Fundamental bound on time signal generation

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\[ \sigma_x \cdot \sigma_p = \frac{\hbar}{2} \]

\[ H(X) + H(P) = \log(e \pi) \]

Two types of "clock":

- stopwatch

- ticking clock

\[ t_2 - t_1 \]

\[ \Rightarrow k \Delta T \Delta T' \]
Examples:
- classical pendulum clock
- optical clock

\[ T_n = \alpha T_1 + \alpha T_2 + \ldots + \alpha T_n \]

\[ \sigma_{\Delta T}^2 = n \sigma_{\Delta T}^2 \]

\[ \sigma_{\Delta T} = \sqrt{n} \sigma_{\Delta T} \]

\[ \mu = \langle \Delta T_i \rangle \]

\[ \sigma = \sqrt{\langle (\Delta T_i - \mu)^2 \rangle} \]
\[ G_{tn} \geq N \]
\[ n6^2 = \kappa^2 \]
\[ n = \frac{\kappa^2}{6^2} = R \]

**Idea:** 
- \( d_{ctri} \geq \) number of dirt states that \( S \) admits during its time evolutions.

**Formally:**
\[ d_{ctri} = \max 2^{I(Ref:S)} \]
\[ 8_{SRef} = S_{d+\Omega_3(t)} \cdot 1t=1/Ref \]
I(Ref; S) = \( H(S) - H(S/Ref) \)

\[ x \sim e \]

\( k < x < 1 \)

\[ \sigma x \sim \frac{k}{n} \]

# disks slabs \( \sim \frac{\sigma x}{\Delta x} \sim e \frac{1}{n} \)

\( \text{Ref} \)

\( \text{S} \)

\( \text{SAF} \)
\[ u \sim \frac{v}{\hbar} \]
\[ \Rightarrow \ell \sim \frac{\hbar}{v} \]
\[ \Rightarrow d_{_{\text{ch}}} \sim \sqrt{n} \frac{\hbar}{v^2} \]

Optical clock: \( n \) photons
\[ d_{_{\text{ch}}} \sim \sqrt{n} \quad \text{for classical light} \]
\[ d_{_{\text{ch}}} \sim n \quad \text{for squeezed light} \]

Thm: \( R \leq 2\pi e d_{_{\text{ch}}}^2 \)

Conversely, the bound is approximately achievable!
Remark about proof:

Data Privacy Inequality

\[ I(A:B) = I(A':B') \]

Conclusions: To build a good signal generator (tickling clock) one has to look for a system with large data, e.g., a fully controlled quantum device.

For n-qubit device, \( d_{cb} = 2^n \)

\[ R \sim 2^{2n} \]