Operational advantage of quantum resources in subchannel discrimination

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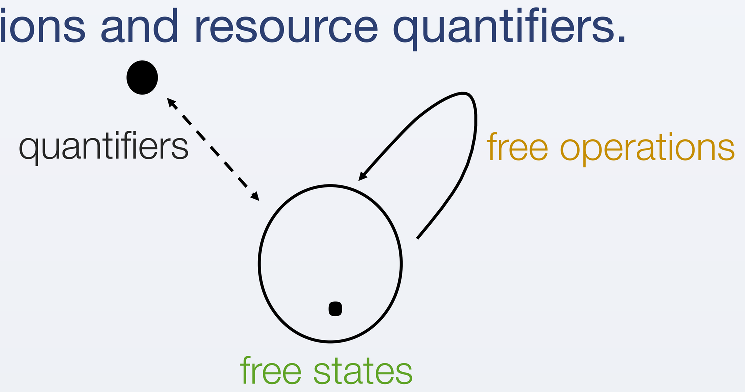
Introduction

(1) Resource theory is a framework to deal with quantification and manipulation of physical quantities considered “precious”. Three ingredients in this framework: free states, free operations and resource quantifiers.



Advantage of resource theory:

- applicable to various settings (coherence etc.)
- tool to understand resources in a unified way

(2) Subchannel discrimination: the task of distinguishing the branches of quantum evolution which a quantum system undergoes.

Subchannels $\{\Psi_i\}$: each subchannel is a CP trace and non-increasing map

$\Lambda = \sum_i \Psi_i$: CPTP

A probe state ρ undergoes an action of one of the subchannels $\{\Psi_i\}$.

The goal of this task: use the Positive Operator Valued Measurement $\{M_i\}$ to identify which one of the subchannels was applied.

(3) Our contribution in this work:

Under the assumption of the convexity and closedness of the set of free states $\mathcal{F}$, we show that the following two results hold in any convex resource theory.

- Existence of an advantage

  Every resource state for **any** convex theory is useful for some subchannel discrimination task.

- Quantification of the advantage

  Generalized robustness serves as an exact quantifier in a class of subchannel discrimination for **any** finite-dimensional convex theory.

Result 1: Existence of an advantage

Theorem 1

For any $\rho \notin \mathcal{F}$, there exists a set of subchannels such that

$$\frac{\max_{\{M_i\}} p_{\rm succ}(\{\Psi_i\},\{M_i\},\rho)}{\max_{\sigma\in\mathcal{F}}\max_{\{M_i\}} p_{\rm succ}(\{\Psi_i\},\{M_i\},\sigma)} > 1 ,$$

where the success probability is defined as

$$p_{\rm succ}(\{\Psi_i\},\{M_i\},\rho) = \sum_j {\rm Tr}\left[M_j \Psi_j(\rho)\right] .$$

- Every resource state in any convex theory is useful.
- The **only** assumption here is the convexity and closedness of the set of free states $\mathcal{F}$.
- This theorem gives operational interpretation to **every** resource state including bound states (e.g. bound magic/non-Gaussian).

Example: coherence

Here, we give a simple example to show the the existence of subchannels to demonstrate the advantage of resource states.

Take $\mathcal{F} = \mathcal{I}$, i.e. the set of incoherent states, and $\rho = |+\rangle\langle+|$. Define $\Psi_{0,1} = \frac{1}{2}\Lambda_{0,1}$ by

$$\begin{cases} \Lambda_0(|0\rangle\langle 0|) = \Lambda_0(|1\rangle\langle 1|) = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| \\ \Lambda_0(|0\rangle\langle 1|) = \Lambda_0(|1\rangle\langle 0|) = -\frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| \end{cases}$$

$$\begin{cases} \Lambda_1(|0\rangle\langle 0|) = \Lambda_1(|1\rangle\langle 1|) = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|2\rangle\langle 2| \\ \Lambda_1(|0\rangle\langle 1|) = \Lambda_1(|1\rangle\langle 0|) = 0 \end{cases}$$

Thus, the corresponding success probability is

$$\max_{\{M_i\}} p_{\rm succ}(\{\Psi_i\},\{M_i\},|+\rangle\langle+|) = 1$$

$$> \max_{\{M_i\}} p_{\rm succ}(\{\Psi_i\},\{M_i\},\sigma) = 3/4$$

$$\forall\sigma\in\mathcal{F}$$

Result 2: Robustness as a quantifier

Theorem 2

The generalized robustness quantifies the maximal achievable advantage:

$$\max_{\{\Psi_i\},\{M_i\}} \frac{p_{\rm succ}(\{\Psi_i\},\{M_i\},\rho)}{\max_{\sigma\in\mathcal{F}} p_{\rm succ}(\{\Psi_i\},\{M_i\},\sigma)} = 1 + R_{\mathcal{F}}^{\mathbb{D}}(\rho),$$

where the generalized robustness is defined as

$$R_{\mathcal{F}}^{\mathbb{D}}(\rho) = \min_{\omega\in\mathbb{D}}\left\{\lambda \mid \frac{\rho+\lambda\omega}{1+\lambda}\in\mathcal{F}\right\} .$$

- $R_{\mathcal{F}}^{\mathbb{D}}$ gains a universal operational interpretation

Theorem 3: Condition for robustness to be quantifier

Let $X = \sum_i x_i|e_i\rangle\langle e_i|$ be the optimal witness for ρ . If there exists a set of unitaries $\{U_i\}_{i=1}^d$ such that $\sum U_i|e_j\rangle\langle e_j|U_i^\dagger = I, \forall j$ and $U_i\sigma U_i^\dagger = U_j\sigma U_j^\dagger, \forall\sigma\in\mathcal{F}$, then

$$\max_{\{\Psi_i\}} \frac{\max_{\{M_i\}} p_{\rm succ}(\{\Psi_i\},\{M_i\},\rho)}{\max_{\sigma\in\mathcal{F}}\max_{\{M_i\}} p_{\rm succ}(\{\Psi_i\},\{M_i\},\sigma)} = 1 + R_{\mathcal{F}}^{\mathbb{D}}(\rho) .$$

Let’s consider a little more restrictive but still natural case, with free measurements. That is,

$$M_i \propto \sigma_i \in \mathcal{F}$$

Theorem 4: Coherence with free measurement

Let $\mathcal{M}_{\mathcal{F}}$ be a set of free measurements with respect to $\mathcal{F}$. If $\mathcal{F} = \mathcal{I}$, then

$$\max_{\{\Psi_i\}} \frac{\max_{\{M_i\}\in\mathcal{M}_{\mathcal{F}}} p_{\rm succ}(\{\Psi_i\},\{M_i\},\rho)}{\max_{\sigma\in\mathcal{F}}\max_{\{M_i\}\in\mathcal{M}_{\mathcal{F}}} p_{\rm succ}(\{\Psi_i\},\{M_i\},\sigma)} = 1 + R_{\mathcal{F}}^{\mathbb{D}}(\rho) .$$

Theorem 5: Magic with free measurement

Let $\mathcal{M}_{\mathcal{F}}^1$ be a set of rank-one free measurements with respect to $\mathcal{F}$ and $\mathcal{F} = {\rm STAB}_{2,1}$. Then for any pure state ρ

$$\max_{\{\Psi_i\}} \frac{\max_{\{M_i\}\in\mathcal{M}_{\mathcal{F}}^1} p_{\rm succ}(\{\Psi_i\},\{M_i\},\rho)}{\max_{\sigma\in\mathcal{F}}\max_{\{M_i\}\in\mathcal{M}_{\mathcal{F}}^1} p_{\rm succ}(\{\Psi_i\},\{M_i\},\sigma)} = 1 + R_{\mathcal{F}}^{\mathbb{D}}(\rho),$$

where

$${\rm STAB}_{d,n} = \{\sum_i p_i|\phi_i\rangle\langle\phi_i| \mid \forall i,\ |\phi_i\rangle\in\mathcal{V}_{d,n}\},$$

$$\mathcal{V}_{d,n} = \{|\phi\rangle\langle\phi| \mid |\phi\rangle = U|0\rangle ,\ U\in\mathcal{C}_{d,n}\},$$

$\mathcal{C}_{d,n}$: Clifford gates on n qudits.

Summary

- In a convex resource theory, every resource state is useful for a subchannel discrimination task.
- Generalized robustness serves as an exact quantifier for a class of subchannel discriminations. This gives **a universal operational interpretation** of the generalized robustness measure.
- When measurement constraints are relaxed, generalized robustness still serves as a quantifier for the theory of entanglement, coherence, and magic.

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