Abstract: I will describe in my talk two roles of prolate spheroidal functions in the spectral interpretation of zeros of the Riemann zeta function. The first part (joint work with C. Consani) will concern the low lying part of the spectrum and will give an operator theoretic incarnation of the Riemann-Siegel formula. The second part (joint work with H. Moscovici) will handle the ultraviolet part of the spectrum.