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Mathematical Picture Language Seminar

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Zoom QR Code & Link:

https://harvard.zoom.us/j/779283357?pwd=MitXVm1pYUlJVzZqT3lwV2pCT1ZUQT09

Random forests and the OSp(1|2) nonlinear sigma model

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Abstract: Given a finite graph, the arboreal gas is the measure on forests (subgraphs without cycles) in which each edge is weighted by a parameter β greater than 0. Equivalently this model is bond percolation conditioned to be a forest, the independent sets of the graphic matroid, or the q→0 limit of the random cluster representation of the q-state Potts model. Our results rely on the fact that this model is also the graphical representation of the non-linear sigma model with target space the fermionic hyperbolic plane H0|2, whose symmetry group is the supergroup OSp(1|2).

The main question we are interested in is whether the arboreal gas percolates, i.e., whether for a given β the forest has a connected component that includes a positive fraction of the total edges of the graph. We show that in two dimensions a Mermin-Wagner theorem associated with the OSp(1|2) symmetry of the non-linear sigma model implies that the arboreal gas does not percolate for any β greater than 0. On the other hand, in three and higher dimensions, we show that percolation occurs for large β by proving that the OSp(1|2) symmetry of the non-linear sigma model is spontaneously broken. We also show that the broken symmetry is accompanied by massless fluctuations (Goldstone mode). This result is achieved by a renormalisation group analysis combined with Ward identities from the internal symmetry of the sigma model.

https://mathpicture.fas.harvard.edu/seminar