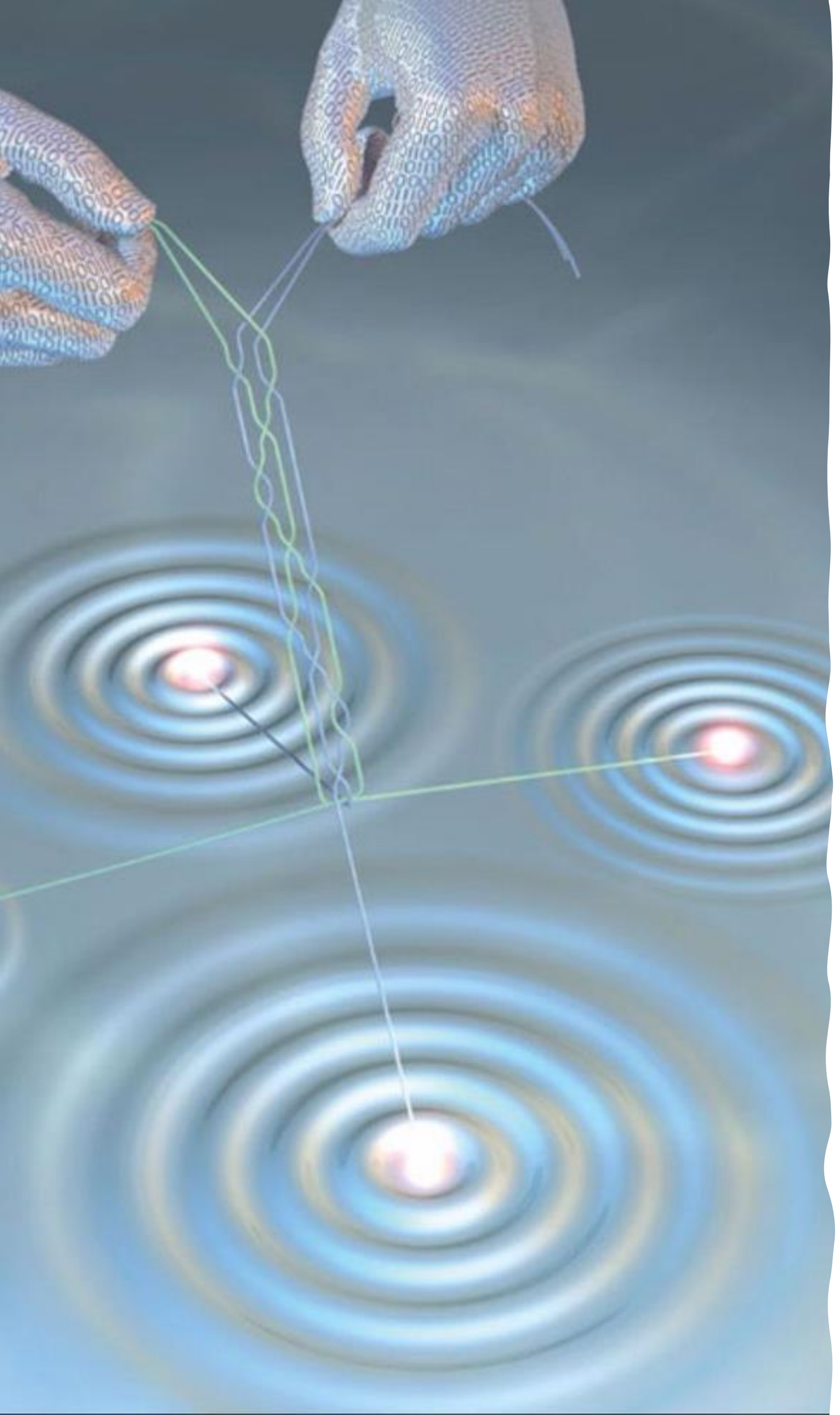
  Multi-qutrit gate
- Circuits for $\mathcal{D}(S_3)$ on qutrits and qubits

  Coherent movement and error correction
- Active Error Correction

  Circuit-level noise ⇔ anyon errors
- Local Error Suppression

  A family of ECCs

  All logical gates in the circuits of $\mathcal{D}(S_3)$



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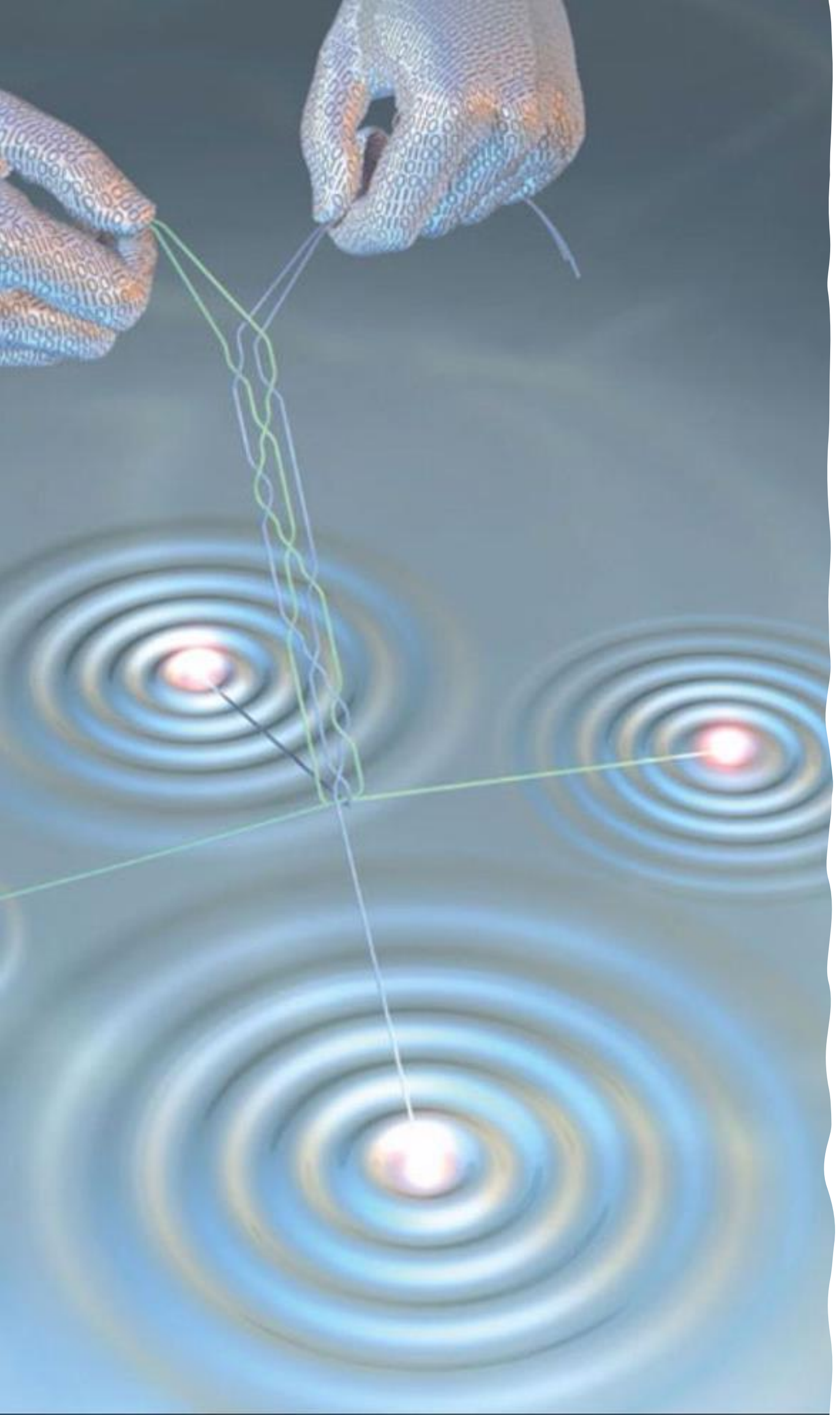

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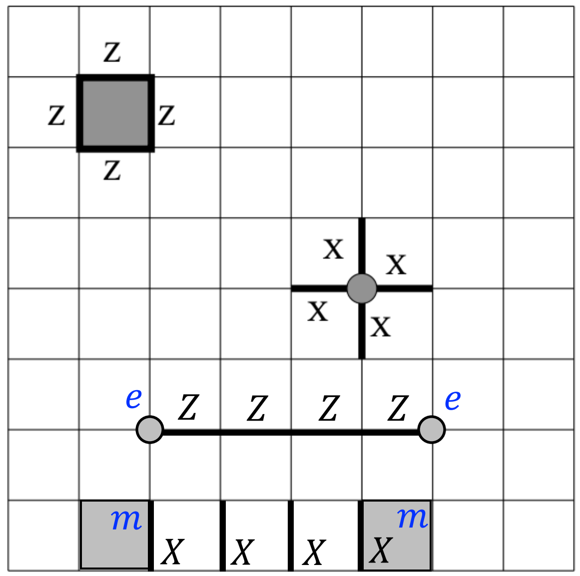
$\mathcal{D}(S_3)$ Quantum Double Model

- Toric Code — $\mathcal{D}(\mathbb{Z}_2)$

Kitaev, 2003
Dennis et al., 2001

Qubit



Edge : $\mathcal{H} = \mathrm{span}\{|0\rangle, |1\rangle\},\ \mathbb{Z}_2$

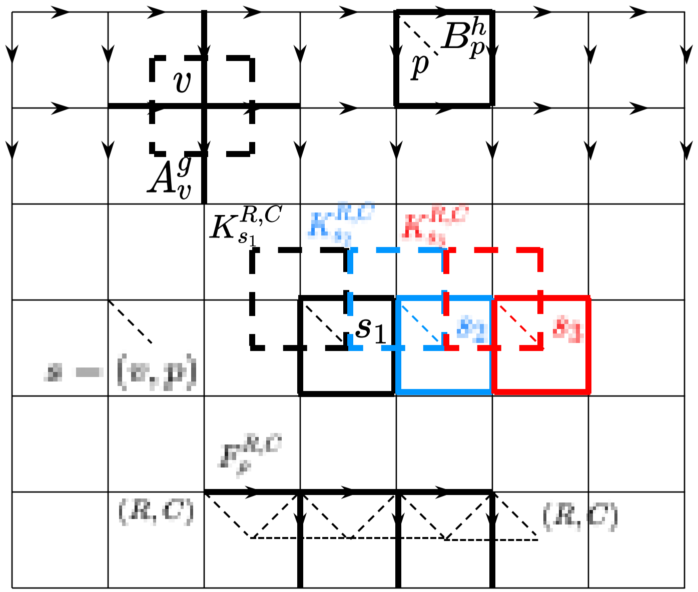
Stabilizers: Plaquette and Vertex operators

Excitations: String operators

$1, e, m, em$ Abelian

- Quantum Double Model— $\mathcal{D}(S_3)$

Qudit



$s = (v, p)$
$A_v^g B_p^h \in \mathcal{D}(S_3)$

$(R, C) \in \mathrm{Irrep}(\mathcal{D}(S_3))$

1-dim'l: Abelian
Higher: non-Abelian

Edge : $\mathcal{H} = \mathrm{span}\{|g\rangle \mid g \in S_3\}$

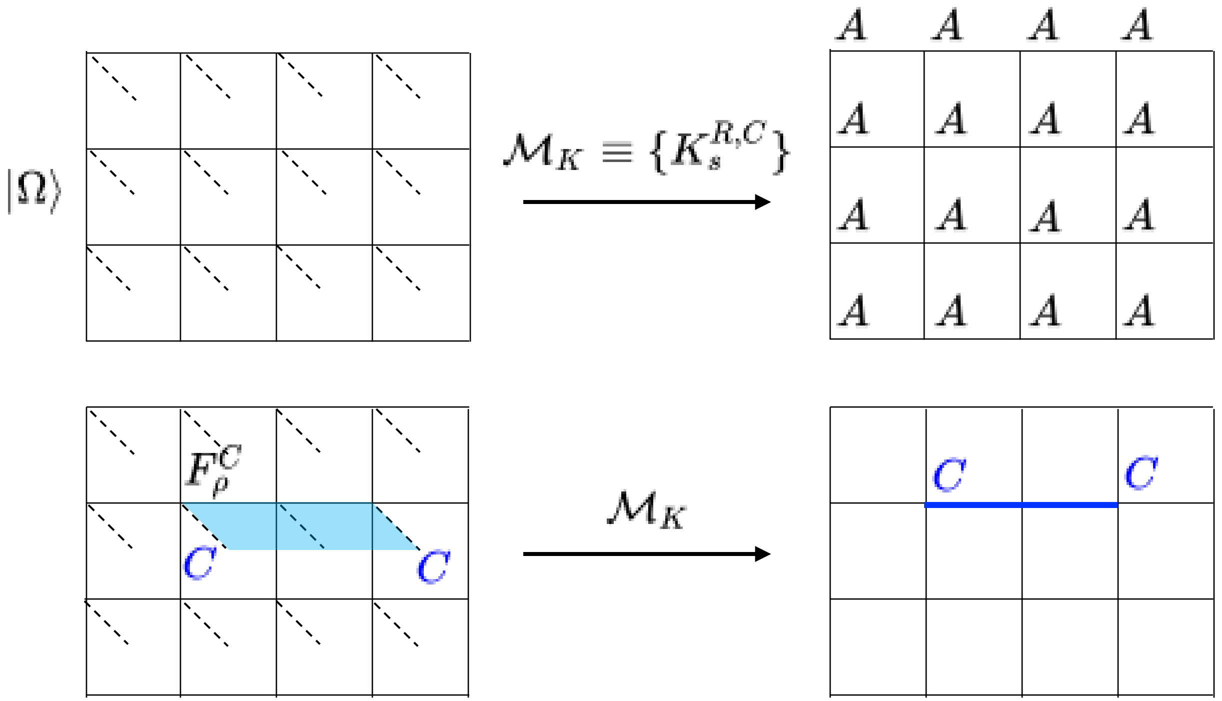
Stabilizers: Plaquette + Vertex operators $K_s^{R,C}$

Excitations: Ribbon operators $F_\rho^{R,C}$

8 Anyons (R, C) — Abelian: A, B; Non-Abelian: $C, D, \cdots, H$

$\mathcal{D}(S_3)$ Anyon Configuration Picture

- Anyon Configuration Picture (canvas)



$\mathcal{M}_K \equiv \{K_s^{R,C}\}$

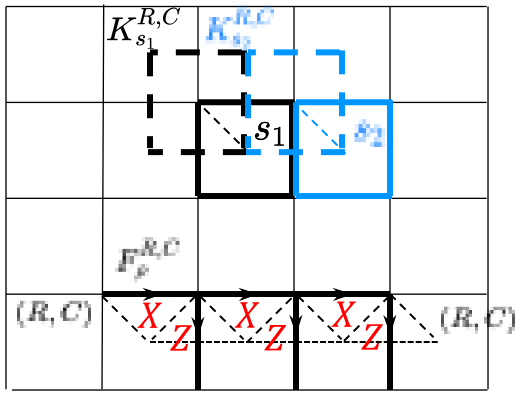
Syndrome Measurement

Lattice model ⇔ Anyon worldlines

Focus on Anyons (Category Theory)

Adaptive Movement Protocol

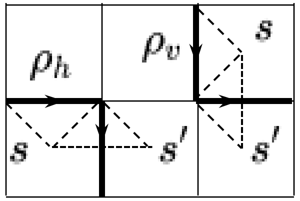
- Quantum Double $\mathcal{D}(S_3)$



Long-range entanglement

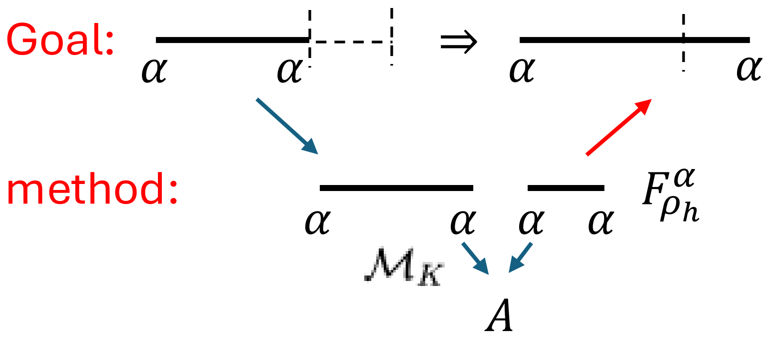
Error out of control!

- Shortest ribbons $F_{\rho_h}^{R,C}, F_{\rho_v}^{R,C}$



Maximally mixed local d.o.f.

- Move by fusion



- (Protocols) Moving non-Abelian anyons

$$\Pr(\text{moving } X) = \begin{cases} 1\,, & X = A, B \\ 1 - \left(\frac{1}{2}\right)^{n}\,, & X = C, F, G, H \\ 1 - \frac{8}{9}\left(\frac{1}{2}\right)^{n-1}\,, & X = D, E \end{cases}$$

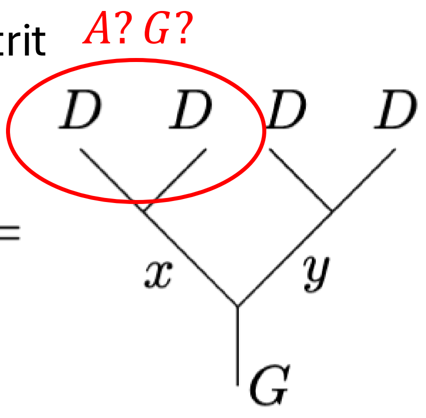
n – number of rounds of $\mathcal{M}_K$

Universal Quantum Computation

- U-model   Cui et al., 2015

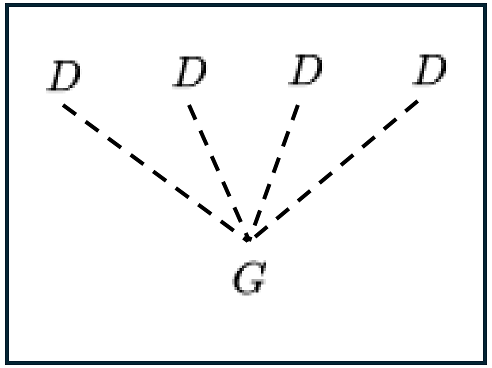
Logical qutrit

$|xy\rangle =$



$U = \mathrm{span}\{|GG\rangle, |AG\rangle, |GA\rangle\}$

$U^{\perp} = \mathrm{span}\{|FC\rangle, |CF\rangle, \cdots\}$



V_G^{DDDD}

Universal Logical Gate Set:

(1) Braiding of D anyons (movement)

(2) Measurements $\mathcal{M}_A, \mathcal{M}_U$ (?)

(3) Fusion space transition

Remotely

- Anyon type measurements

$\mathcal{M}_U = \{\Pi_U, \Pi_{U^{\perp}}\}$ Back to U Preserve coherence

$\mathcal{M}_A = \{\Pi_A, \Pi_{A^{\perp}}\}$ in $U = \mathrm{span}\{|GG\rangle, |AG\rangle, |GA\rangle\}$

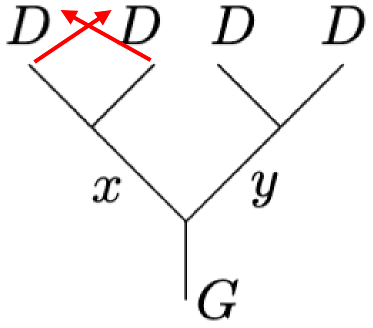
$\{|0\rangle, |0\rangle^{\perp}\}$ Computational power for qutrit

Anyon Interferometer

Anyon Interferometer

- Anyon Interferometer

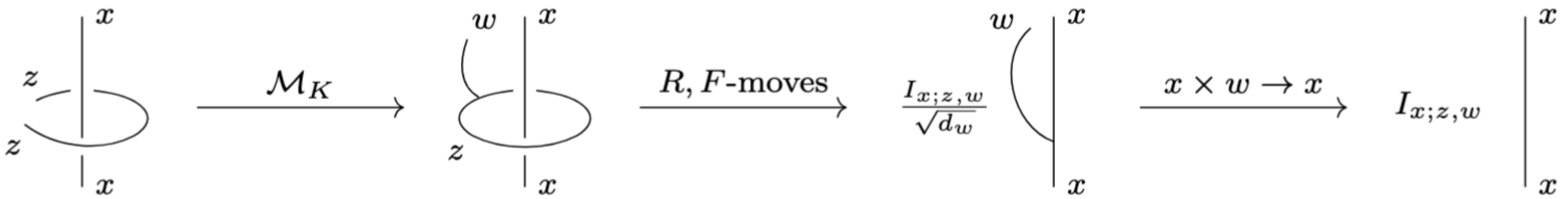
Goal:
Measure x
Remotely



$$\Rightarrow \quad R_x^{DD} = \begin{pmatrix} \omega^2 & & \\ & 1 & \\ & & \omega^2 \end{pmatrix} \begin{matrix} |GG\rangle \\ |AG\rangle \\ |GA\rangle \end{matrix}$$

Logical error

- Procedure



z anyon pair
Circle around

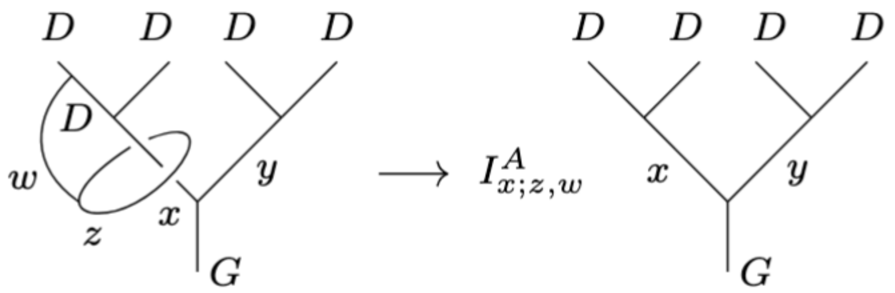
Fuse z pair
A w anyon

Resolve the z loop
A phase factor

Fuse w and x
A diagonal gate

Anyon Interferometer – Measurements

- Measurement 1 $\mathcal{M}_A = \{\Pi_A, \Pi_{A^\perp}\}$ in $U = \mathrm{span}\{|GG\rangle, |AG\rangle, |GA\rangle\}$

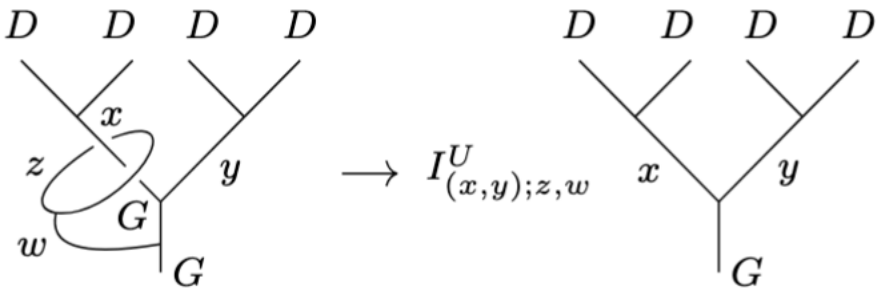


$z = D$

$$w = \begin{cases} A, & x = A \\ G, & x = G \end{cases}$$

Distinguish A or G by w

- Measurement 2 $\mathcal{M}_U = \{\Pi_U, \Pi_{U^\perp}\}$



$z = H$

$w = A \quad w = B$

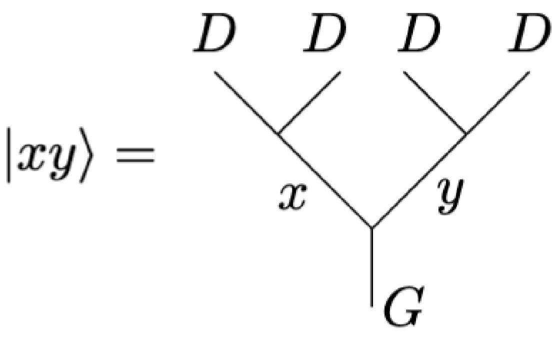
Prob. Proj. into U and $U^\perp$

Accuracy: exp → 1

All local errors ⇒ Global phases indep. of x, y

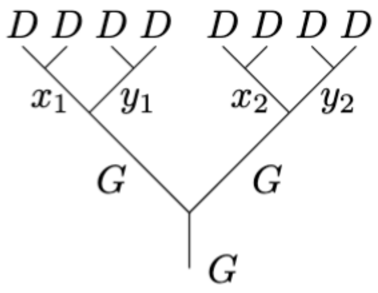
The multi-qutrit gate

- U-model   Cui et al., 2015



- Multi-qutrit CZ

Braiding of D in



Universal Logical Gate Set:

(1) Braiding of D anyons (✓)

(2) Measurements $\mathcal{M}_A, \mathcal{M}_U$ (✓)

(3) Fusion space transition

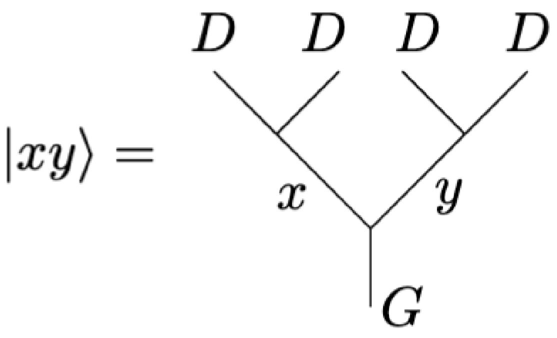
Movement + interferometer

→ Transitions

Remotely

Universal Quantum Computation

- U-model   Cui et al., 2015



Universal Logical Gate Set:

(1) Braiding of D anyons (movement)

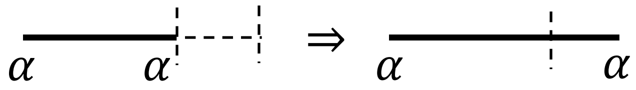
(2) Measurements $\mathcal{M}_A, \mathcal{M}_U$ (interferometer)

(3) Fusion space transition (movement+interferometer)

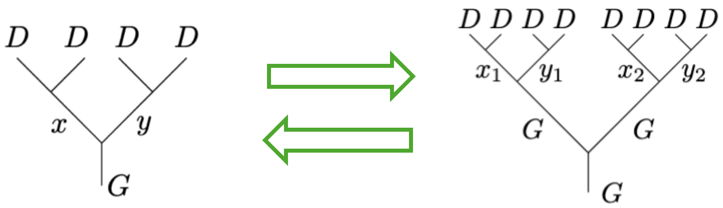
We implement all operations remotely

Realization?

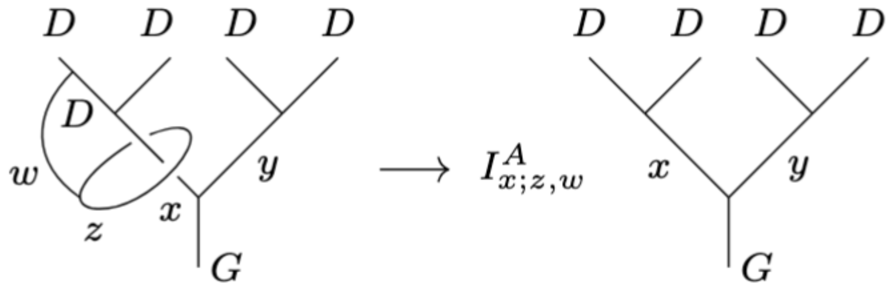
- Adaptive movement – braiding

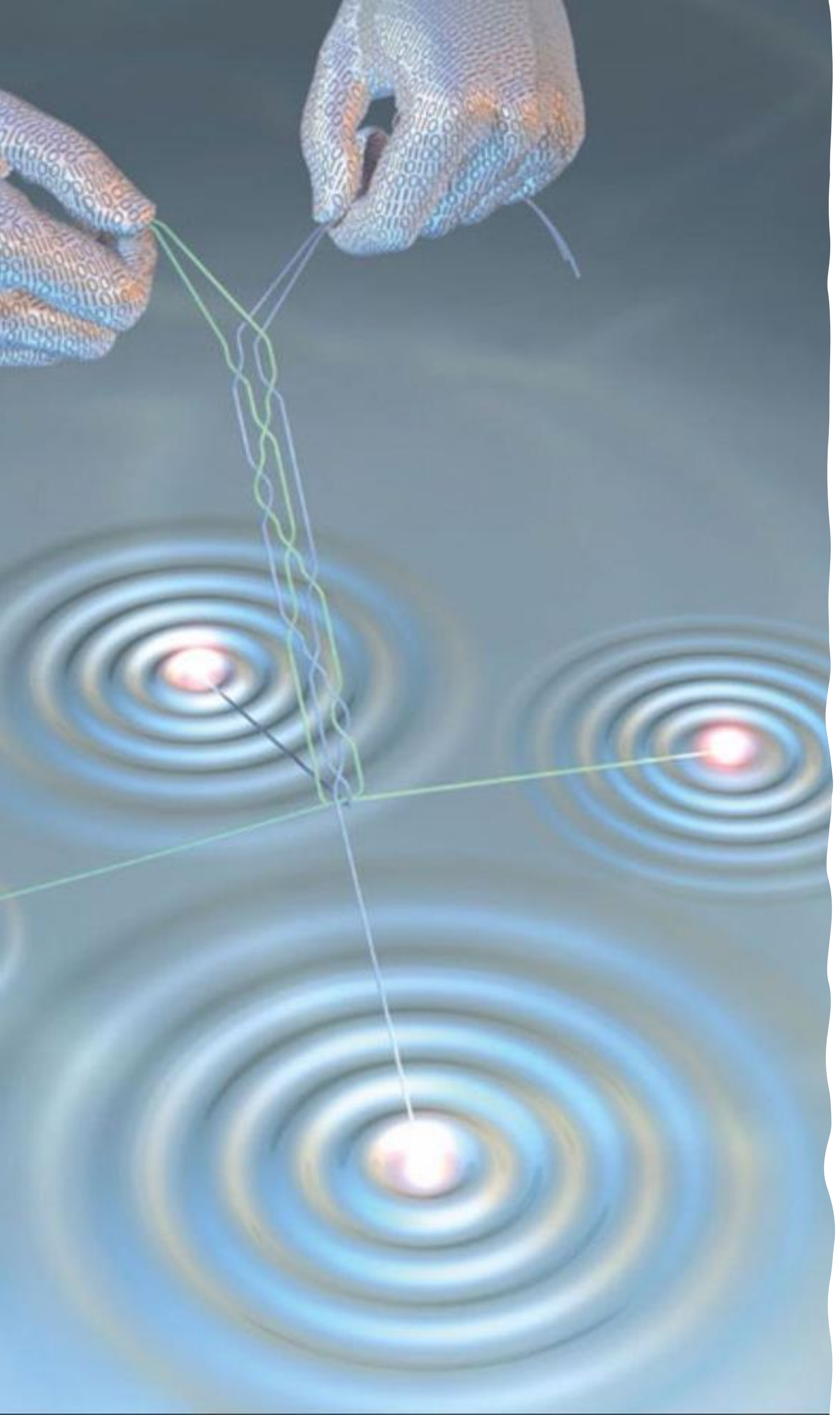


- Fusion space transition



- Interferometry – measurements





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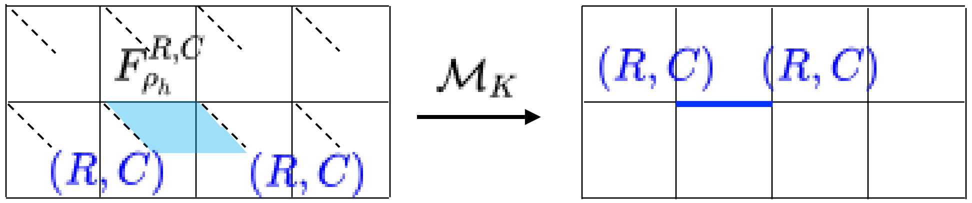
Circuits realization of $\mathcal{D}(S_3)$

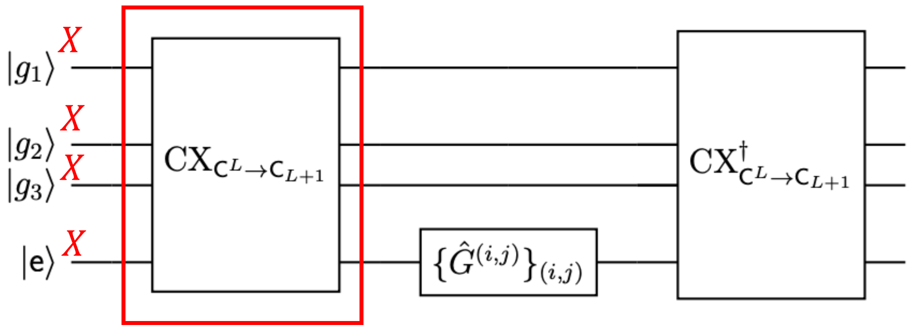
- Computation

  Universal Logical Gate Set

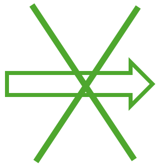
- Basic Operation



- Circuit Realization (noiseless)



Bravyi et al., 2022



- Noisy environment

  Single-site error → high density of anyon errors

  $\mathcal{M}_K$ syndrome measurement

  Solution: shortest ribbon + fusion + QEC

- Fusion

  Unknown local d.o.f.

  Solution: maximally mixed local d.o.f.

Circuits for anyon manipulation

- Circuits for $F_{\rho_h}^{R,C}, F_{\rho_v}^{R,C}$

$S_3 = \mathbb{Z}_3 \rtimes \mathbb{Z}_2 \qquad \mu \in \mathbb{Z}_3 \qquad \sigma \in \mathbb{Z}_2$

Edge $\quad |g\rangle = |\hat{k}, l\rangle \quad$ qutrit-qubit pair

Circuits for all anyons: $A \sim H$

Semidirect Product $\quad \sigma\mu\sigma = \mu^{-1}$

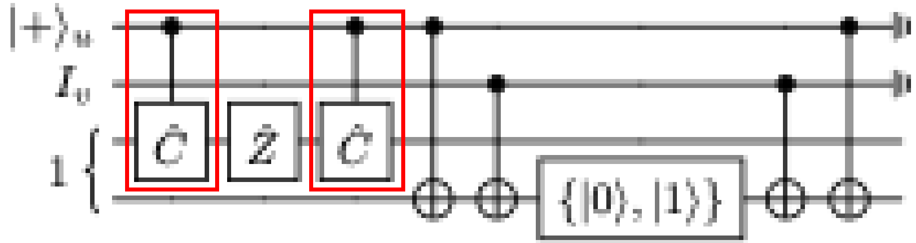
Controlled charge conjugation

$C\hat{C}|\hat{k}, l\rangle = |(-1)^l \hat{k}, l\rangle$

- Maximally mixed local d.o.f.

Bell pair + trace out

Example: $F_{\rho_h}^{C}$

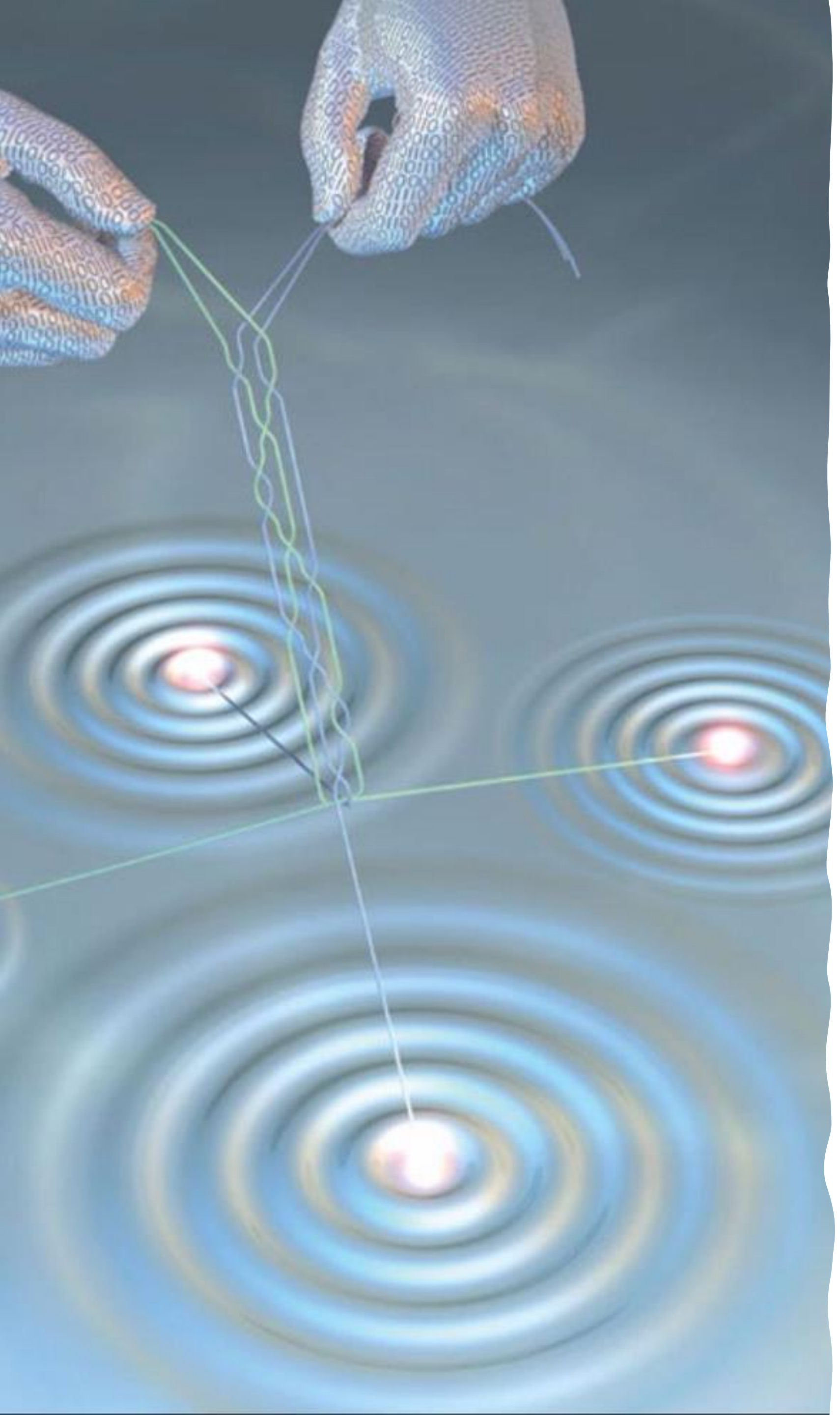


- Circuits for $K_s^{R,C}$

Circuits for manipulating anyons compatible with fusion and QEC

$C\hat{C}$ is the only non-Clifford gate!

Important for error control



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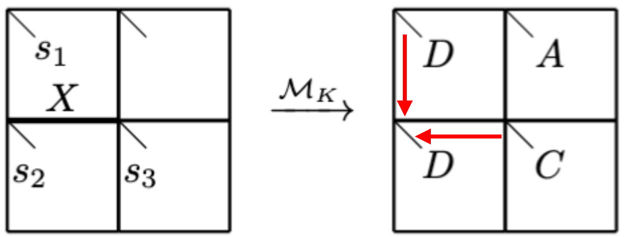
Circuit-level noise correction

- Basic Operation

$$F^{R,C}_{\rho_h/\rho_v} + \mathcal{M}_K$$

Anyon Configuration Picture



Correct anyon errors ⇒ Correct circuit-level noise?

Yes

- Circuit-level noise $X, Z, \hat{X}, \hat{Z}$

(1) Idling qubits/qutrits
(2) Circuits of $F^{R,C}_{\rho_h/\rho_v}$

$\mathcal{M}_K$ Projection → anyon errors

- Correction of circuit-level noise

Proof: Shortest ribbon ops → complete ONB

QEC in one complete ONB → any error

Algorithm: anyon errors

Anyon Error Suppression

- QEC for non-Abelian anyons

  instantaneous anyon movement

  Error thresholds

  Wootton et al., 2014

  Wootton et al., 2016

  No instantaneous movement

- Continuous Error Correction

  Density of anyon errors → low

- Ising anyons (indep.)

  $p_c \approx 3 \times 10^{-17}$

  Hutter and Wootton., 2016

- Expect: a small but finite p_c for $\mathcal{D}(S_3)$

  Our solution: concatenation

  $|g\rangle \to |\hat{k}, l\rangle \to |\hat{k}, l\rangle_L$

  $p \to p_{\text{logical}}$

Construct qutrit-qubit codes with a fault-tolerant logical $C\hat{C}$ gate

- Fault-Tolerant $C\hat{C}^L$

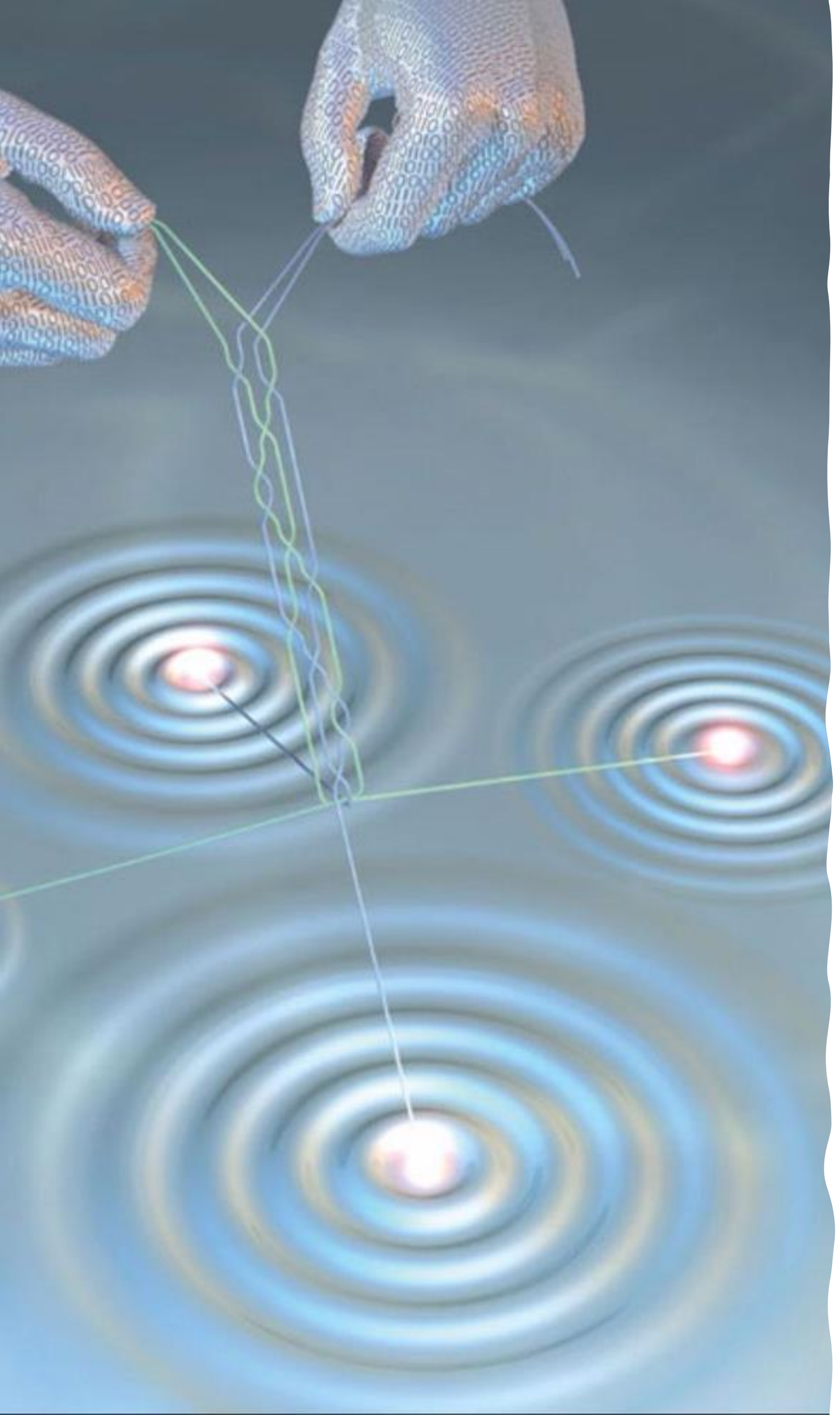
  odd n

  $[[n^2, 1, n]]$ qubit Shor code

  $[[n, 1, d]]$ qutrit CSS code

$$C\hat{C}^L = C\hat{C}^{\otimes n}_{1\sim n} \hat{R} C\hat{C}^{\otimes n}_{n+1\sim 2n} \cdots \hat{R} C\hat{C}^{\otimes n}_{(n-1)n+1\sim n^2}$$

Encoding: $p_{\text{logical}} < p_c$



Outline

- Introduction
- Circuit for $\mathcal{D}(S_3)$ anyon manipulation
- Universal Quantum Computation
- Active Error Correction
- Summary and Outlook

Summary

- Topological Quantum Computation (TQC)

  Quantum Error Correction (QEC)
  Universal Quantum Computation (UQC)
  Experimental feasibility

  $\left.\right\} \Rightarrow \mathcal{D}(S_3)$

- Universal gate set

  Anyon interferometer – remote operations

  Multi-qutrit gate

- Circuit Realization of Quantum Double $\mathcal{D}(S_3)$

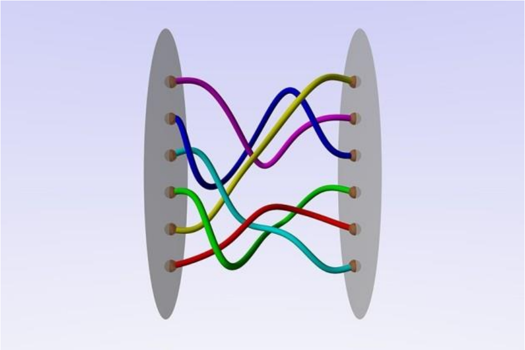
  Locally creating and measuring anyons

  Coherently movement and error correction

- Active Error Correction

  Circuit-level noise $\Leftrightarrow$ anyon errors

  Local codes: anyon density suppression



Microsoft, 2016

A step toward a Practical Topological Quantum Computer

Outlook

- Continuous Error Correction for $\mathcal{D}(S_3)$

  Algorithm

  Threshold

  Measurement errors

- Constant-depth protocol

  Reduce resource overhead

  Error control

  } Better than Toric code and color code

- Practical application

  Experiments: neutral atoms, ion traps…

  Near-term algorithms: efficient by this computer

- Local codes and magic-state distillation

  Better codes with logical $C\hat{C}$

  Magic-state distillation between qubits and qutrits

A protocol for Large-Scale Quantum Computation

A step toward a Practical Topological Quantum Computer