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Quantum Simulation of Renormalized Quantum Field Theory Hamiltonians

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Harvard University Mathematical Picture Language Seminar
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Prelude **Motivation**

Part I **Renormalized Quantum Field Theory Hamiltonians**

Part II **Quantum Simulation**

Part III **Results**

Part IV **Conclusion**

Prelude **Motivation**

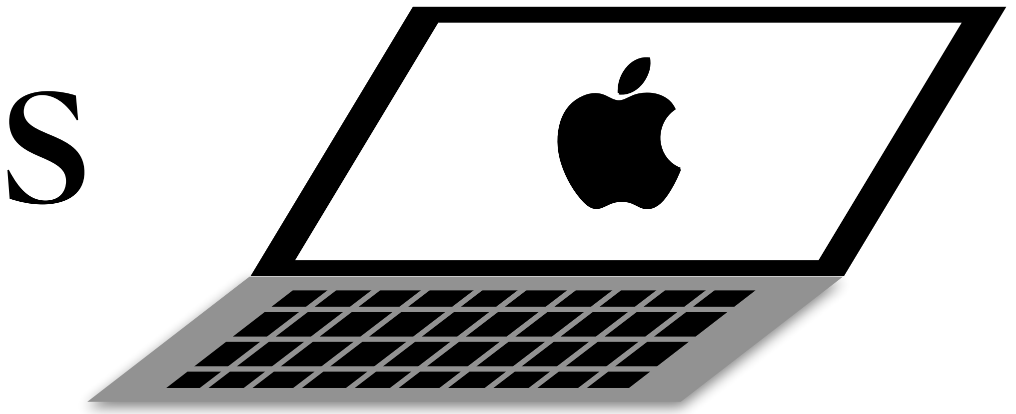
Part I Renormalized Quantum Field Theory Hamiltonians

Part II Quantum Simulation

Part III Results

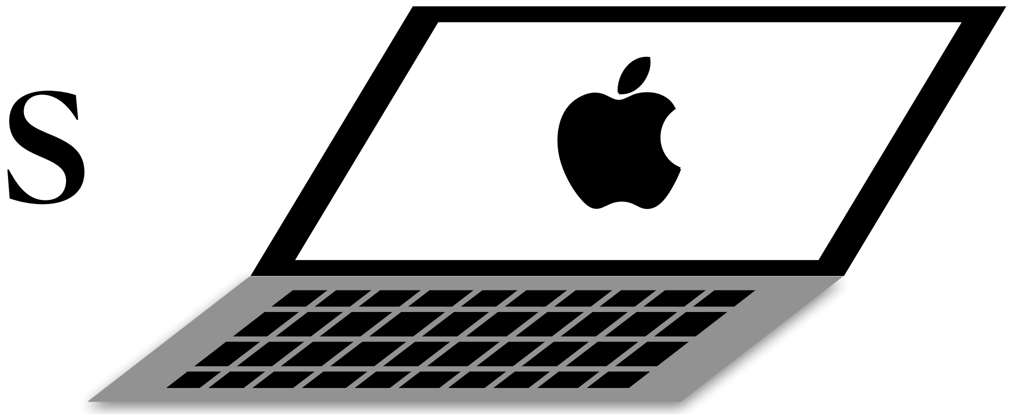
Part IV Conclusion

Simulating Many-Body Quantum Mechanics



$$H|\psi\rangle = E|\psi\rangle$$

Simulating Many-Body Quantum Mechanics

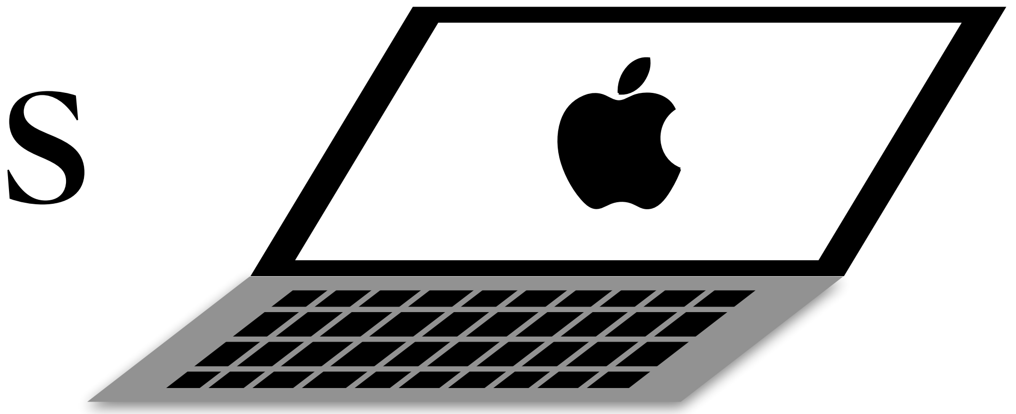


$$H|\psi\rangle = E|\psi\rangle$$

Bottleneck:

$$|\psi\rangle \in \mathcal{H};\ \dim(\mathcal{H}) = \mathcal{O}\Big(\exp(\text{n particles})\Big)$$

Simulating Many-Body Quantum Mechanics



$$H|\psi\rangle = E|\psi\rangle$$

Bottleneck:

$$|\psi\rangle \in \mathcal{H};\ \dim(\mathcal{H}) = \mathcal{O}\Big(\exp(\text{n particles})\Big)$$

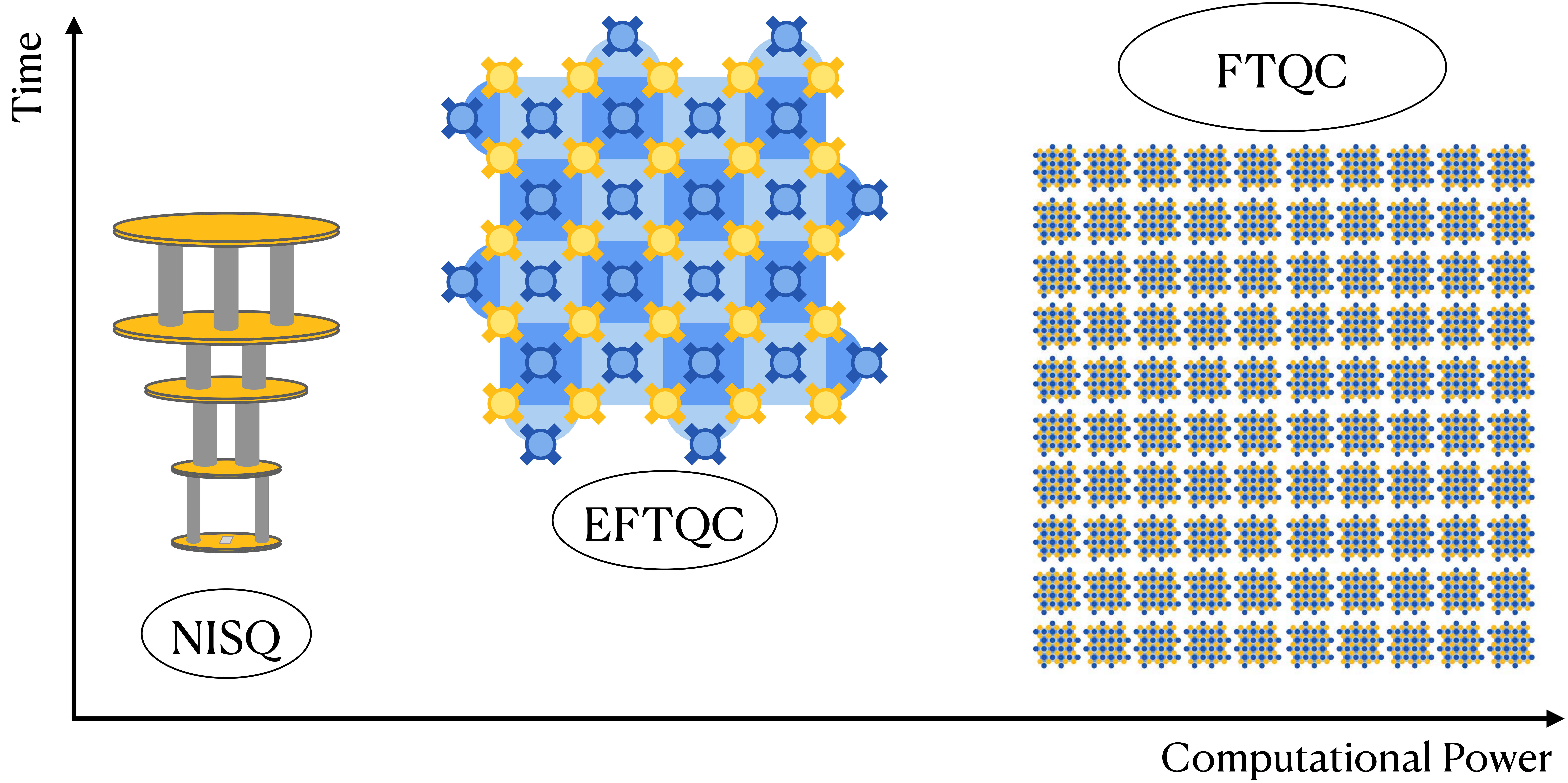
Classically either:

Exponential memory: Store $O(2^n)$ complex amplitudes in classical memory

Or:

Exponential time: Add up $O(2^n)$ amplitudes of separate paths

Quantum Computing Timeline



Time

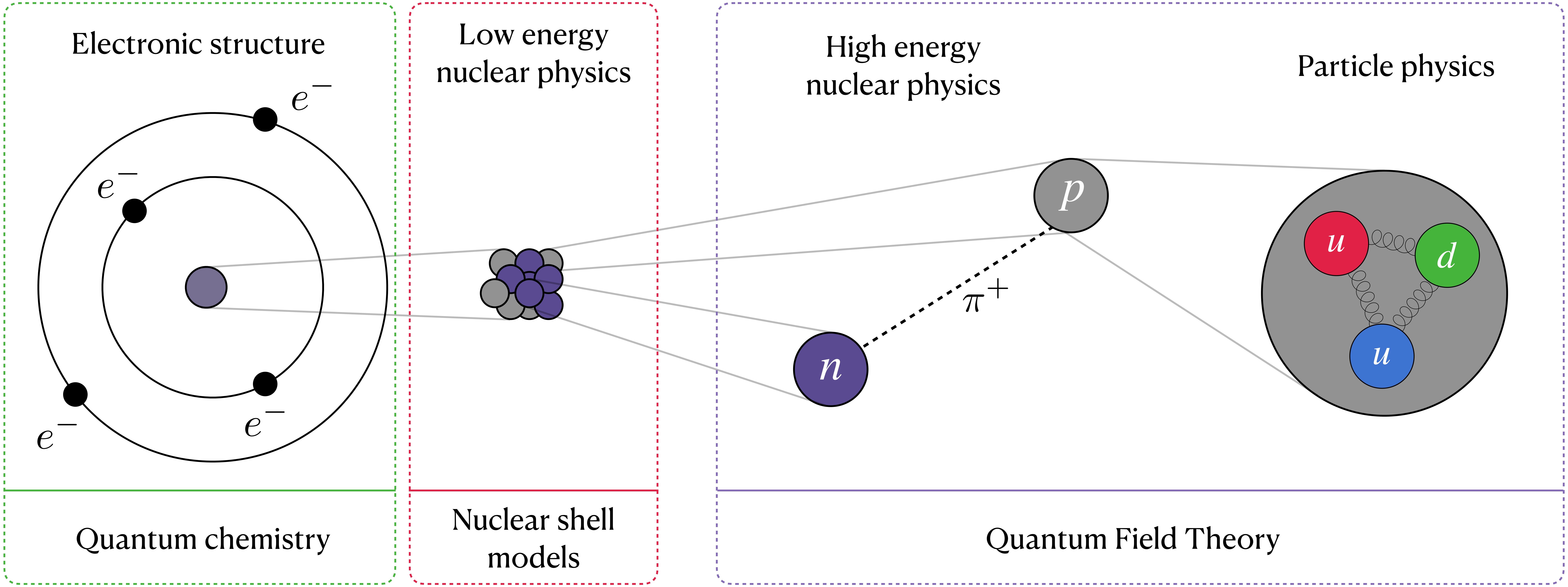
FTQC

EFTQC

NISQ

Computational Power

Looking for Quantum Advantage


Electronic structure

Low energy nuclear physics

High energy nuclear physics

Particle physics

Quantum chemistry

Nuclear shell models

Quantum Field Theory

Quantum Simulation

Dynamics

Statics

Quantum Simulation

Dynamics

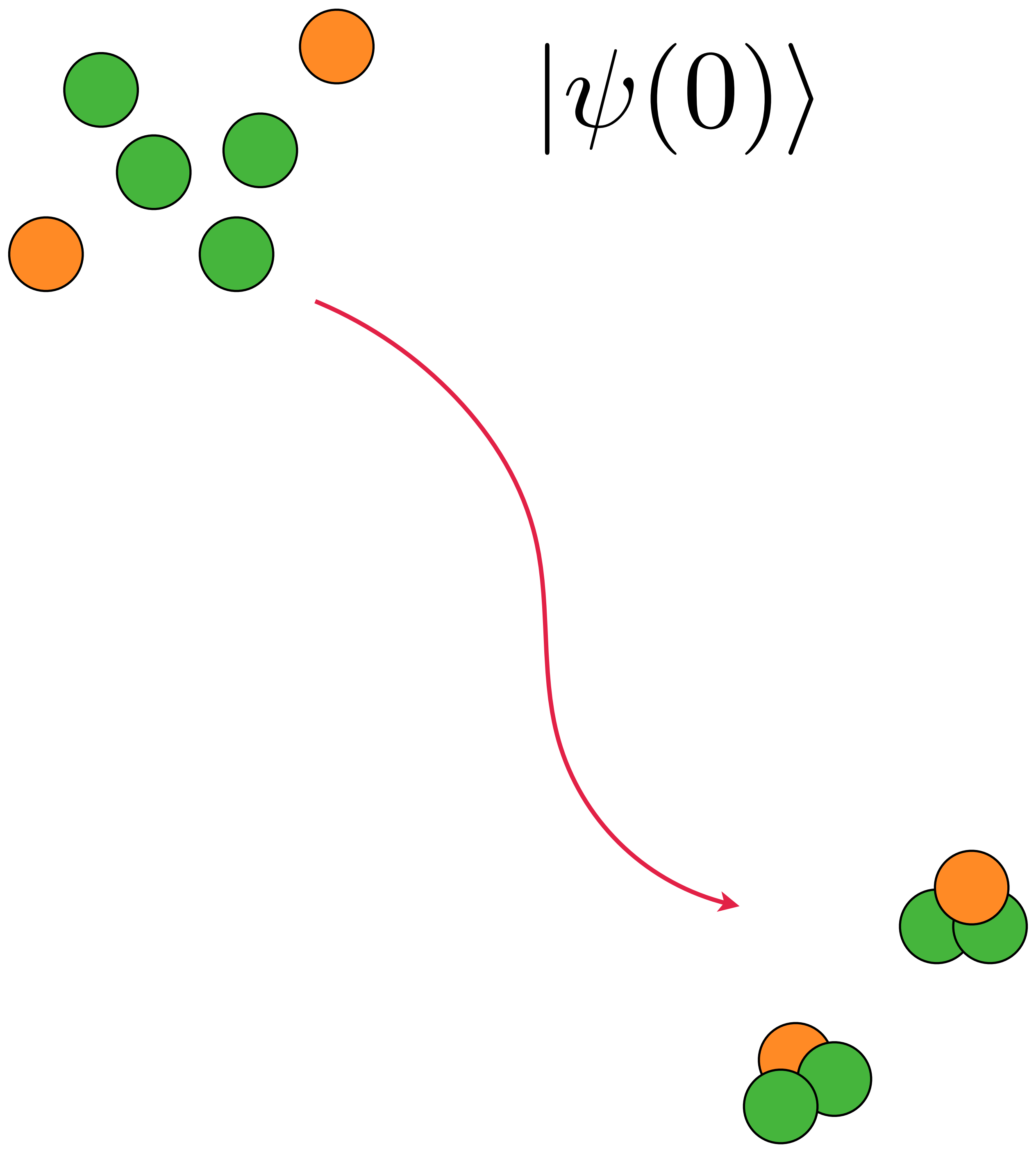
$$|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$$

Statics

Quantum Simulation

Dynamics

$$|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$$



$|\psi(0)\rangle$

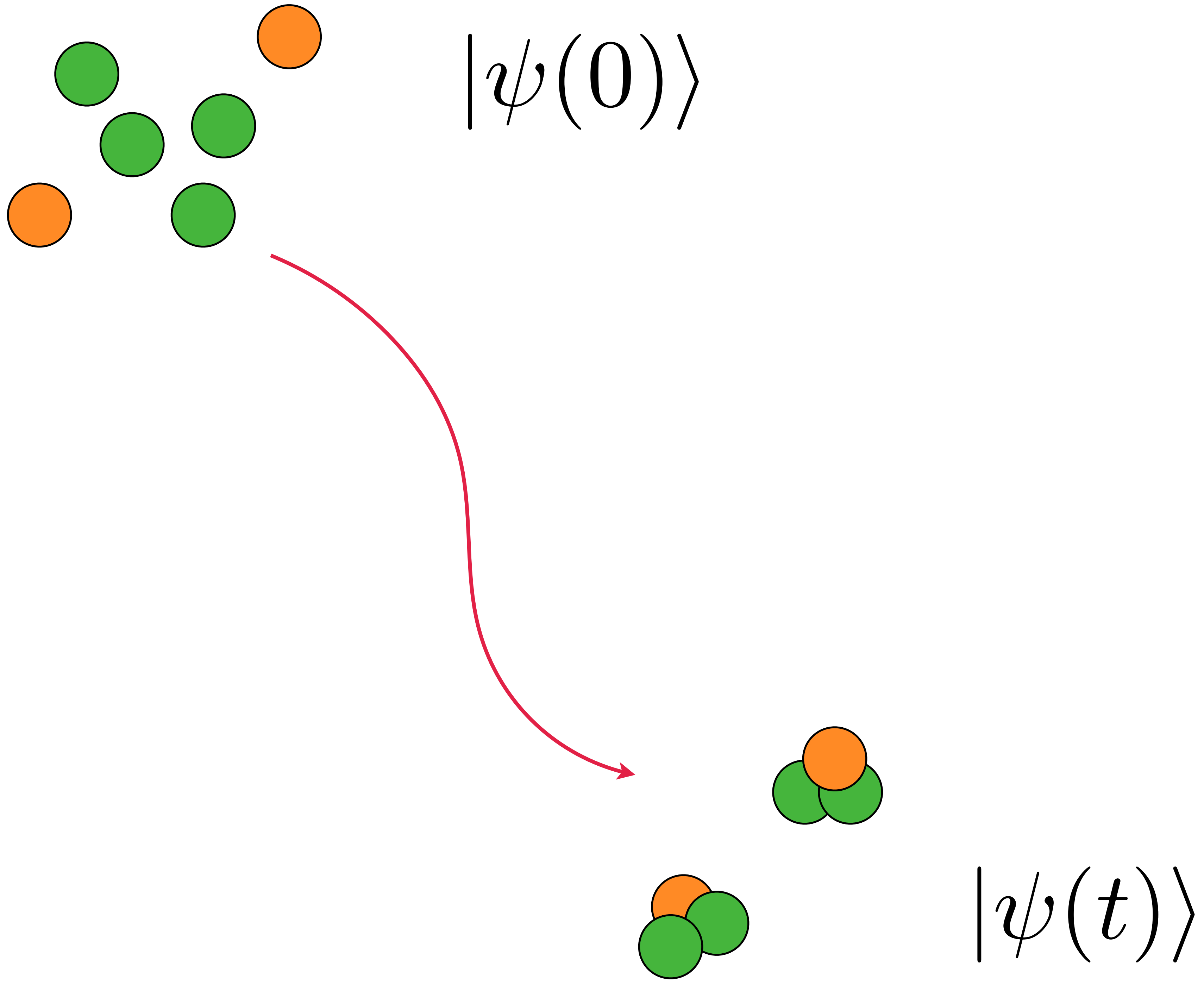
$|\psi(t)\rangle$

Statics

Quantum Simulation

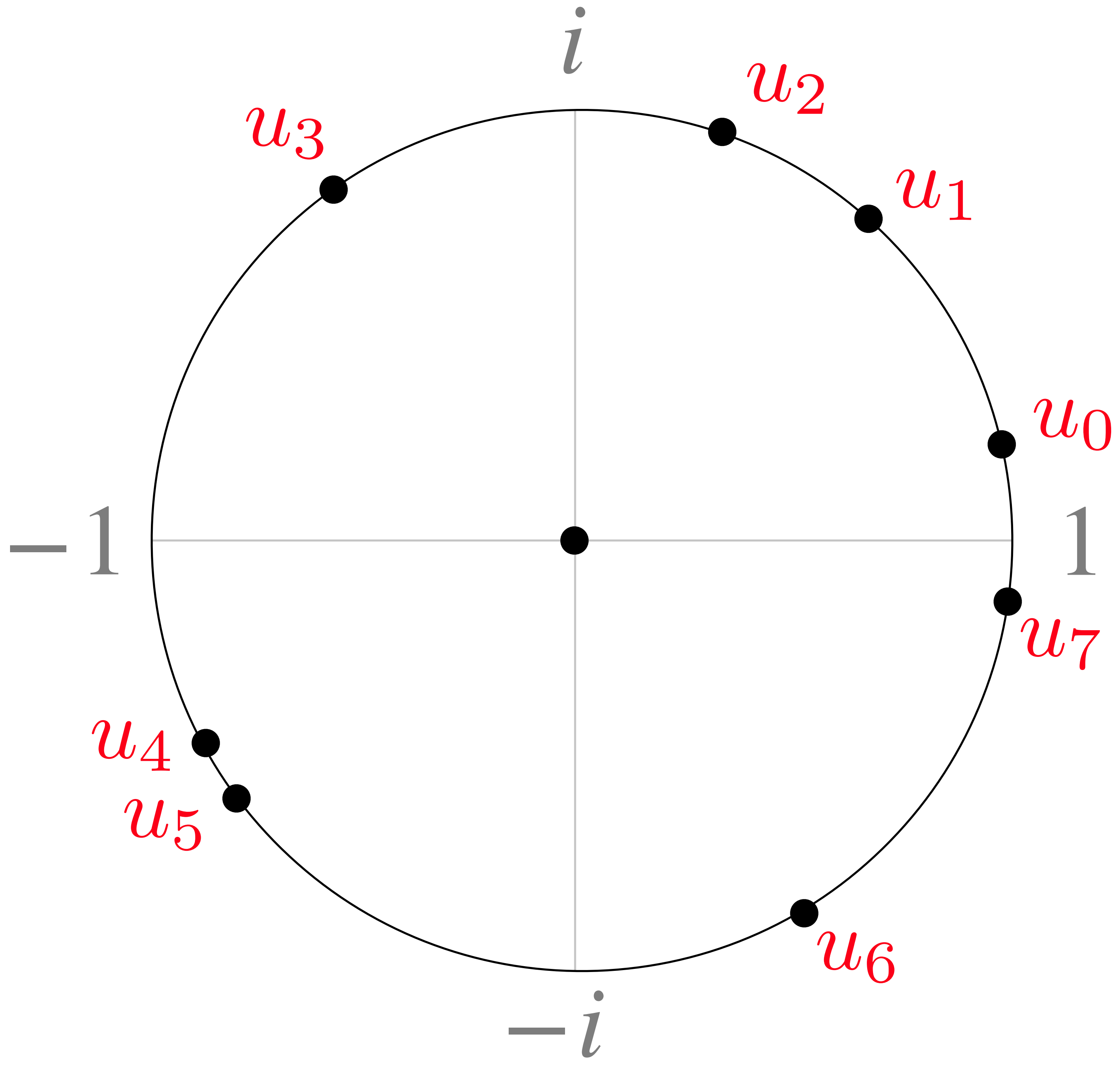
Dynamics

$$|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$$

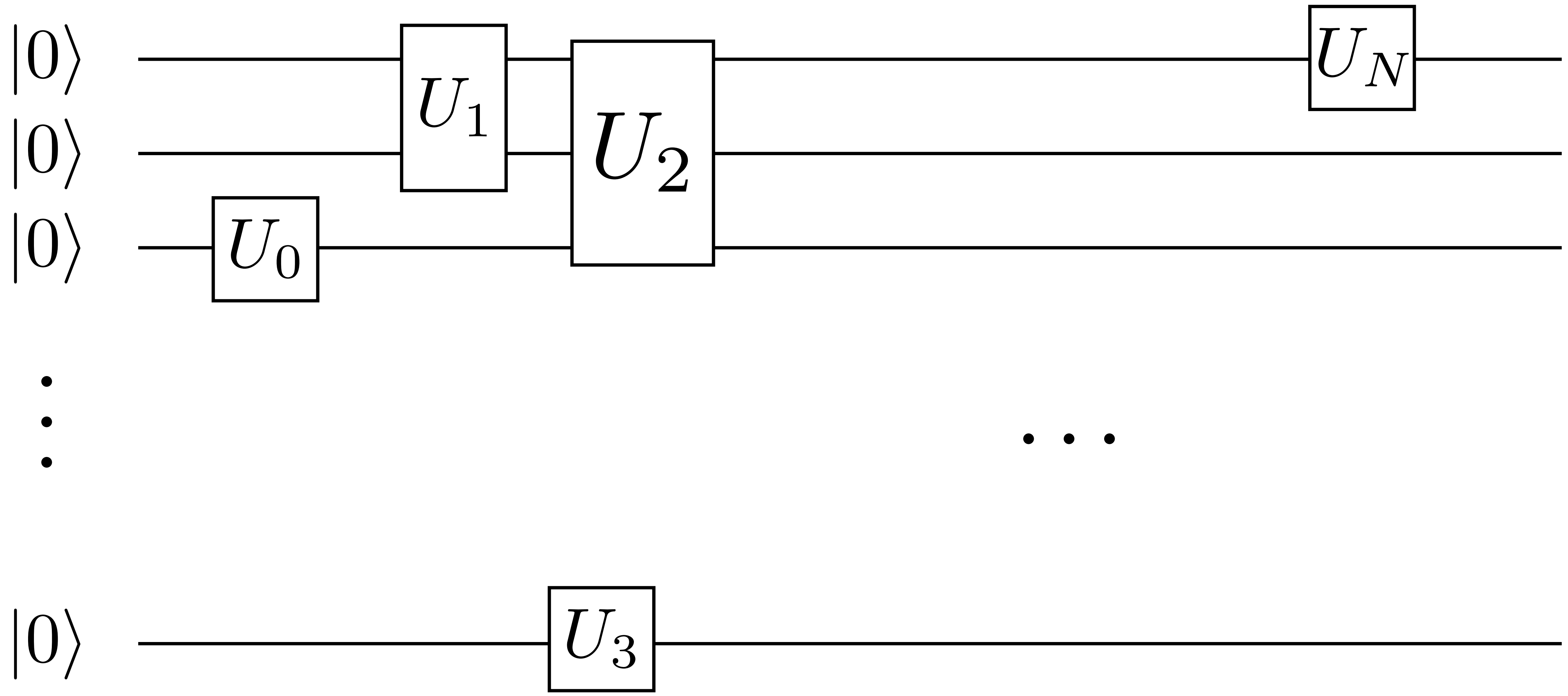


Statics

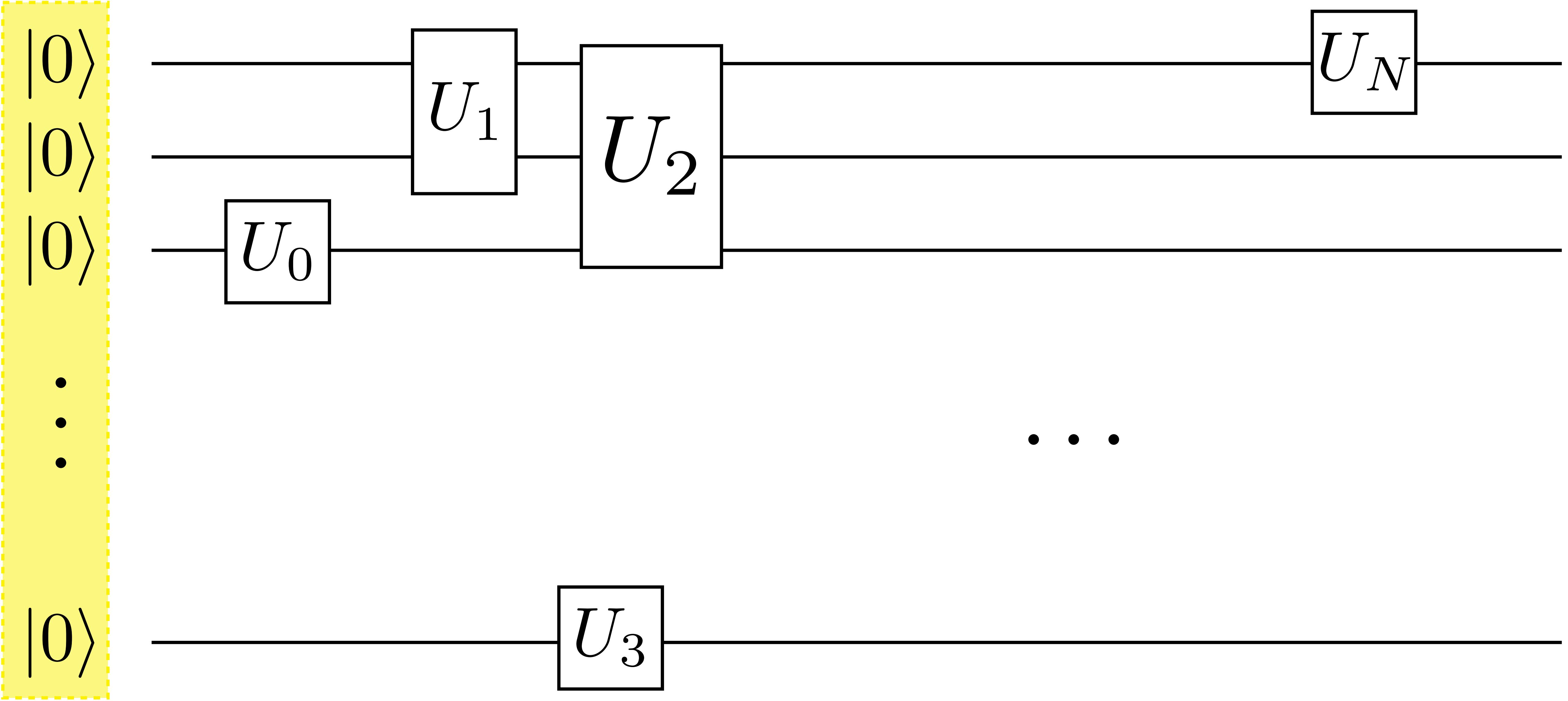
$$U|u\rangle = u\,|u\rangle \qquad |u| = 1$$



Quantum Resource Estimates

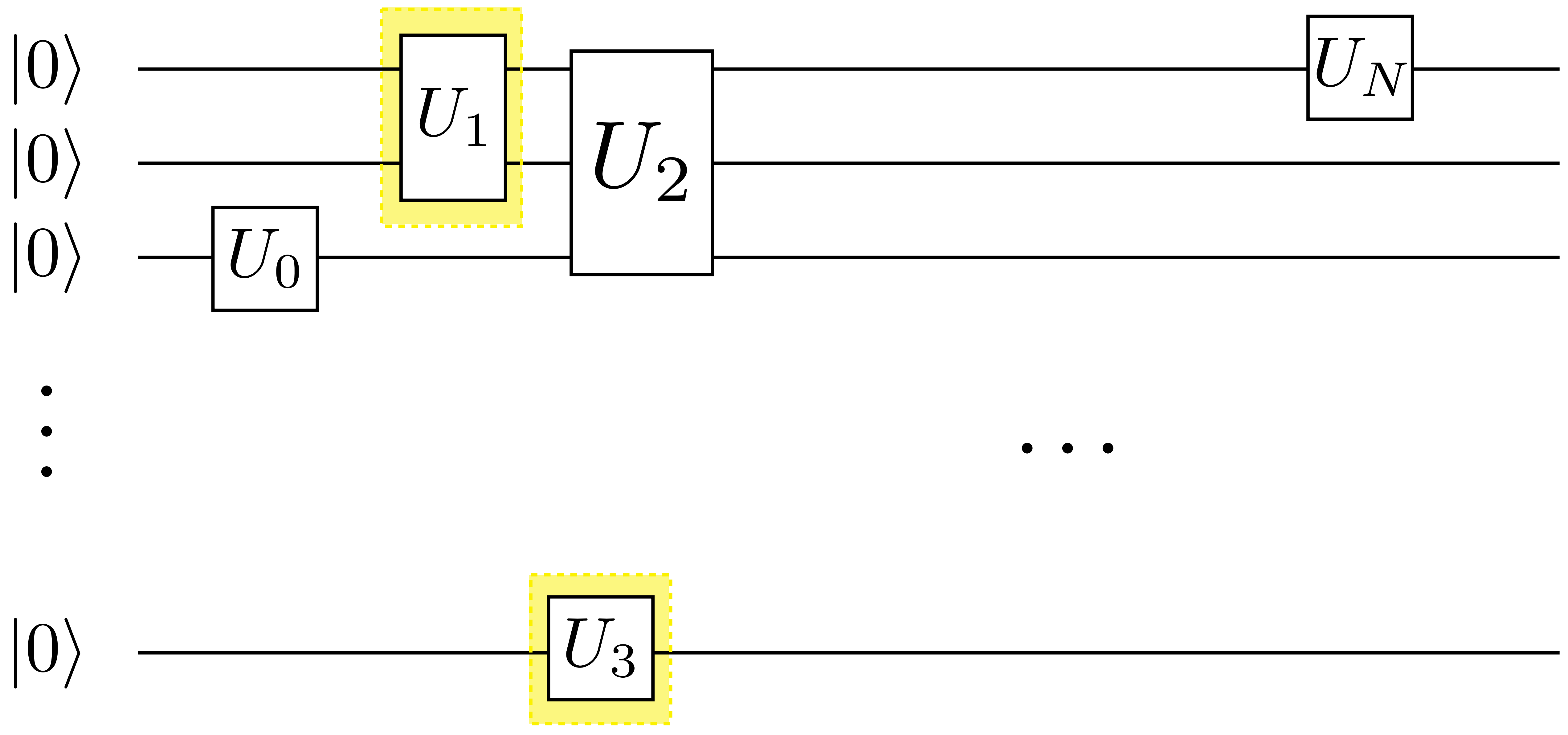


Quantum Resource Estimates



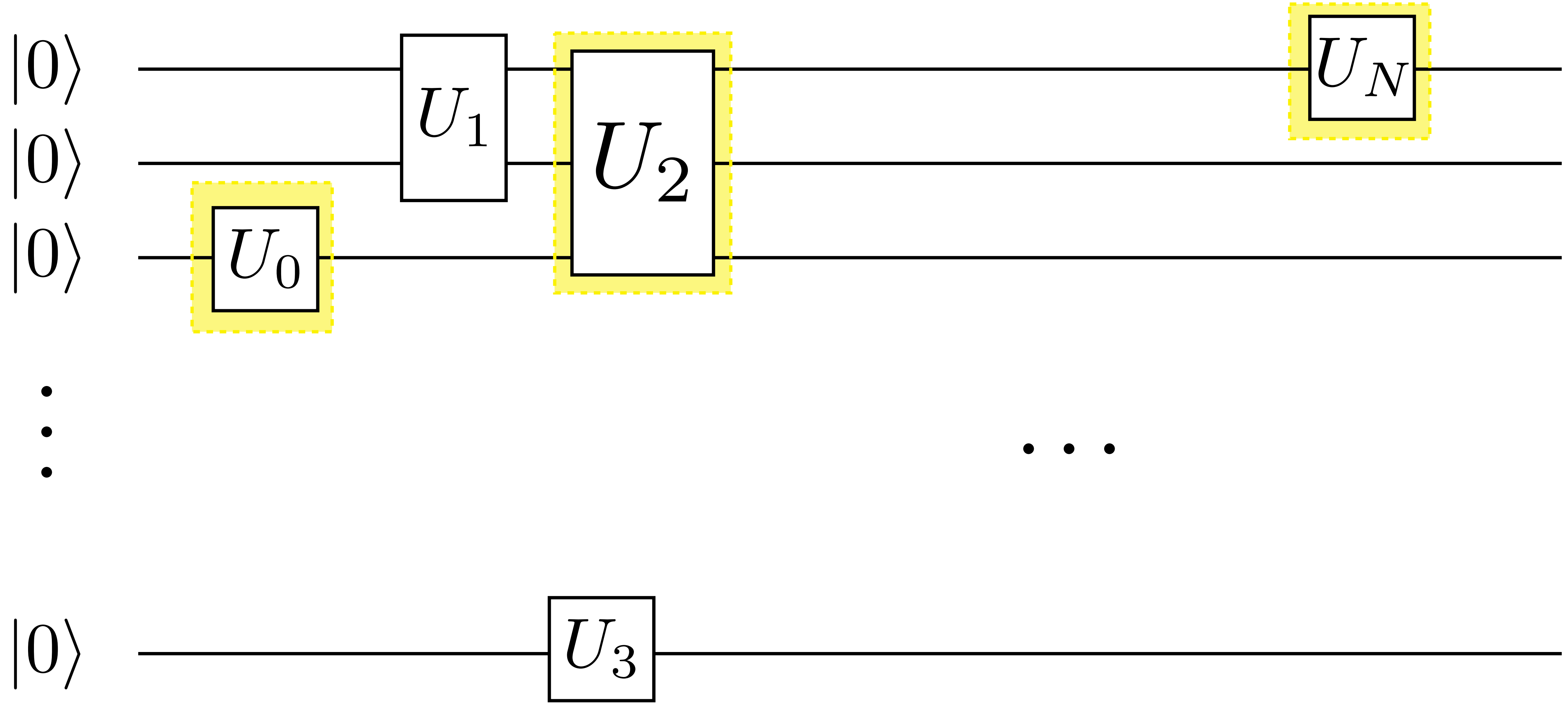
1) Number of qubits

Quantum Resource Estimates



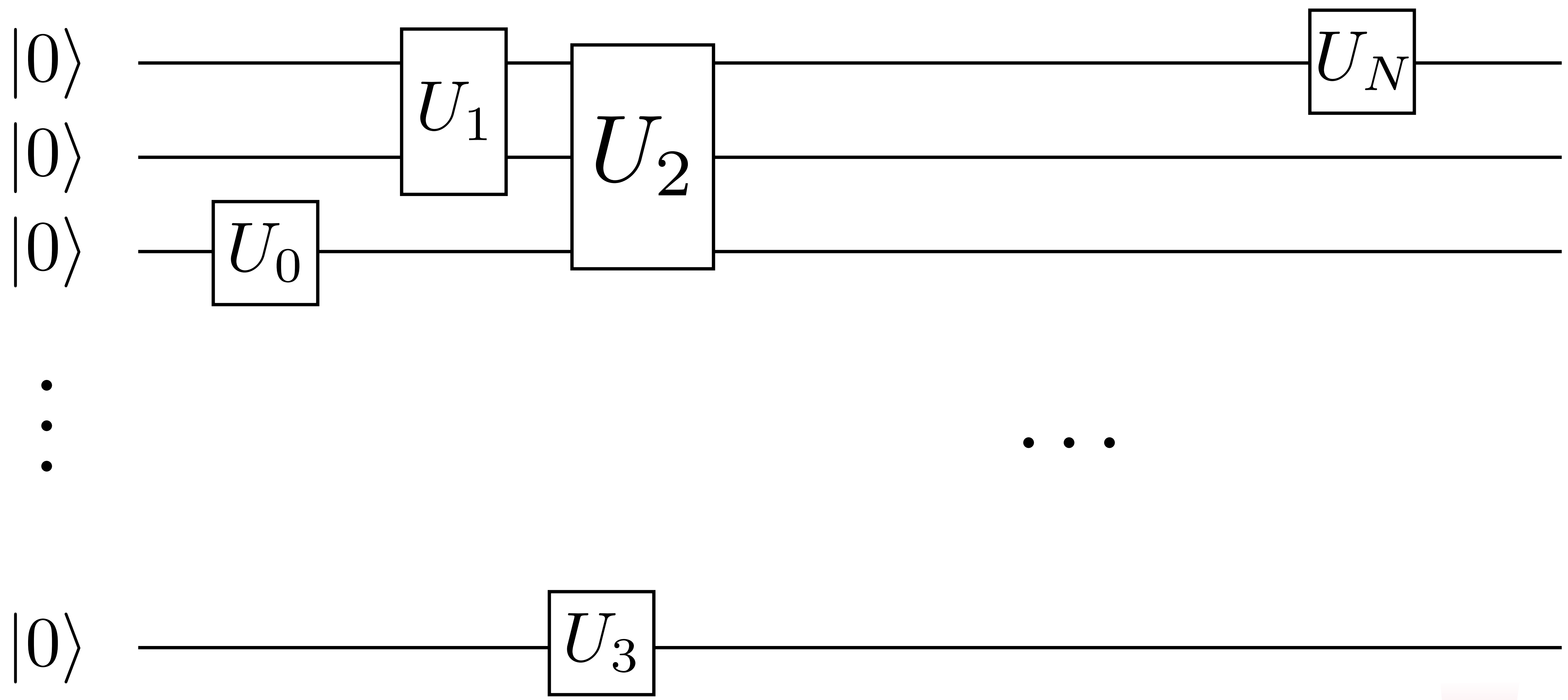
1) Number of qubits
2) Clifford Gates

Quantum Resource Estimates



1) Number of qubits
2) Clifford Gates
3) Non-Clifford gates

Quantum Resource Estimates



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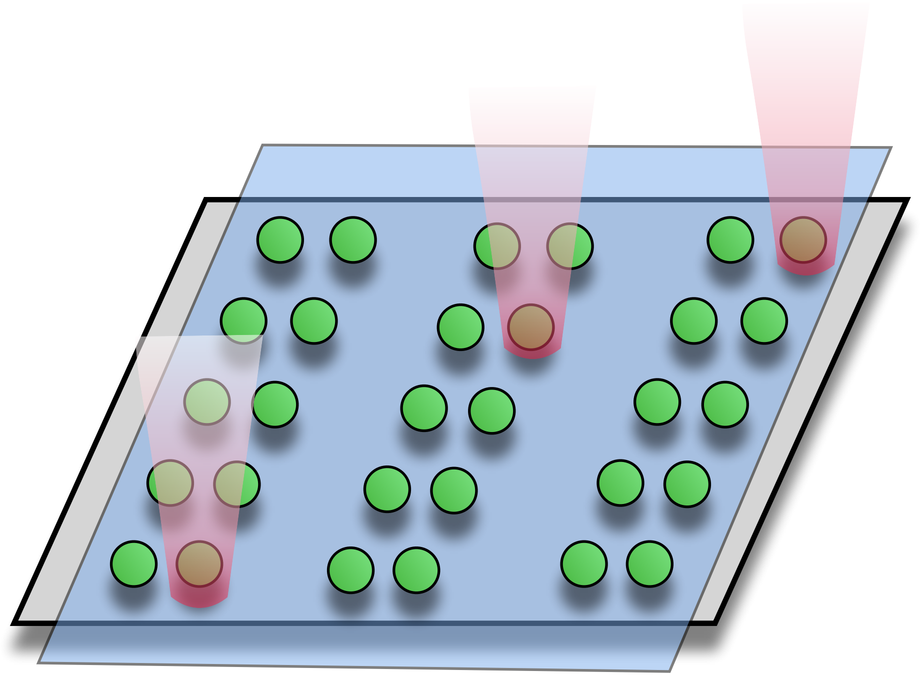
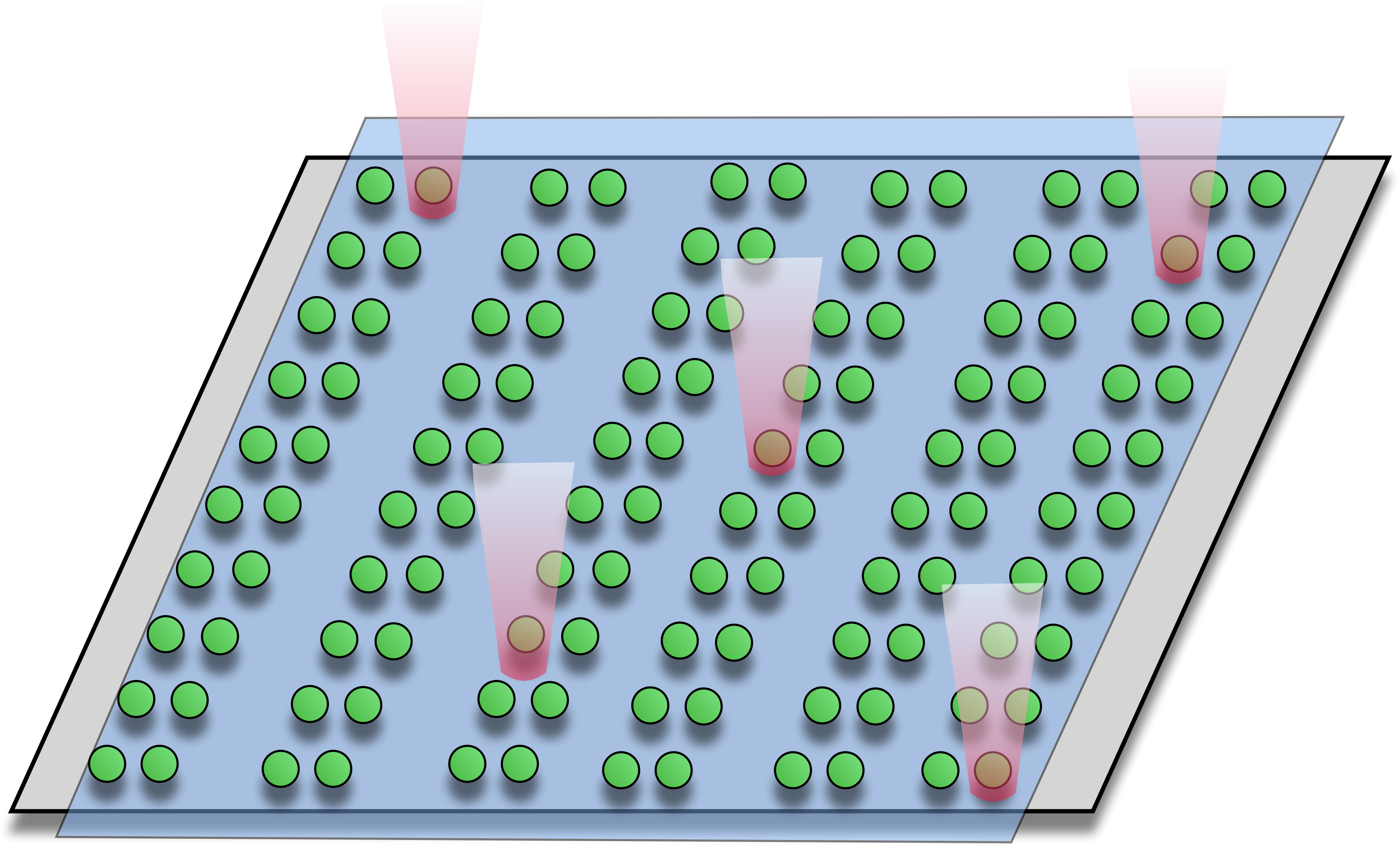


vs.



An end-to-end framework for quantum simulation of QFTs

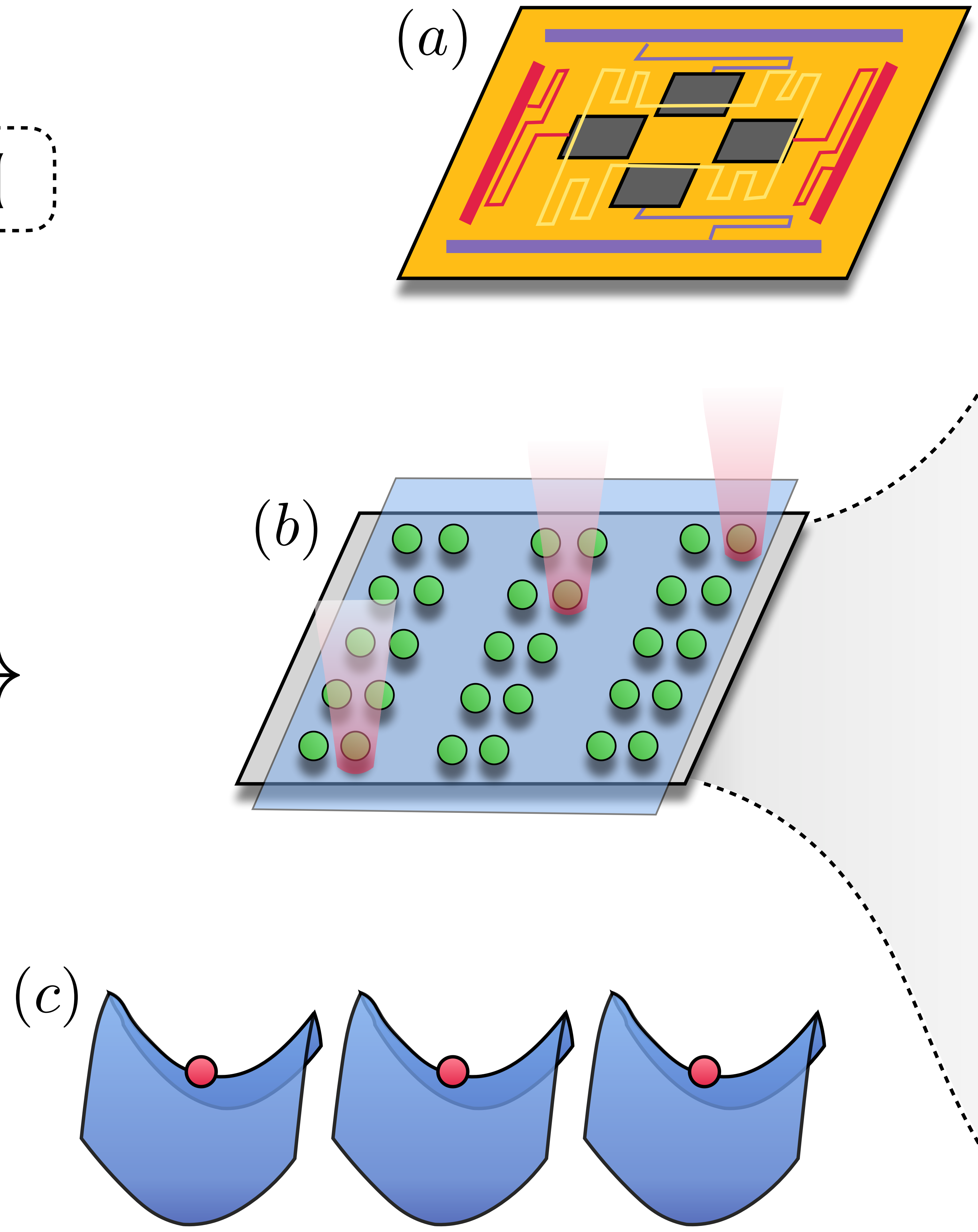
$\mathcal{L}_{\text{QFT}} \rightarrow$? Part I

$\downarrow$

H_{QFT}

$\downarrow$

Part II ? $\rightarrow$

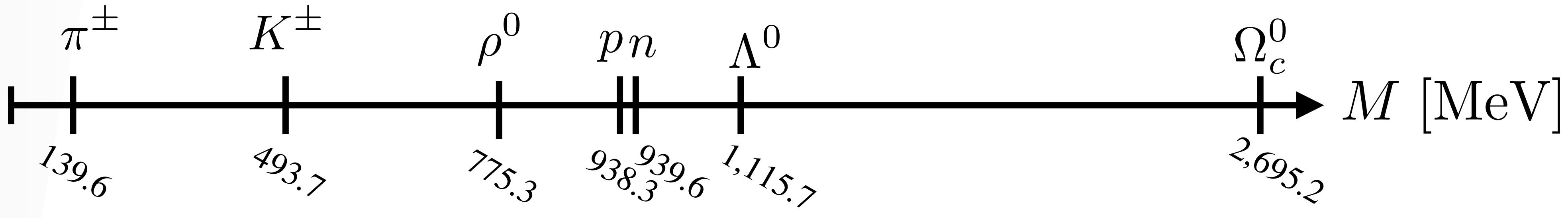


(a) Superconducting

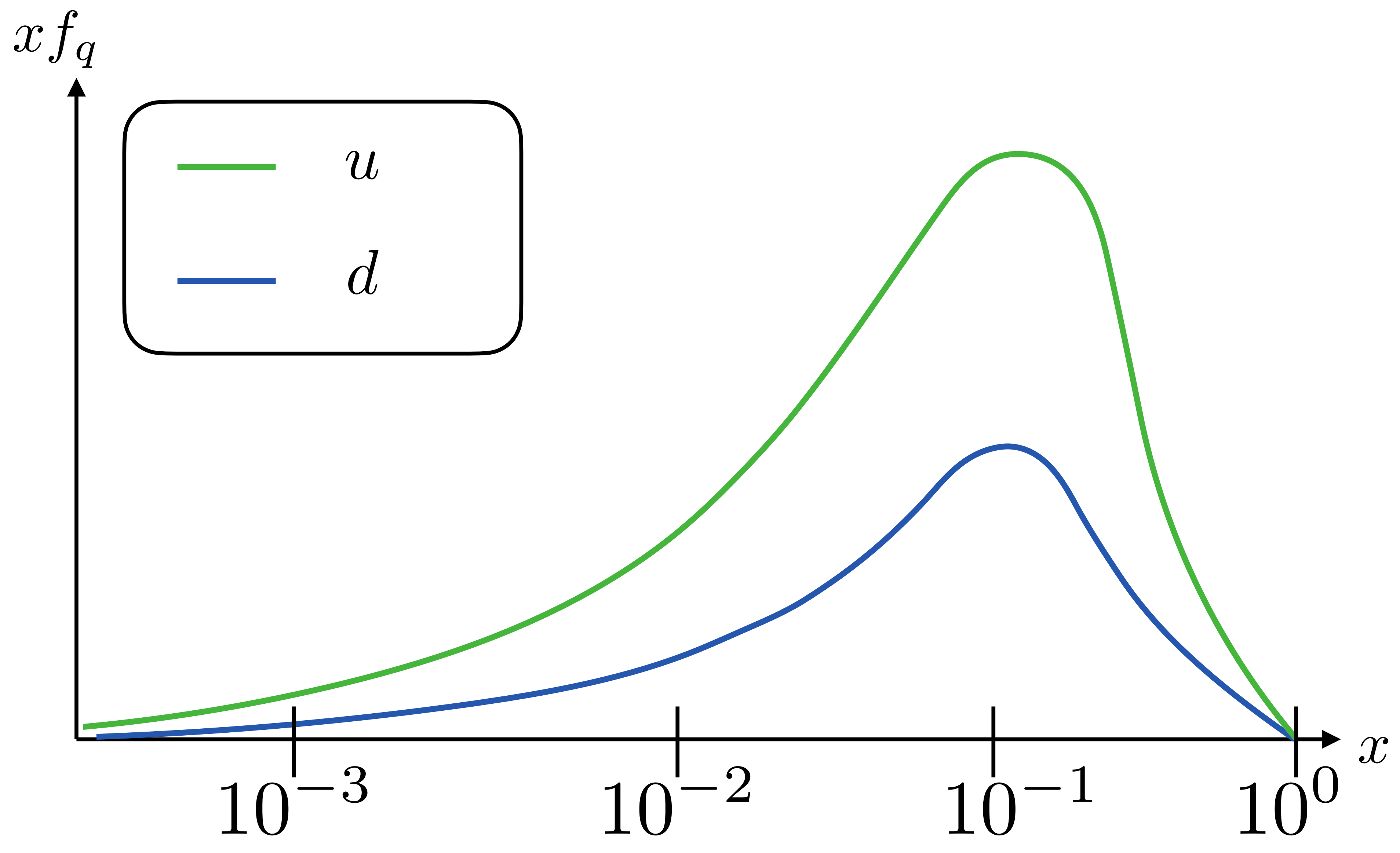
(b) Neutral/Cold Atom

(c) Ion Trap

Hadronic Mass Spectra



Hadronic Distribution Functions



Prelude Motivation

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Constructing Canonical QFT Hamiltonians

Constructing Canonical QFT Hamiltonians

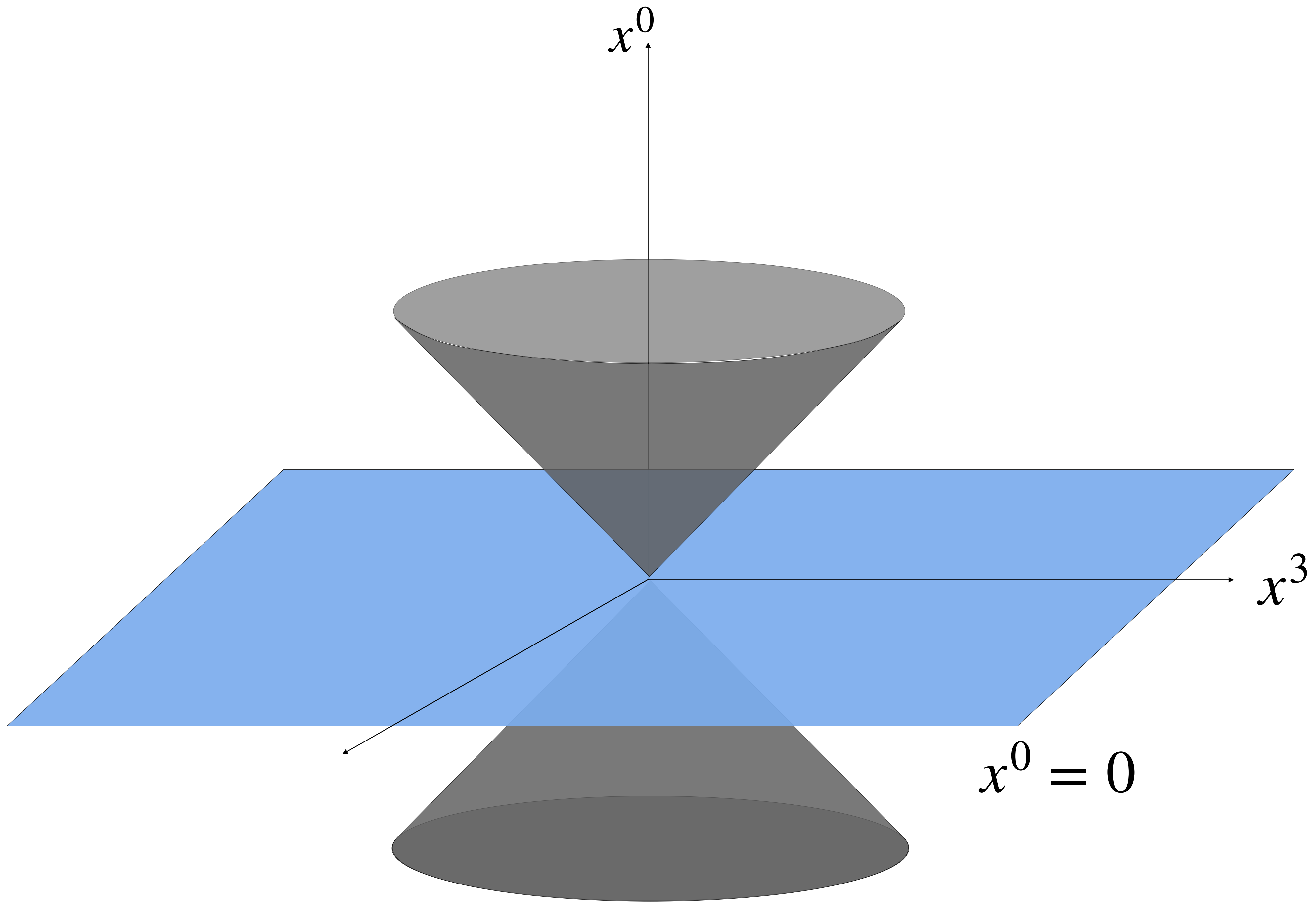
(Lagrangian) $\mathcal{L} \rightarrow \left\{ \begin{array}{c} \text{Noether's theorem} \\ x^\mu \rightarrow x^\mu + \delta x^\mu \end{array} \right\} \rightarrow T^{\mu\nu} \rightarrow \int dV T^{00} = H$ (Hamiltonian)

YES

NO

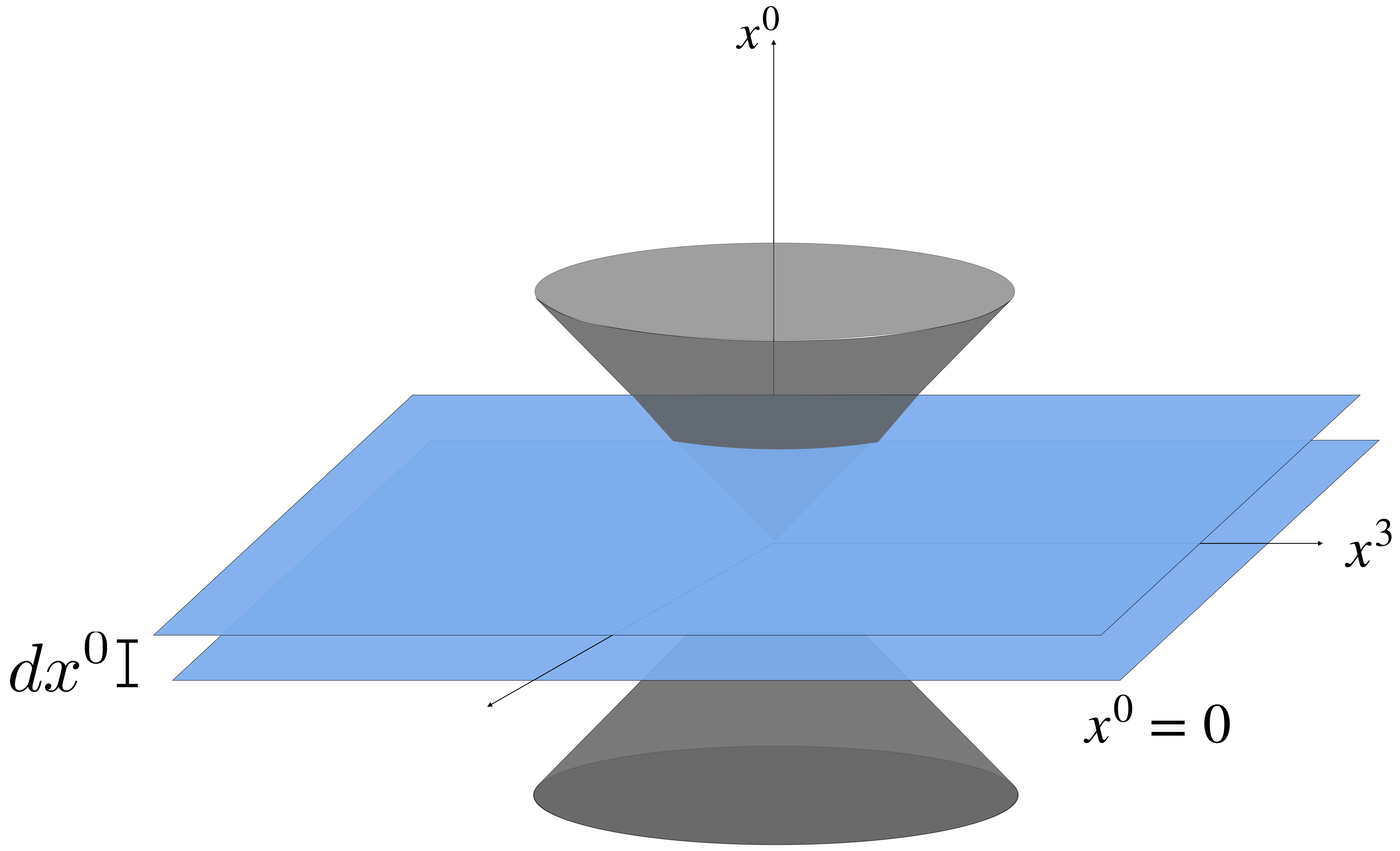
Lorentz Invariant
Quantity?

Forms of Relativistic Dynamics



(a) Instant Form $x^\mu = (x^0, x^1, x^2, x^3)$

Forms of Relativistic Dynamics



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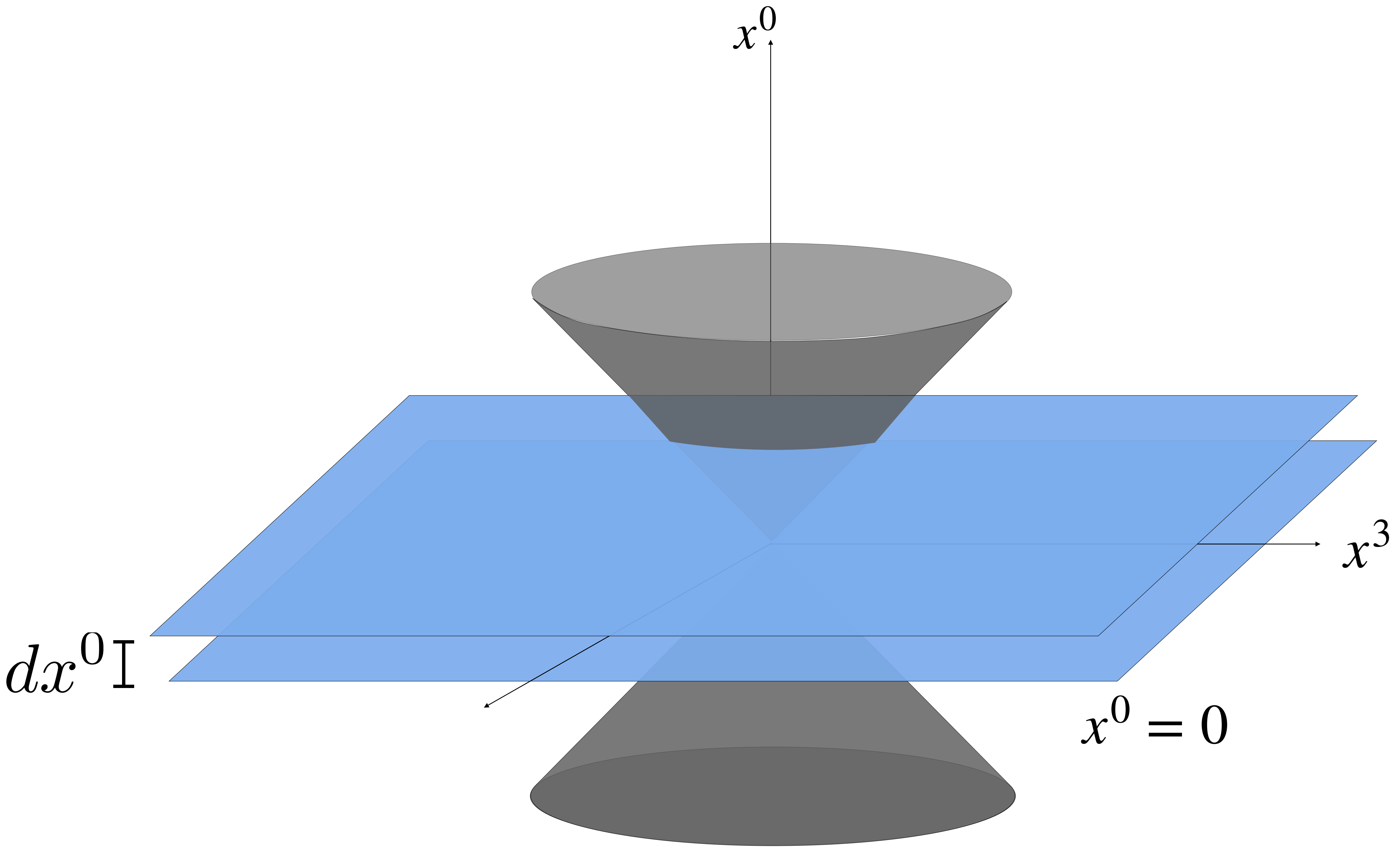

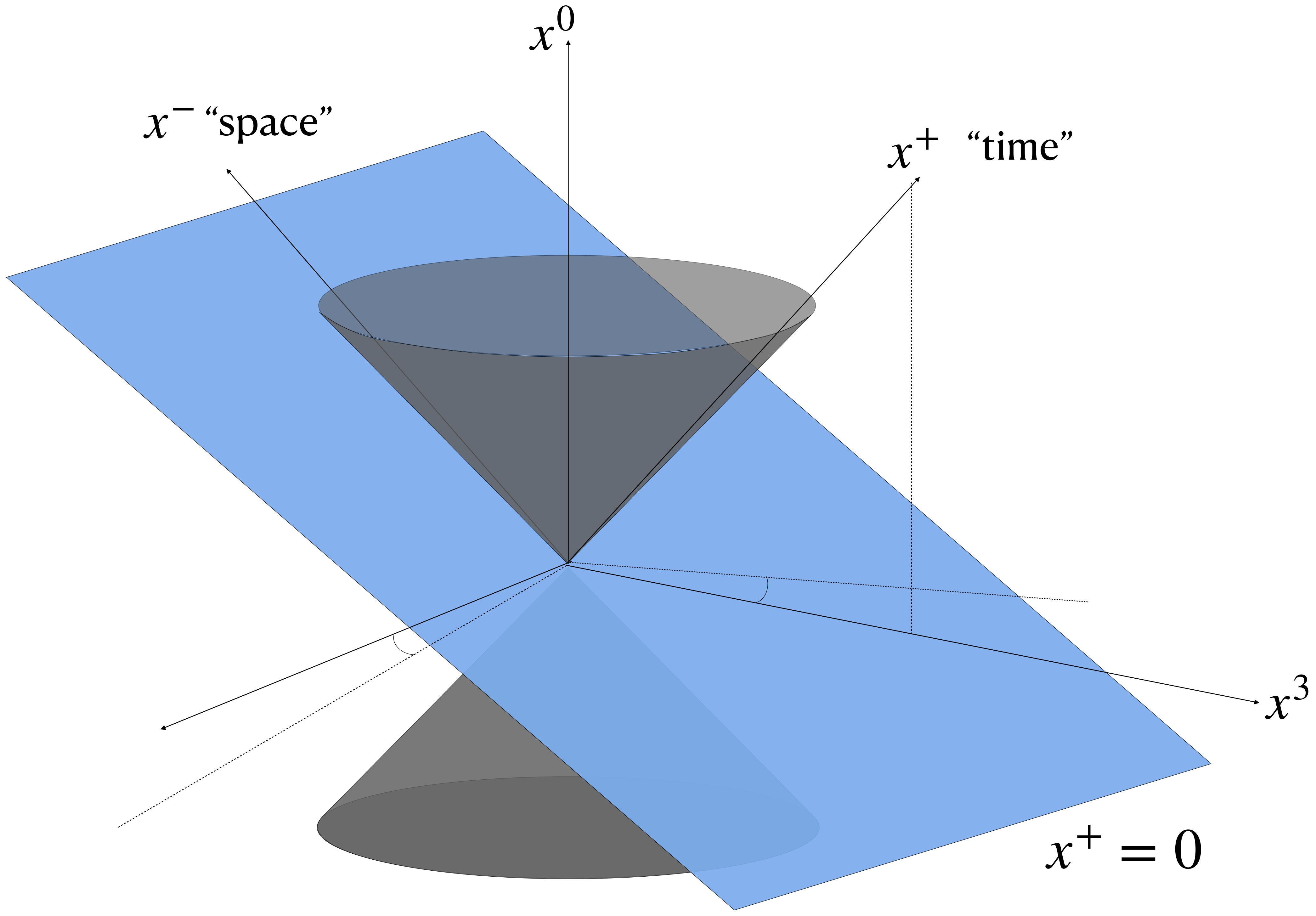

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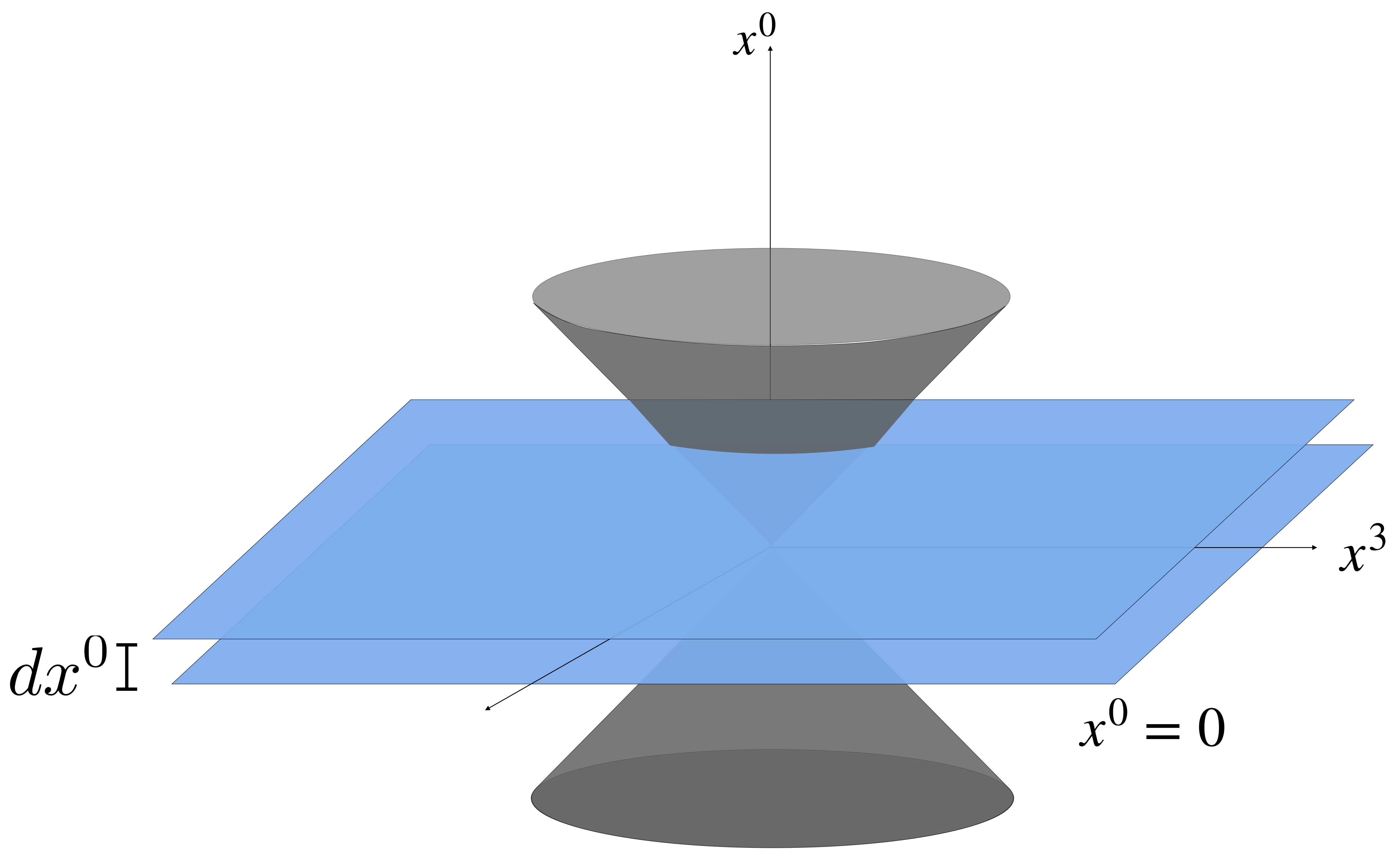
Forms of Relativistic Dynamics

$$\begin{cases} x^+ \equiv x^0 + x^3 \\ x^- \equiv x^0 - x^3 \end{cases}$$



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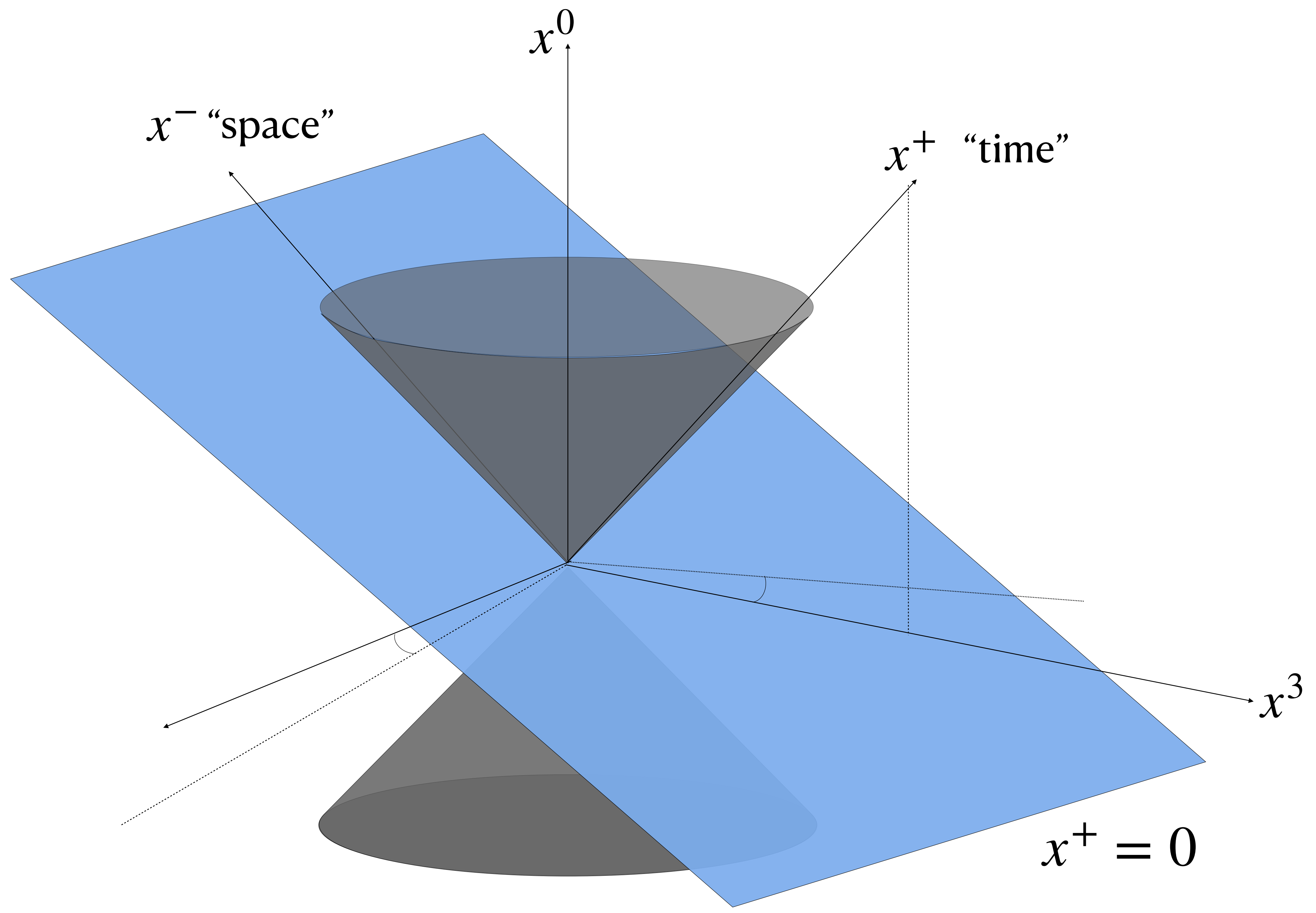

(b) Front Form $x^\mu = (x^+, x^-, \underbrace{x_1, x_2}_{\vec{x}^\perp})$
“Lightfront”

Forms of Relativistic Dynamics. Dirac

Forms of Relativistic Dynamics

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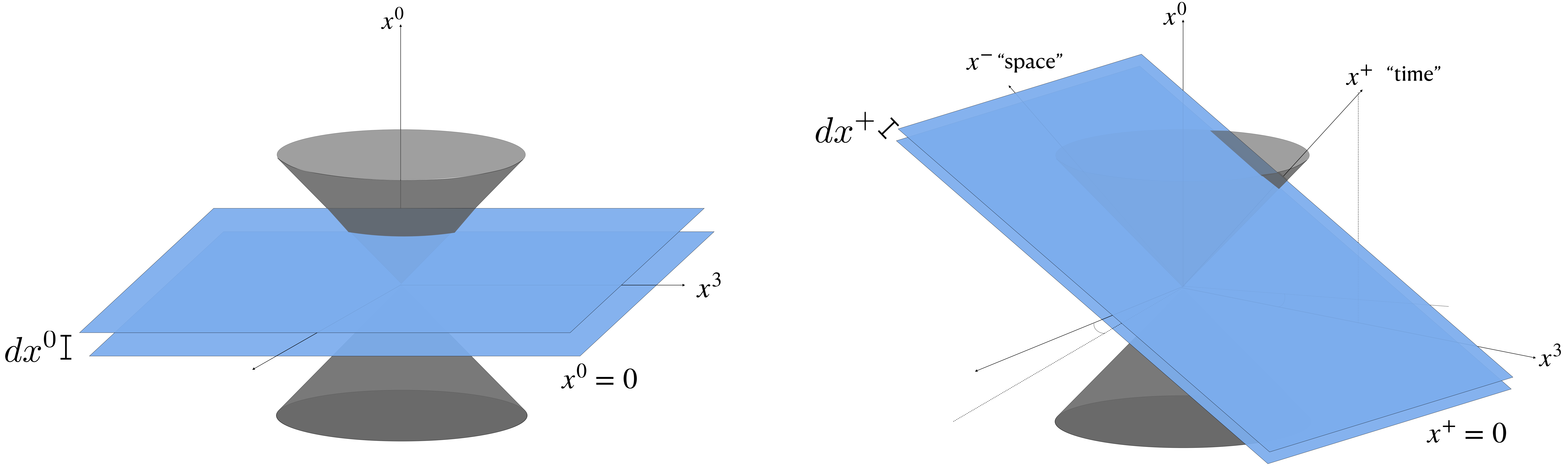

(a) Instant Form $x^\mu = (x^0, x^1, x^2, x^3)$

(b) Front Form $x^\mu = (x^+, x^-, \underbrace{x_1, x_2}_{\vec{x}^\perp})$

"Lightfront"

Energy-Momentum Four Vector

Instant Form

Front Form

Energy-Momentum Four Vector

Instant Form

$$P^\mu = \left(\boxed{P^0}, \boxed{P^1, P^2, P^3} \right)$$

$$(P^0)^2 - (\vec{P})^2 = M^2$$

$$P^0|\psi\rangle = \sqrt{M^2 + \vec{P}^2}|\psi\rangle$$

Front Form

Energy-Momentum Four Vector

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$$P^-|\psi\rangle = M^2|\psi\rangle$$

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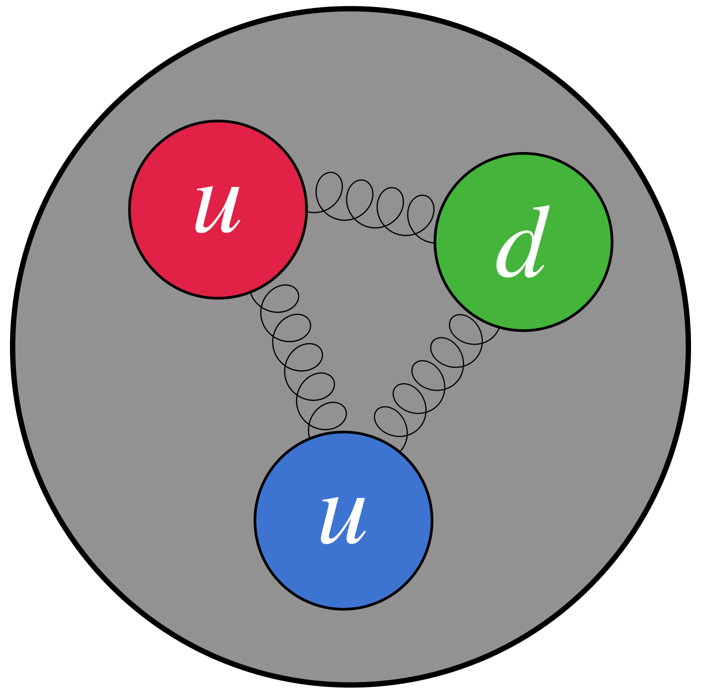
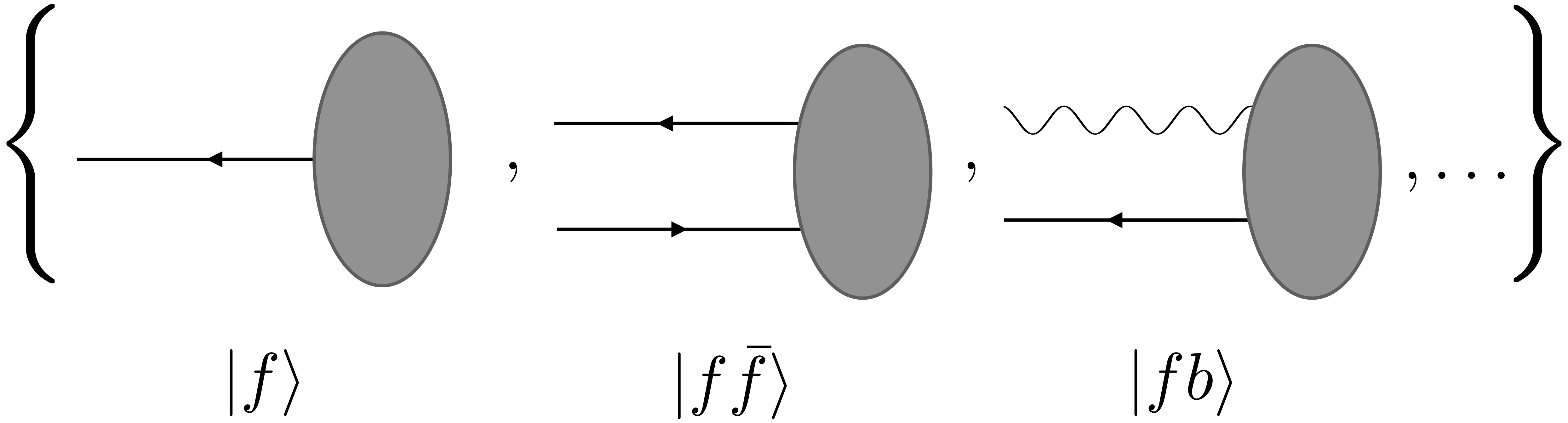
Eigenvectors ⟷ wavefunctions of bound states

Eigenvalues ⟷ M^2 of bound states



Fock Basis

$\mathcal{B} =$



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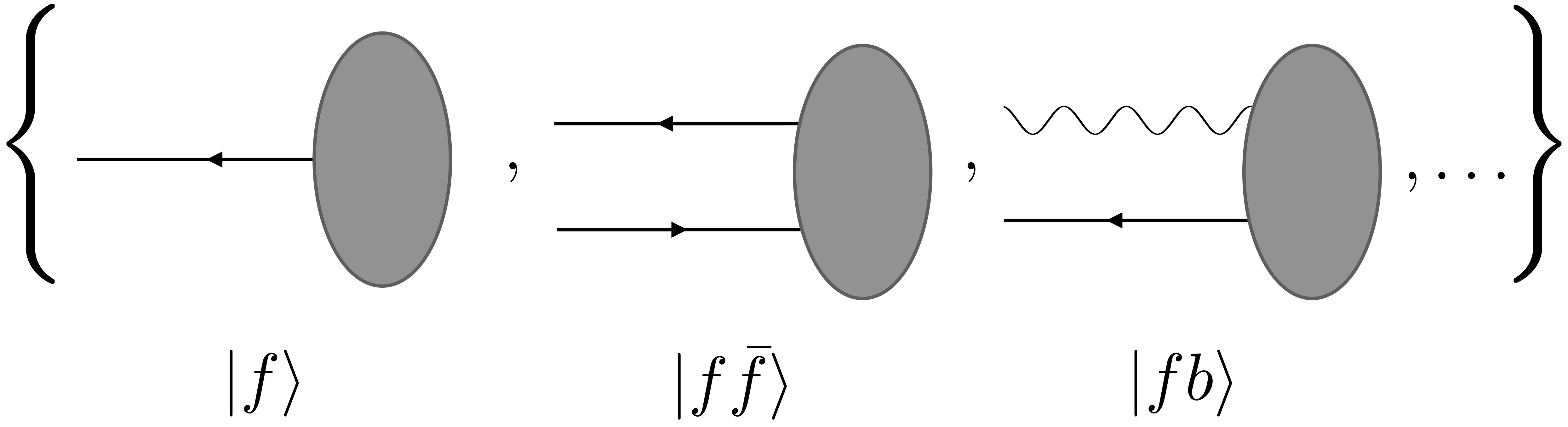


$$|f;\bar{f};b\rangle = |n_1^{m_1} n_2^{m_2} \dots n_N^{m_M}; \bar{n}_1^{\bar{m}_1} \bar{n}_2^{\bar{m}_2} \dots \bar{n}_{\bar{N}}^{\bar{m}_{\bar{N}}}; \tilde{n}_1^{\tilde{m}_1} \tilde{n}_2^{\tilde{m}_2} \dots \tilde{n}_{\tilde{N}}^{\tilde{m}_{\tilde{N}}}\rangle$$

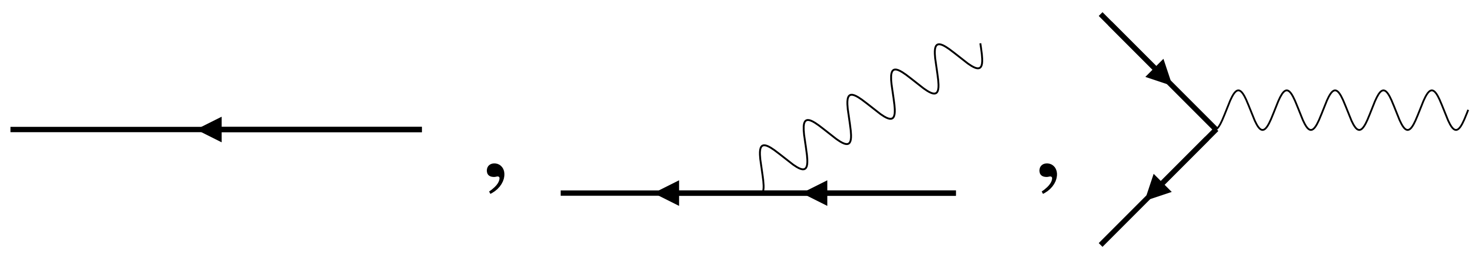
$$n_i = \left\{p_i^+, \vec{p}_i^\perp, h_i, f_i, c_i\right\}$$

$$\left.\begin{aligned} m_i &\in [0,1] \\ \bar{m}_i &\in [0,1] \end{aligned}\right\} \text{Fermi-Dirac Statistics}$$

$$\tilde{m}_i \in [0,1,2,\dots,\Omega] \text{ Bose Statistics}$$

The anatomy of the front form Hamiltonian

$$H = \sum_{l} \int_{1\ldots m} c_l(n_1, \ldots, n_m) O_l(n_1, \ldots, n_m) \qquad O_l \in \Big\{$$

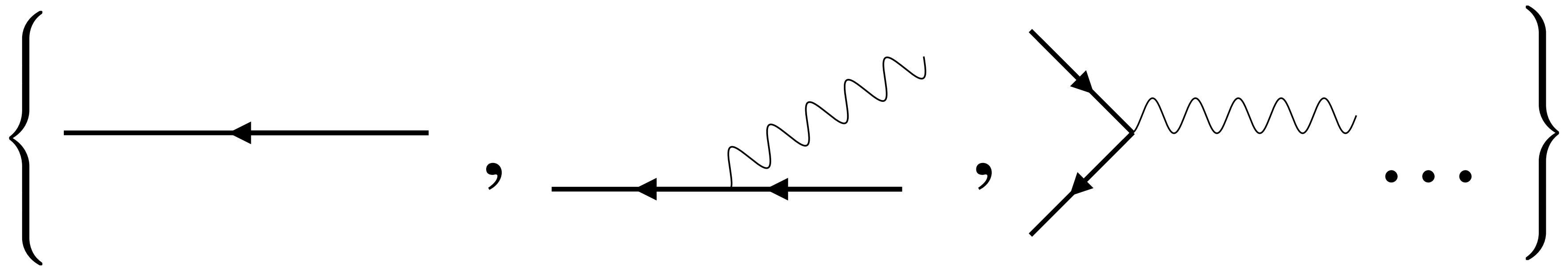


$$\ldots \Big\}$$

Quantum Chromodynamics and Other Field Theories on the Light Cone. Brodsky et. al

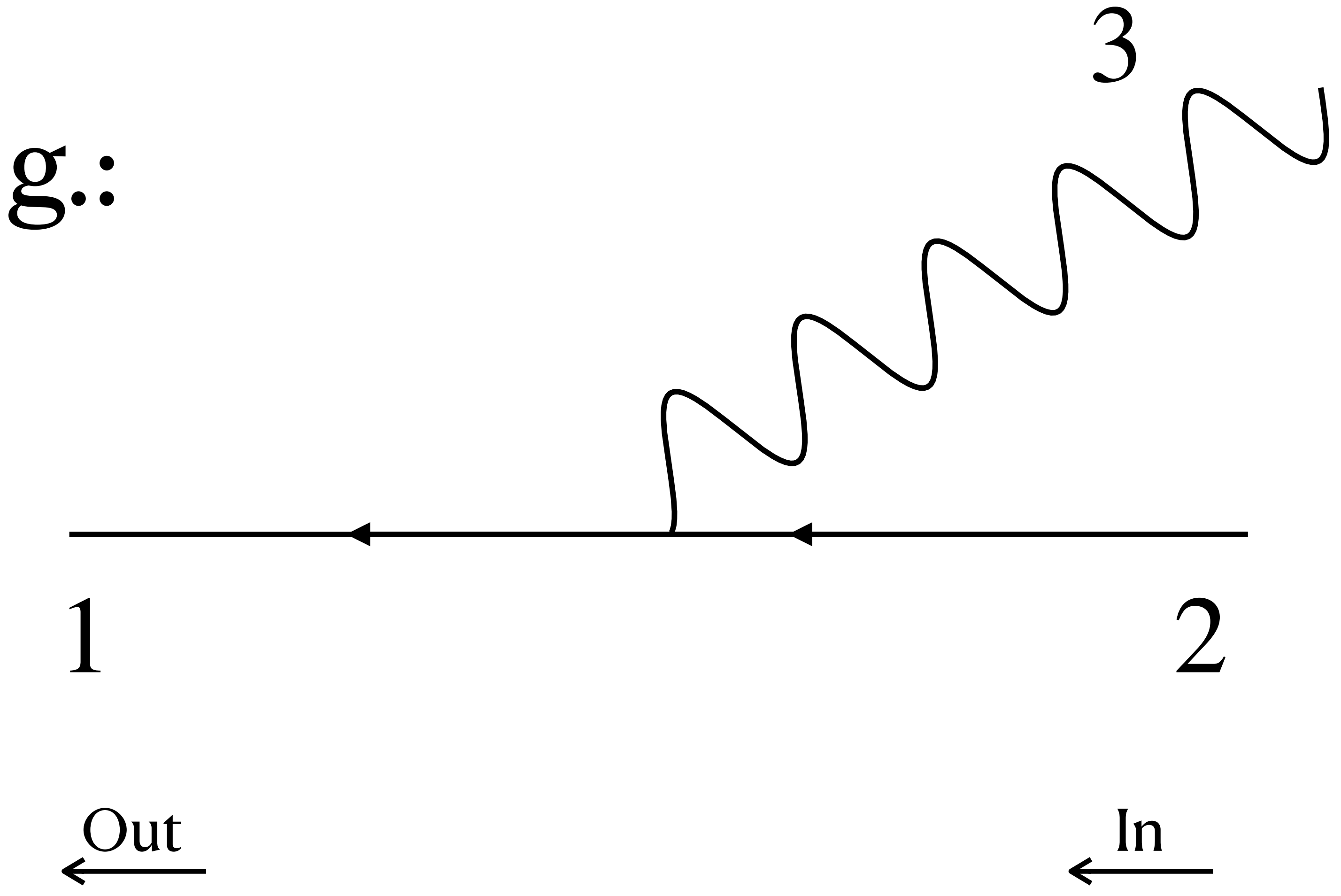
The anatomy of the front form Hamiltonian

$$H = \sum_l \int_{1\ldots m} c_l(n_1,\ldots,n_m)\,O_l(n_1,\ldots,n_m) \qquad O_l \in \Big\{$$



$$\ldots \Big\}$$

e.g.:



Coupling

$$\sim g \int_{123} \underbrace{\bar{u}_1 \gamma^\mu u_2 \epsilon^\sigma_{3\mu}}_{\text{Coefficient ("}c_l\text{")}} \overbrace{b_1^\dagger b_2 a_3}^{\text{Ladder Operators ("}O_l\text{")}}$$

Ladder Operators ("O_l")

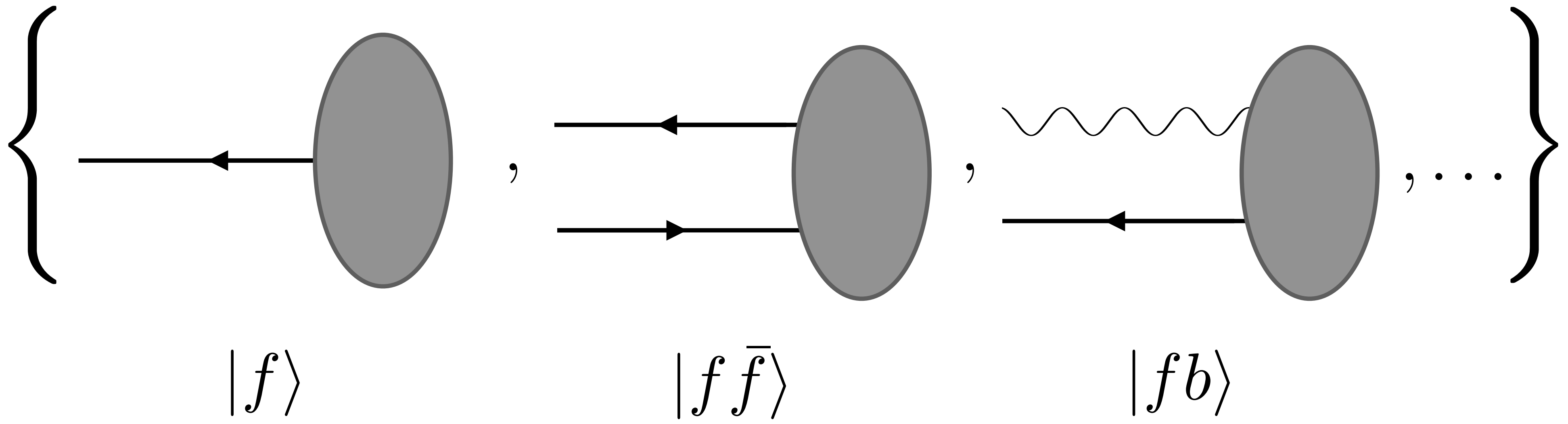
Coefficient ("c_l")

Integrate over all constituent momentum $dp_i^+ d^2\vec{p}_i^\perp$

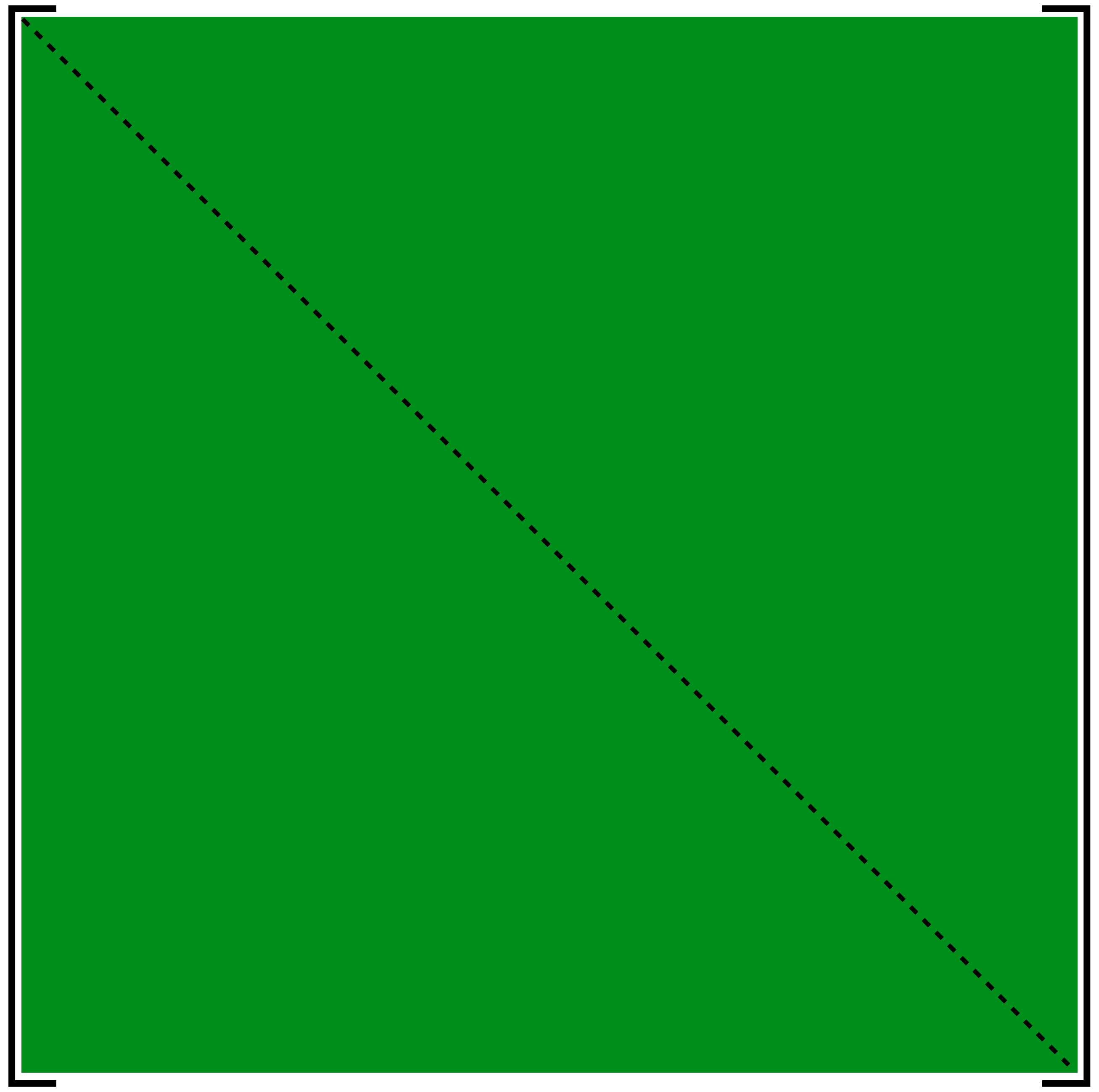
Quantum Chromodynamics and Other Field Theories on the Light Cone. Brodsky et. al

Front Form Hamiltonian Matrix

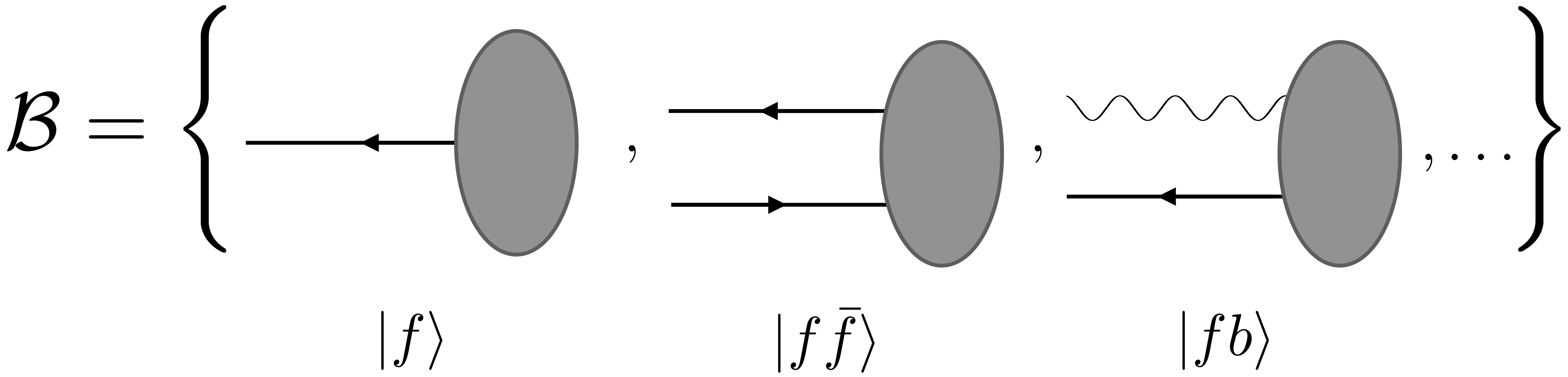
$\mathcal{B} =$



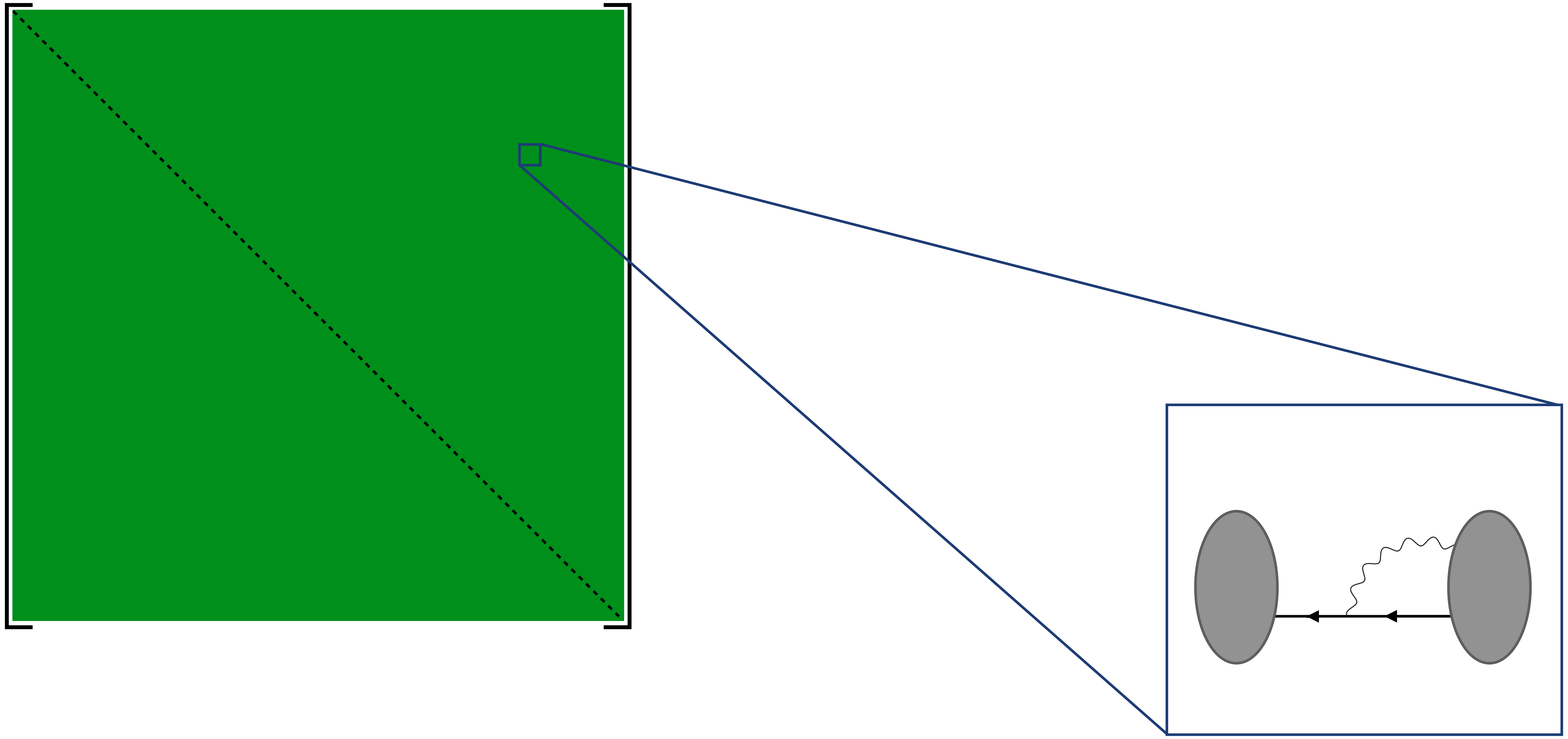
$H =$



Front Form Hamiltonian Matrix



$H =$



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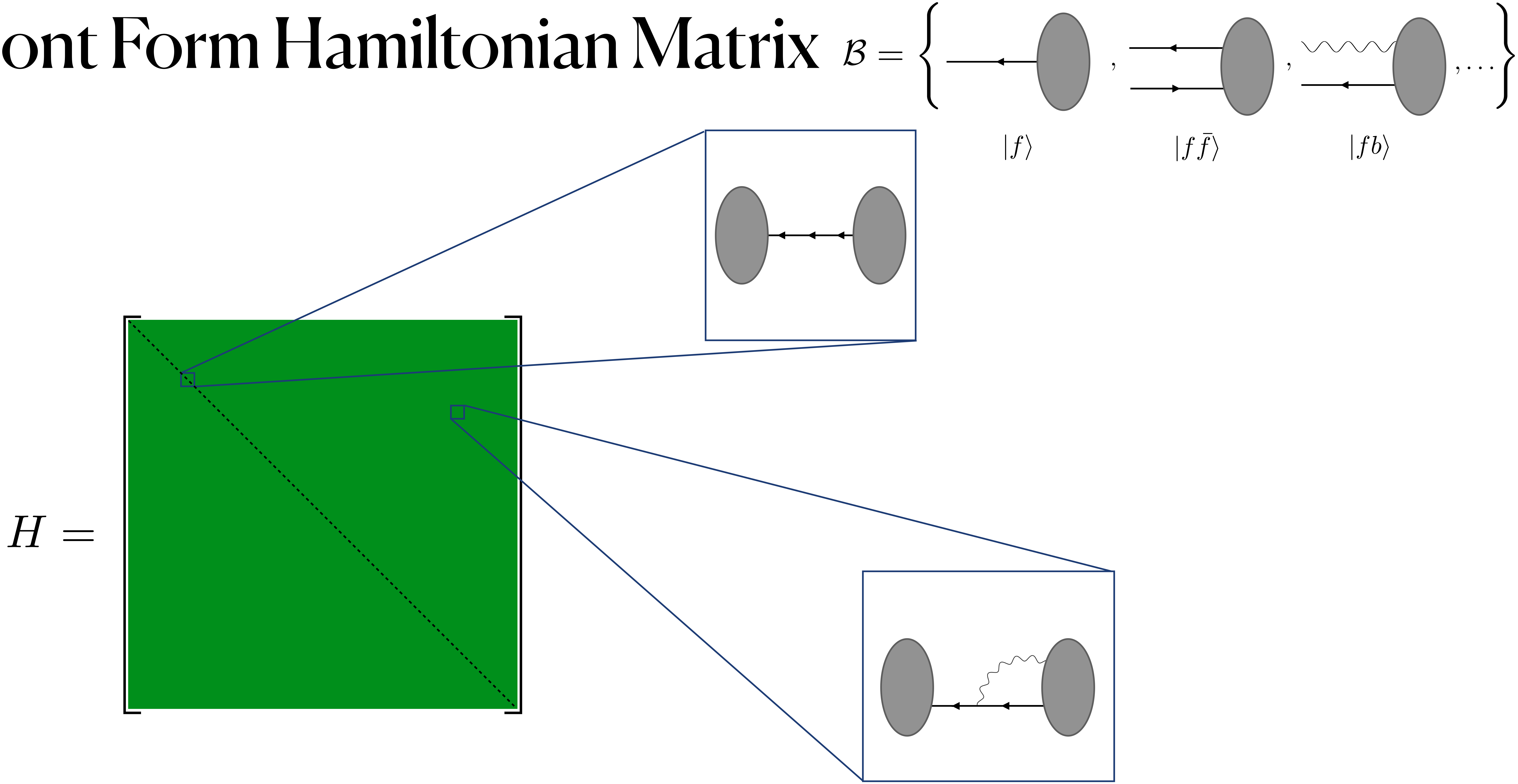

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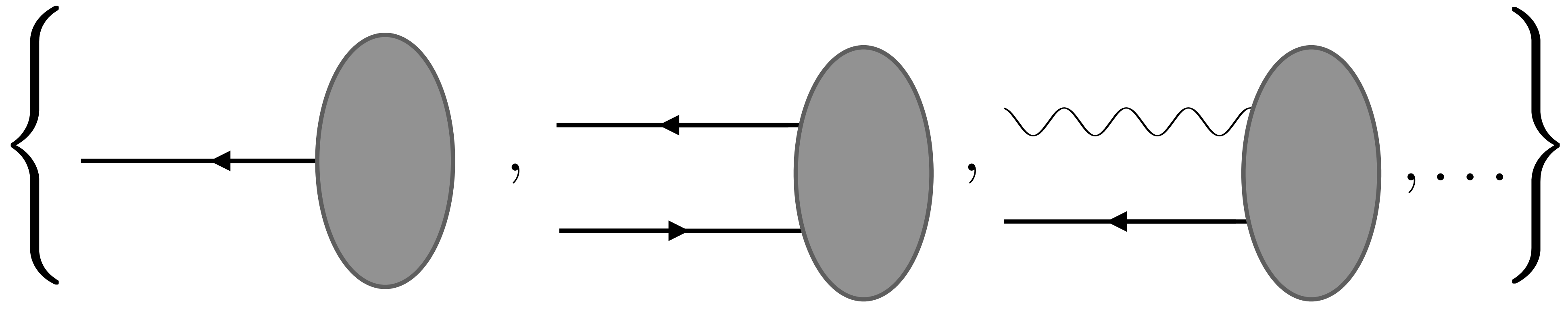
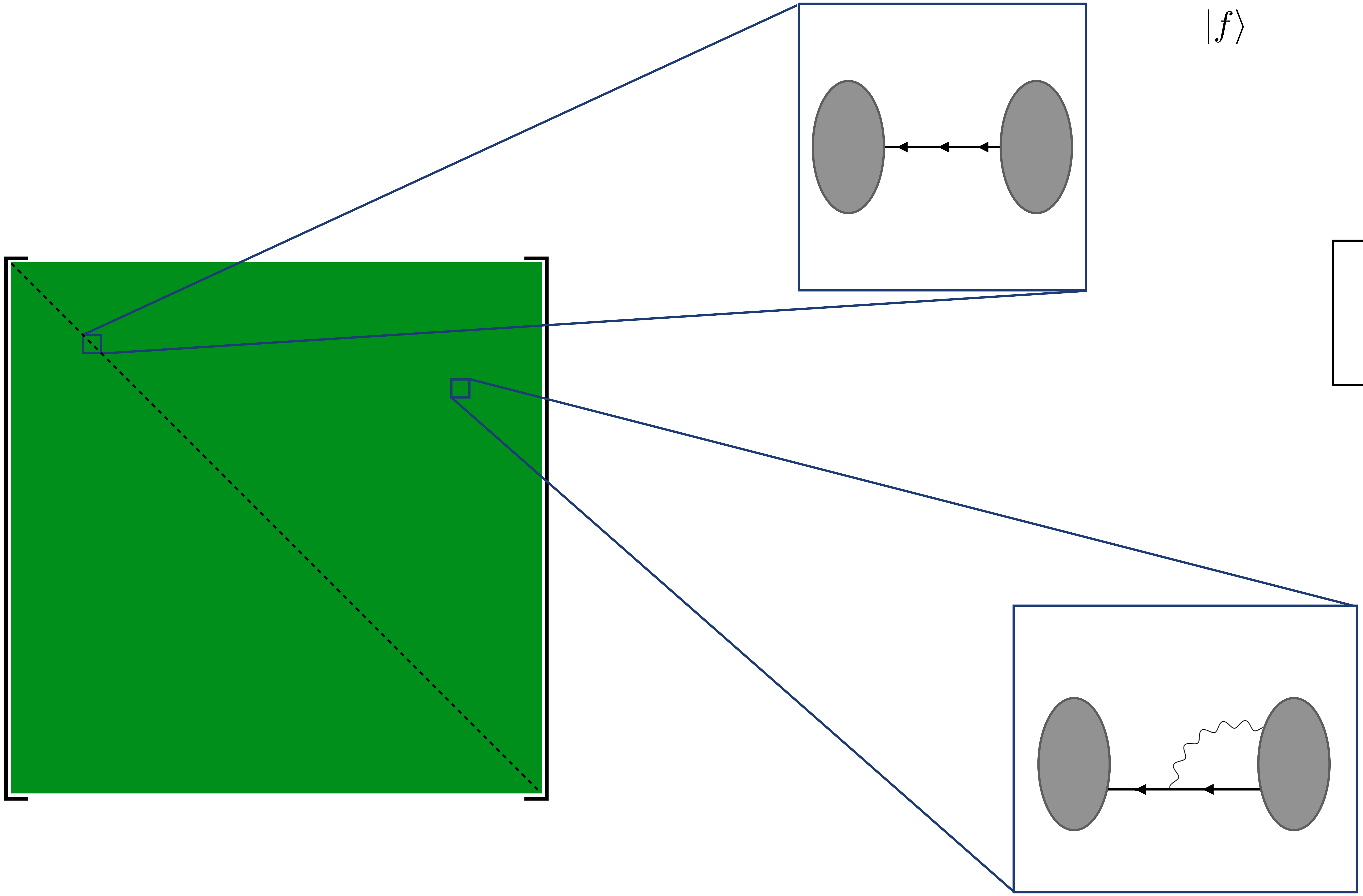


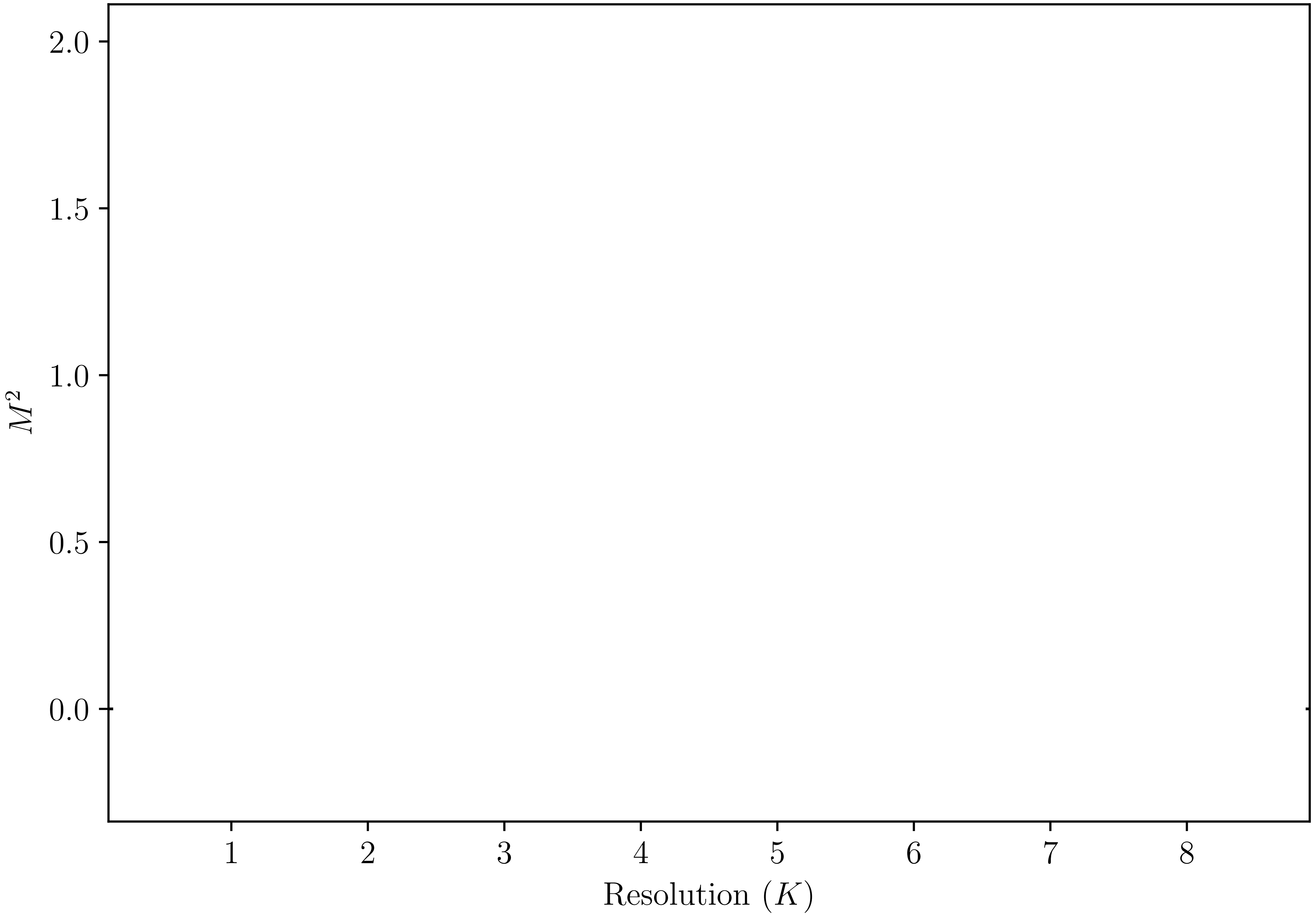
$|f\rangle$ $|f\bar{f}\rangle$ $|fb\rangle$

$H =$

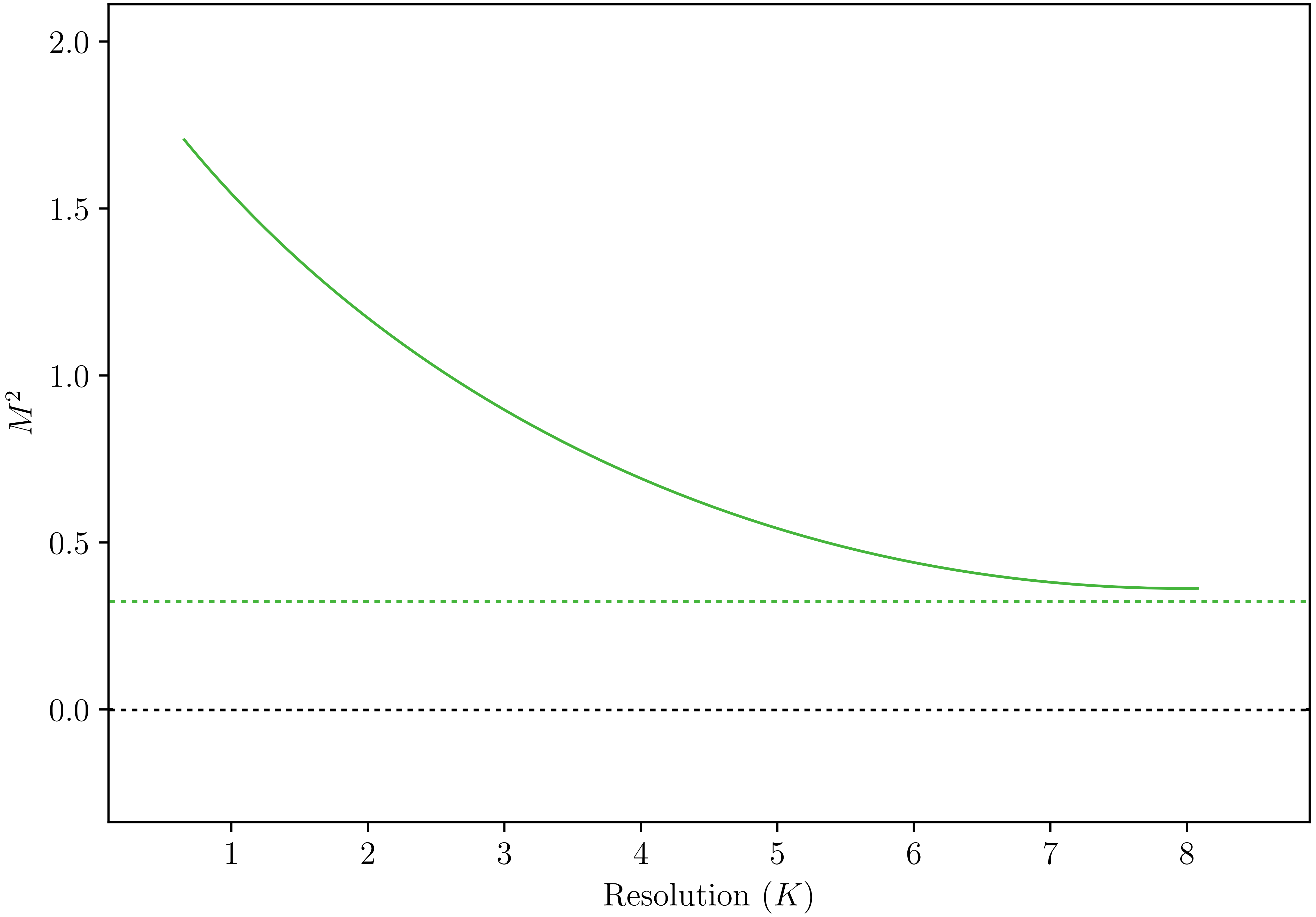


$$h_{ij} < \infty \ \forall i, j$$

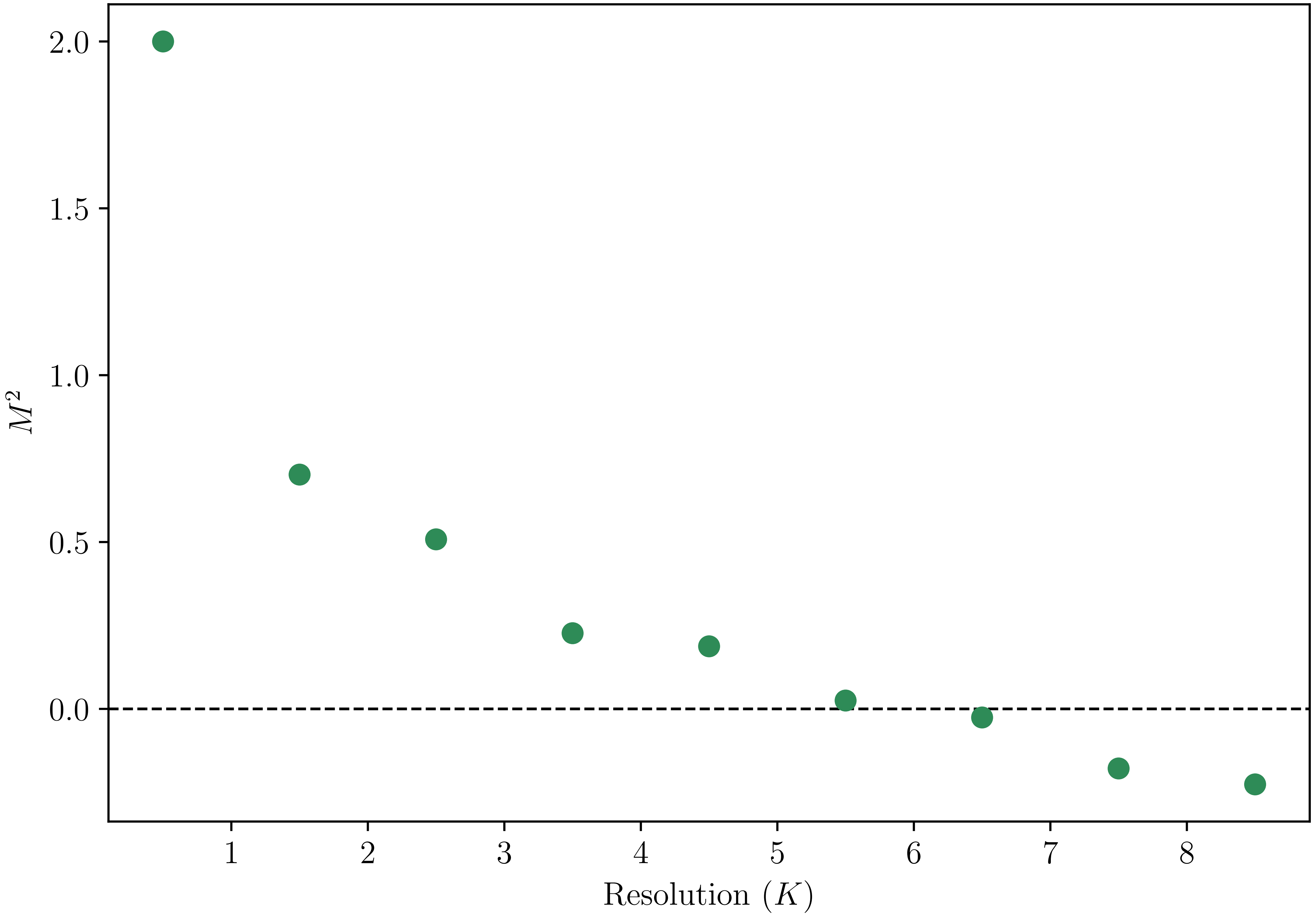
Hamiltonian Spectrum Behavior



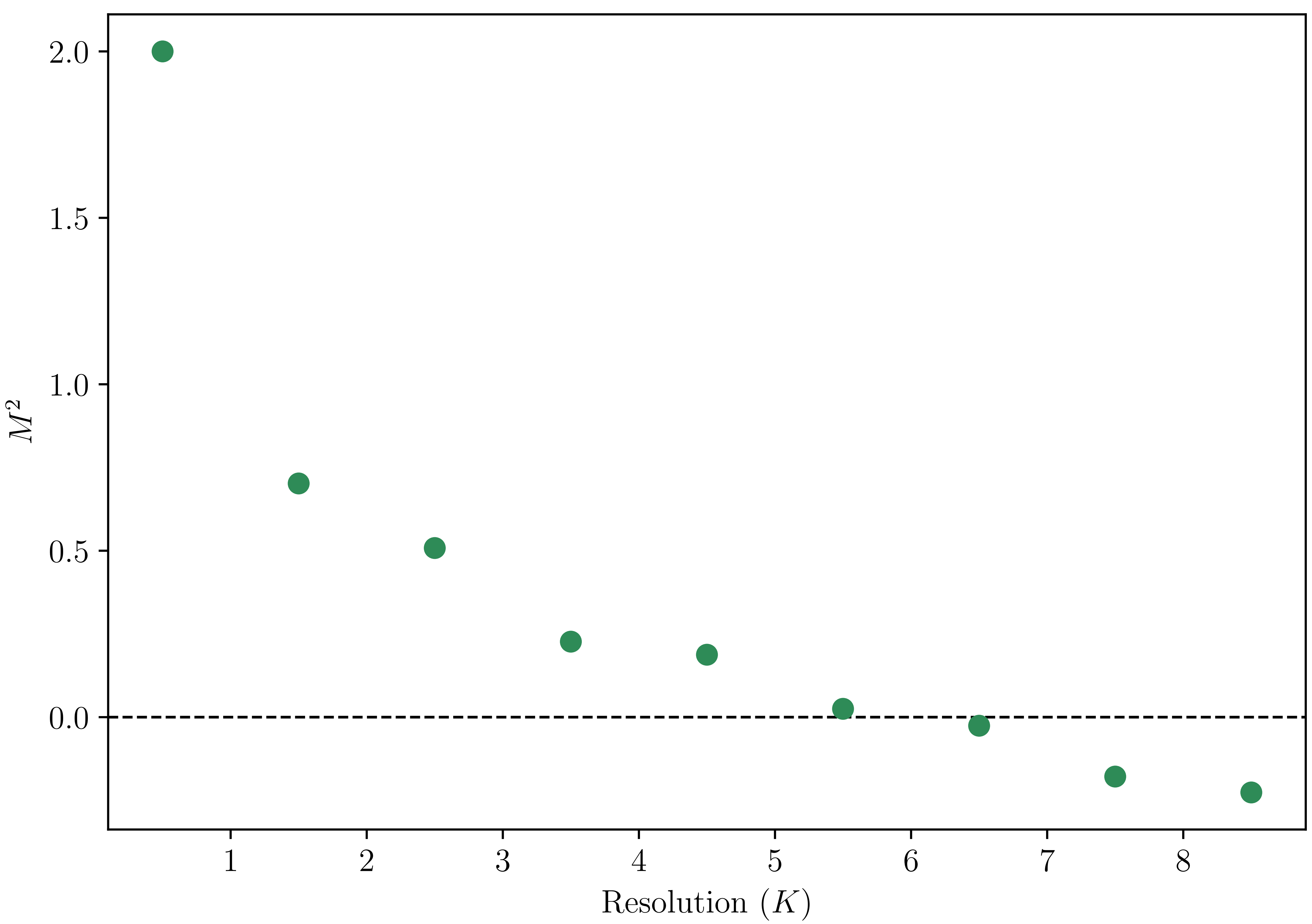
Hamiltonian Spectrum Behavior

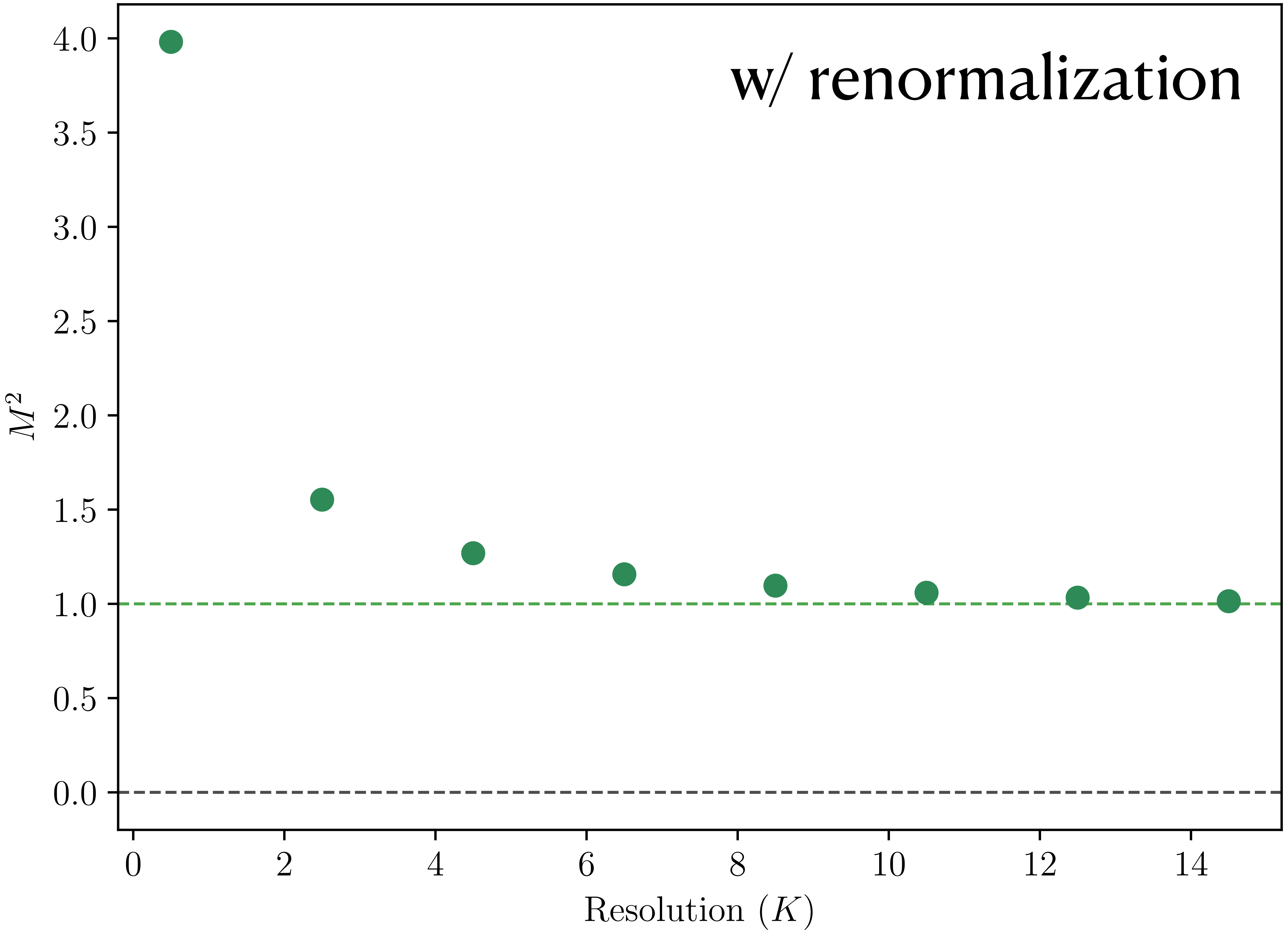


Hamiltonian Spectrum Behavior



Hamiltonian Spectrum Behavior

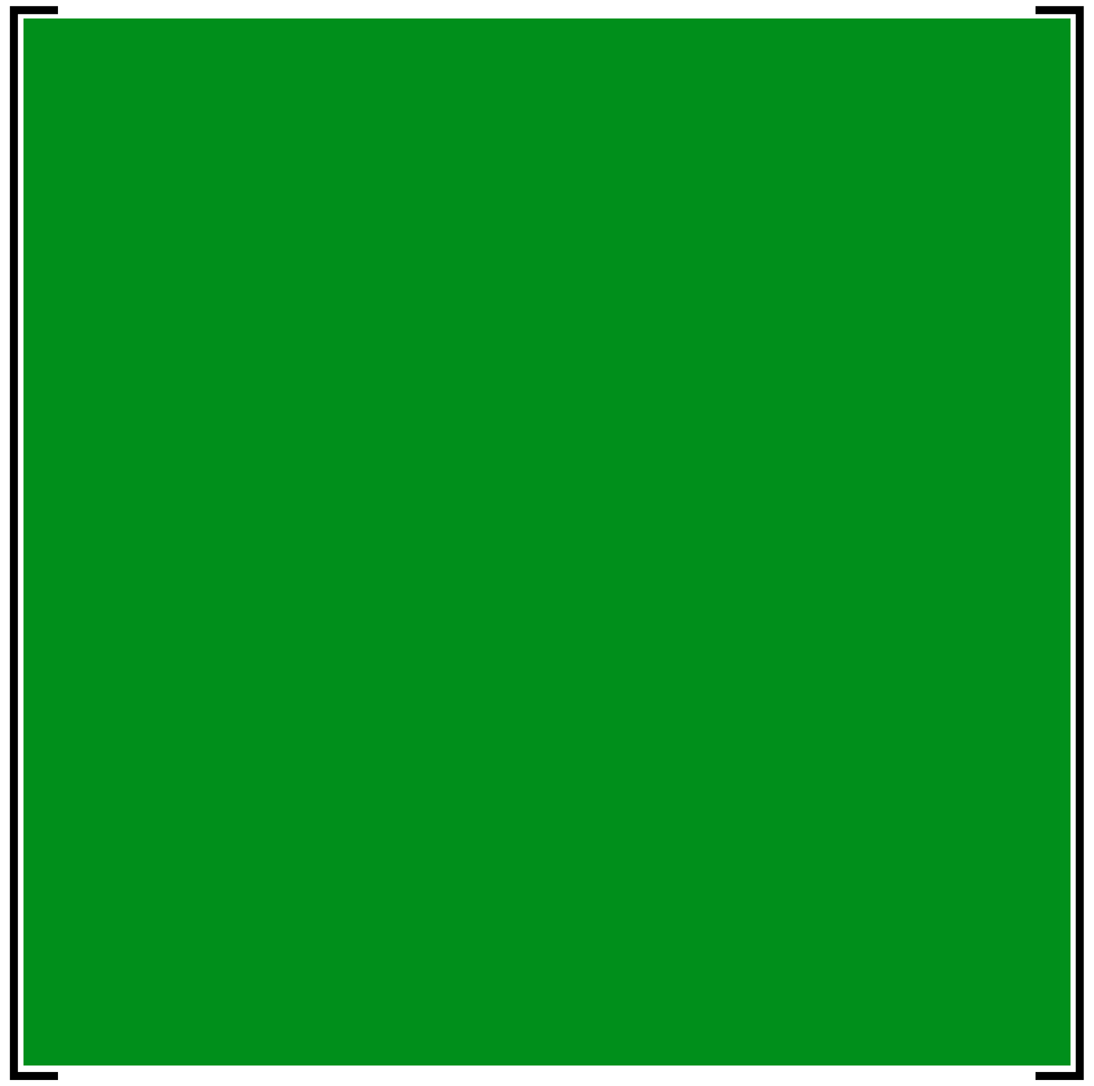




The Similarity Renormalization Group

Fact: Far off-diagonal h_{ij}
cause divergences

$$H$$



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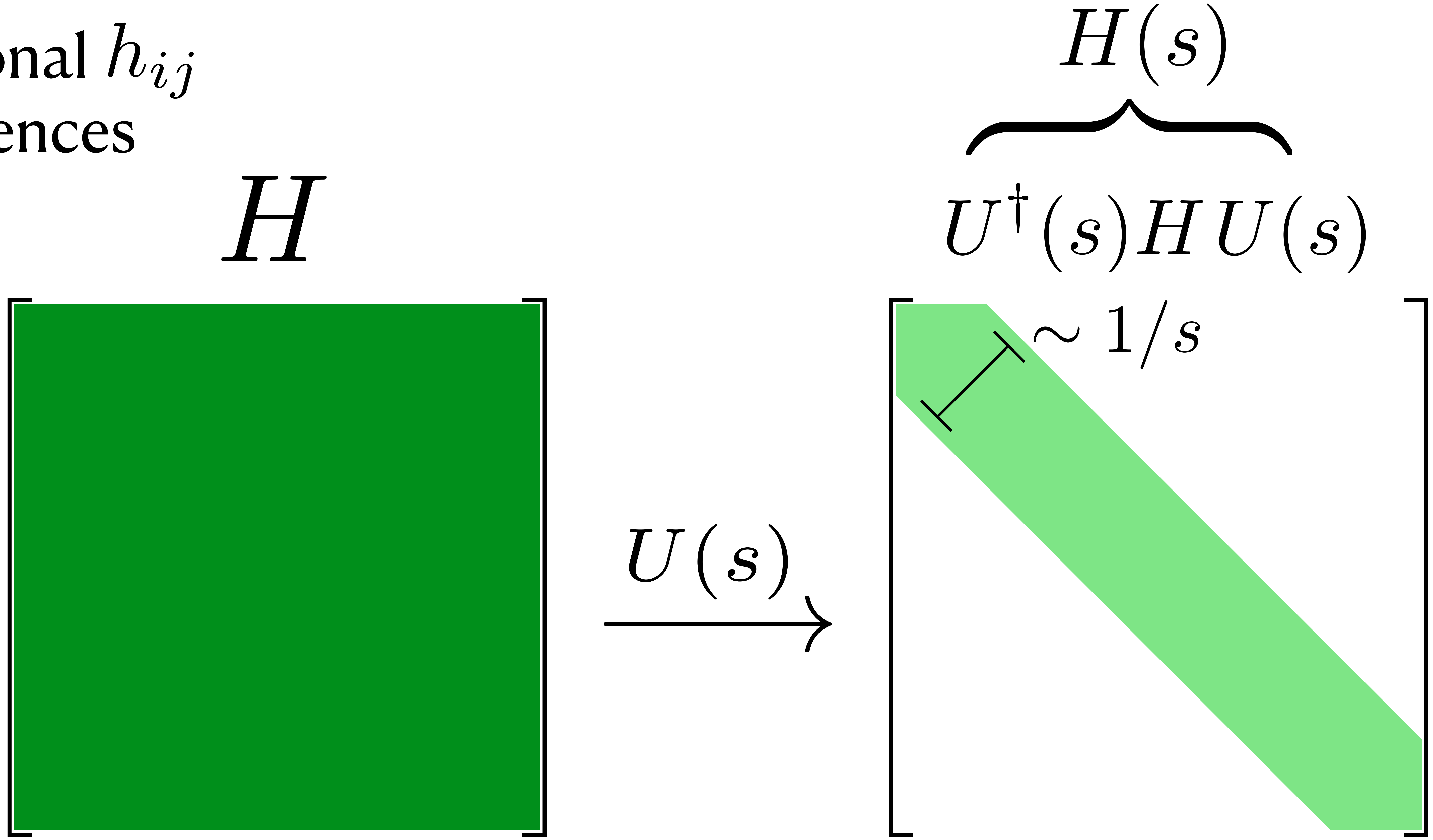

$$\text{spectrum}(H) = \text{spectrum}\Big(H(s)\Big)$$

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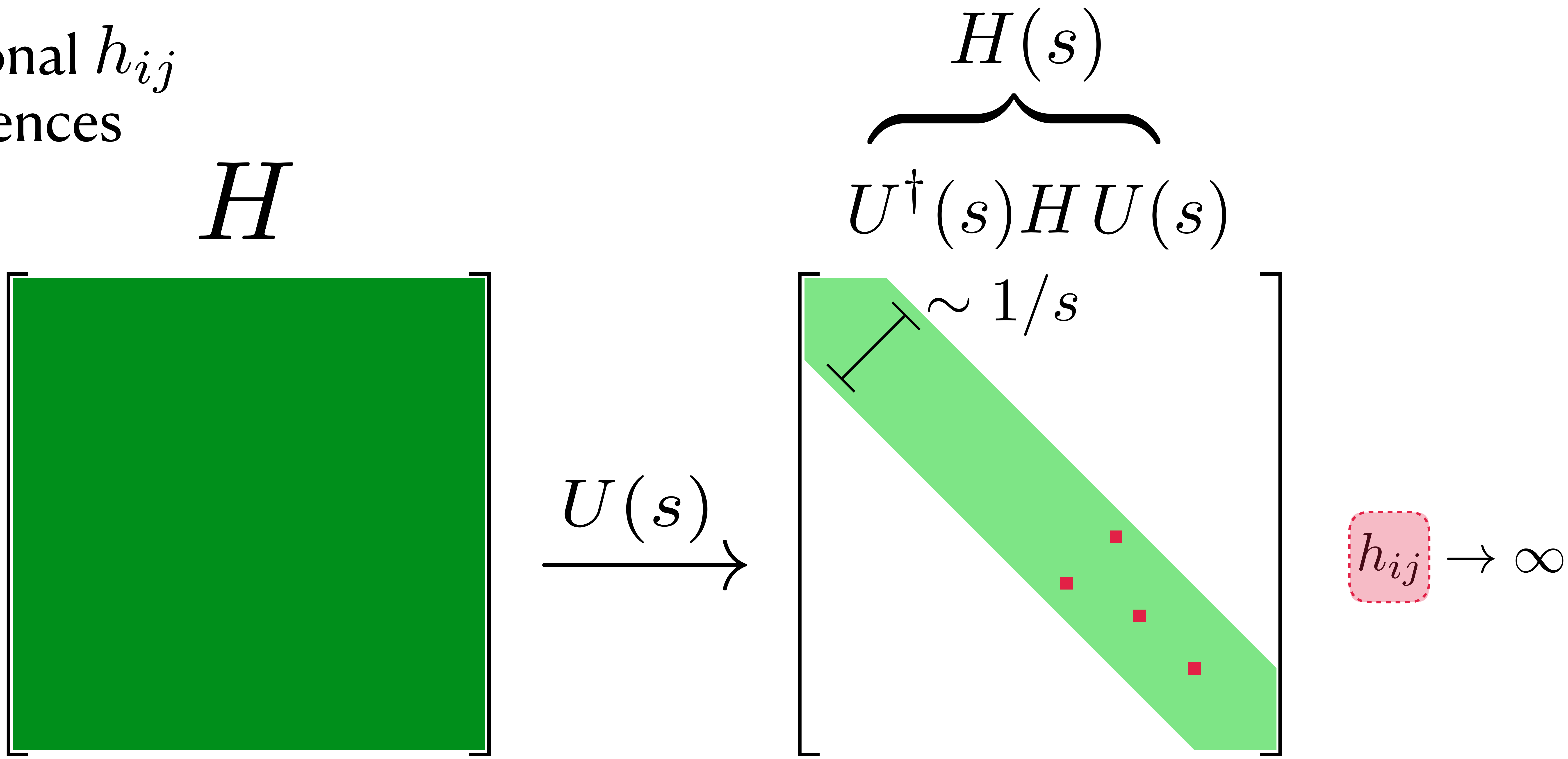
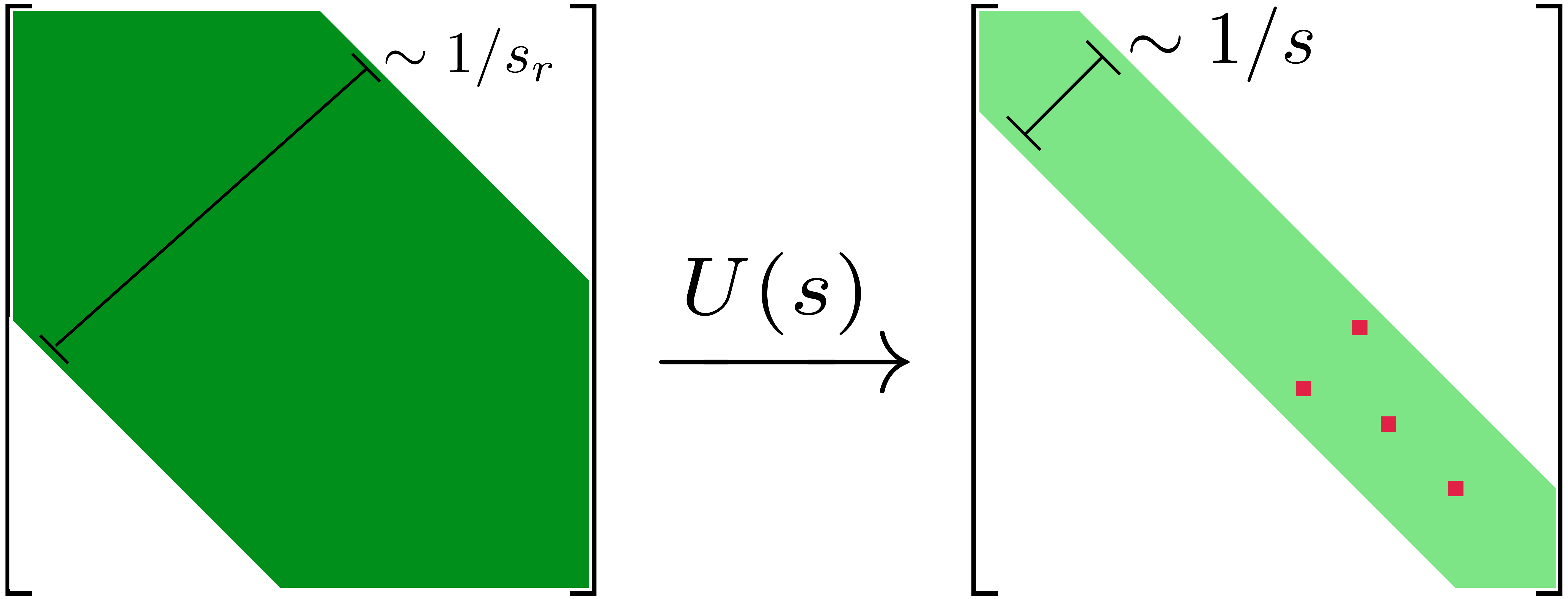


$$\text{spectrum}(H) = \text{spectrum}\Big(H(s)\Big)$$

The Similarity Renormalization Group

Fact: Far off-diagonal h_{ij}
cause divergences

$$H(s_r) \qquad \overbrace{U^\dagger(s)H(s_r)U(s)}^{H(s,s_r)}$$



$$\lim_{s_r\to 0} \boxed{h_{ij}(s,s_r)} \to \infty$$

$$\text{spectrum}\Big(H(s_r)\Big) = \text{spectrum}\Big(H(s,s_r)\Big)$$

The Renormalized Hamiltonian

$$\boxed{\mathcal{H}(s) = H(s, s_r) + X(s_r)}$$

The Renormalized Hamiltonian

$$\boxed{\mathcal{H}(s) = H(s, s_r) + X(s_r)}$$

Unitarily Transformed Hamiltonian

$$H(s, s_r) = U^\dagger(s) H(s_r) U(s)$$

$$\exists (i, j) : \lim_{s_r \to 0} h_{ij}(s, s_r) \to \infty$$

The Renormalized Hamiltonian

$$\boxed{\mathcal{H}(s) = H(s, s_r) + X(s_r)}$$

Unitarily Transformed Hamiltonian

$$H(s, s_r) = U^\dagger(s) H(s_r) U(s)$$

$$\exists (i, j) : \lim_{s_r \to 0} h_{ij}(s, s_r) \to \infty$$

Counterterms

$$X(s_r) = \sum_{ij} x_{ij}(s_r)$$

$$x_{ij}(s_r) : \lim_{s_r \to 0} \Big(x_{ij}(s_r) + h_{ij}(s, s_r) \Big) \underset{\forall s}{< \infty}$$

The Renormalized Hamiltonian

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Similarity Renormalization Group Equation

$$\boxed{\frac{dH(s,s_r)}{ds} = \left[\mathcal{G}(s,s_r), H(s,s_r)\right]}$$

Renormalization of Hamiltonians. Głazek and Wilson
Perturbative formulae for relativistic interactions of effective particles. Głazek

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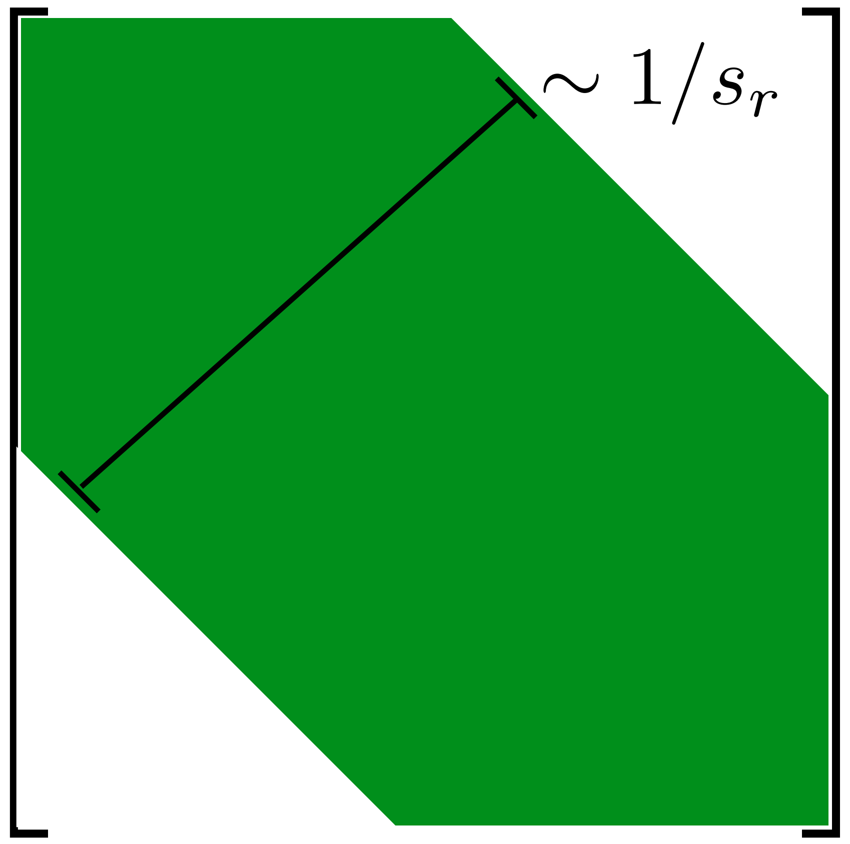
Głazek-Wegner Flow Equation

$H(s_r)$:



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Renormalization of Hamiltonians. Głazek and Wilson
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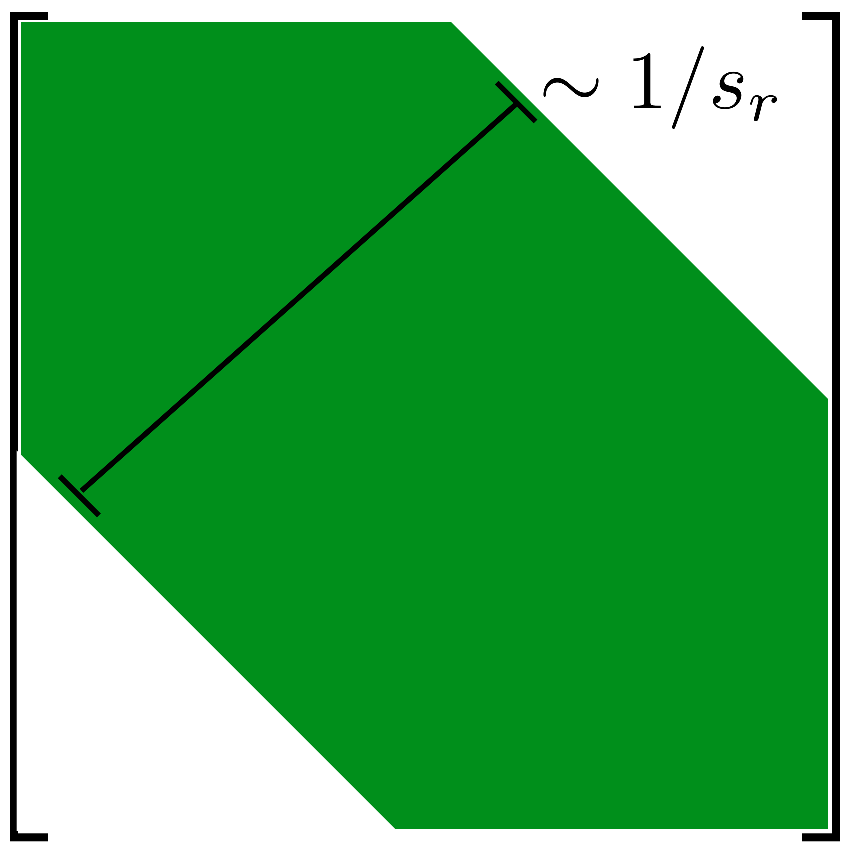

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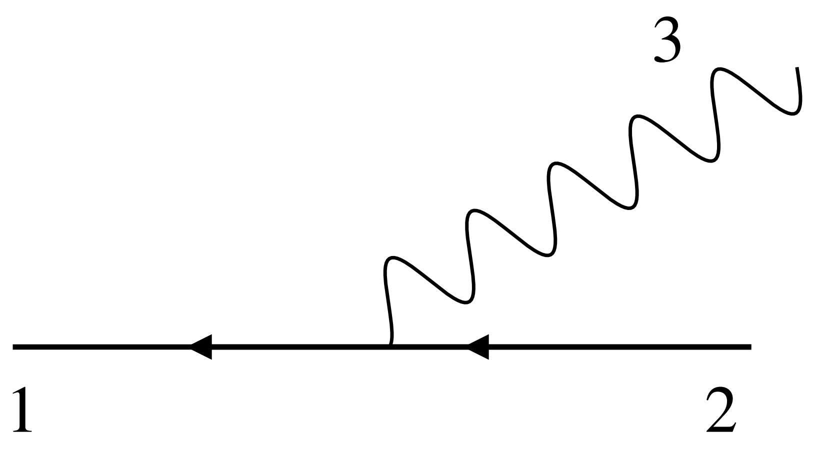
Renormalization Group Procedure for Effective Particles (RGPEP) Equation



$$\sim g \int_{123} \bar{u}_1 \gamma^\mu u_2 \epsilon^\sigma_{3\mu} b_1^\dagger b_2 a_3$$

$$\mathcal{G}(s,s_r) = \Big[H_0, H(s,s_r)\Big]$$

Renormalization of Hamiltonians. Głazek and Wilson
Perturbative formulae for relativistic interactions of effective particles. Głazek

Solving the RGPEP Equation

$$\frac{dH(s,s_r)}{ds} = \left[\left[H_0, H(s,s_r)\right], H(s,s_r)\right]$$

Propose

$$H(s,s_r) = H^{(0)}(s,s_r) + gH^{(1)}(s,s_r) + g^2H^{(2)}(s,s_r) + \mathcal{O}(g^3)$$

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Propose

$$H(s,s_r) = H^{(0)}(s,s_r) + gH^{(1)}(s,s_r) + g^2H^{(2)}(s,s_r) + \mathcal{O}(g^3)$$

Perturbative RGPEP Equations:

$$\mathcal{O}(g^0):\ \frac{dH^{(0)}(s,s_r)}{ds} = 0$$

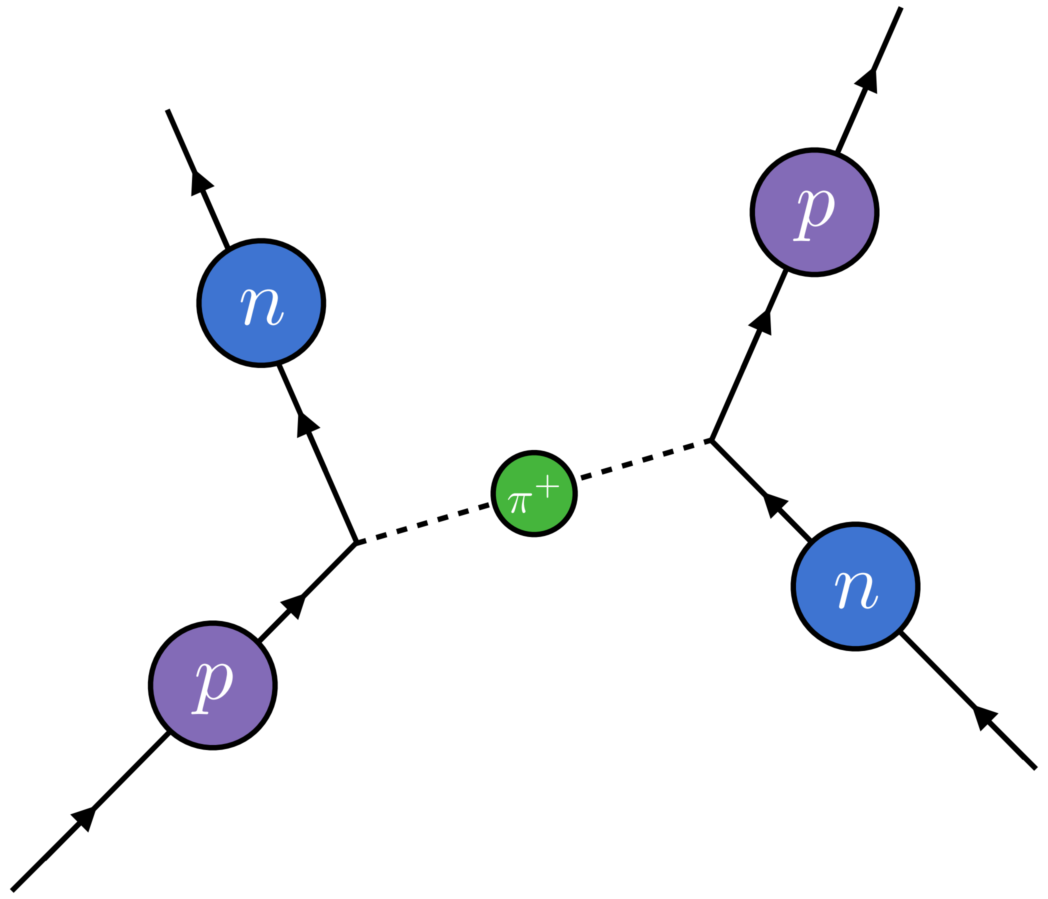
$$\mathcal{O}(g^1):\ \frac{dH^{(1)}(s,s_r)}{ds} = \Big[\big[H^{(0)}(s,s_r), H^{(1)}(s,s_r)\big], H^{(0)}(s,s_r)\Big]$$

$$\mathcal{O}(g^2):\ \frac{dH^{(2)}(s,s_r)}{ds} = \Big[\big[H^{(0)}(s,s_r), H^{(2)}(s,s_r)\big], H^{(0)}(s,s_r)\Big] + \Big[\big[H^{(0)}(s,s_r), H^{(1)}(s,s_r)\big], H^{(1)}(s,s_r)\Big]$$

$$\vdots$$

$\mathcal{O}(g^2)$ Renormalized Yukawa Hamiltonian

$$\mathcal{L}_{\mathrm{Y}} = \bar{\psi}\Big(i\gamma^\mu\partial_\mu - m\Big)\psi + \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}\mu^2\phi^2 - g\bar{\psi}\psi\phi$$



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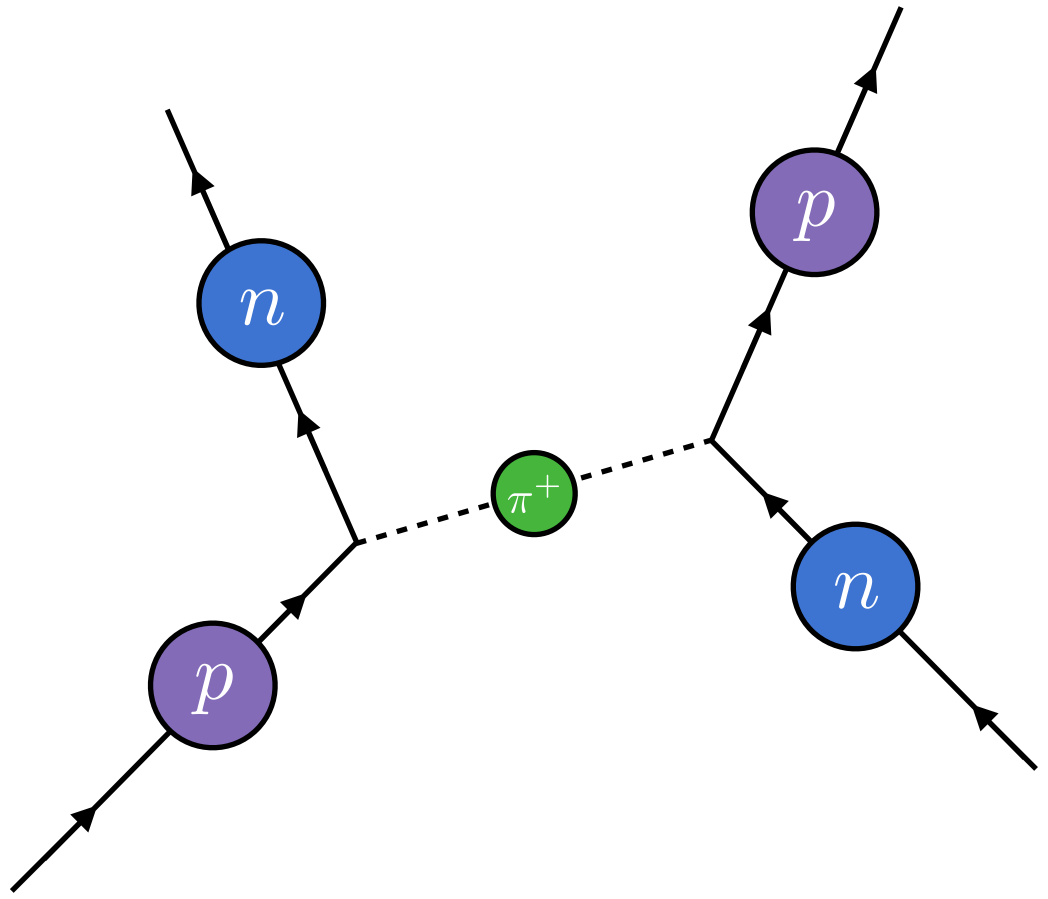
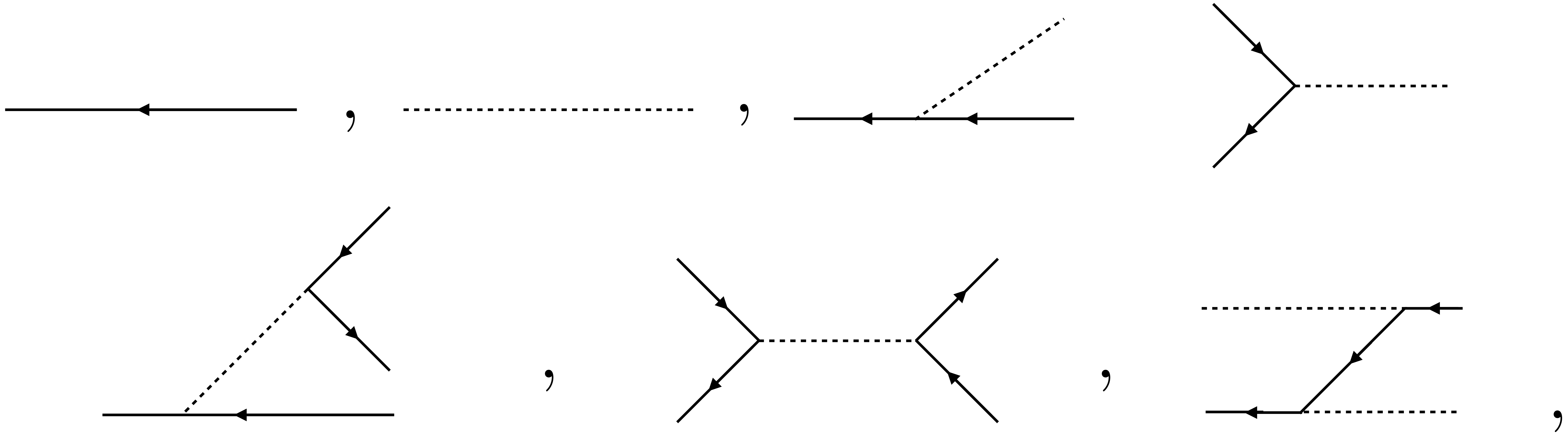
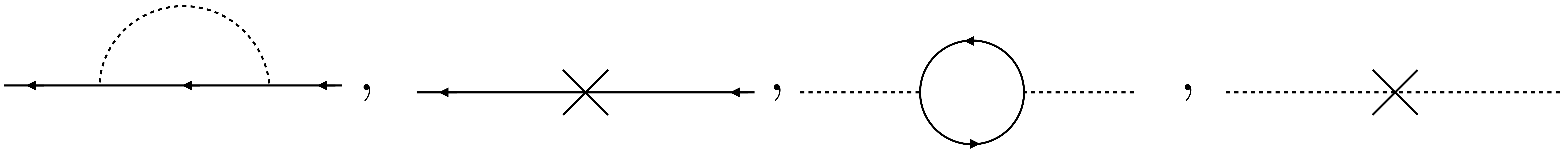


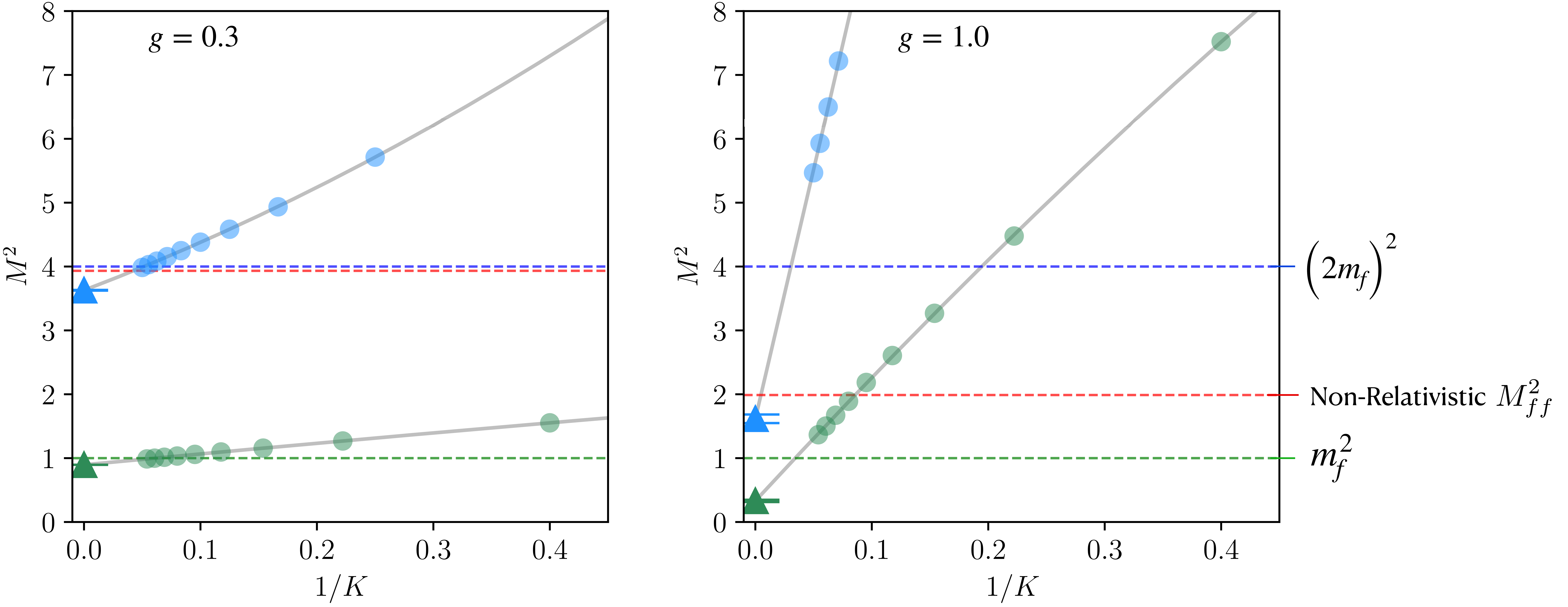
$\mathcal{H}(s) \in \Big\{$





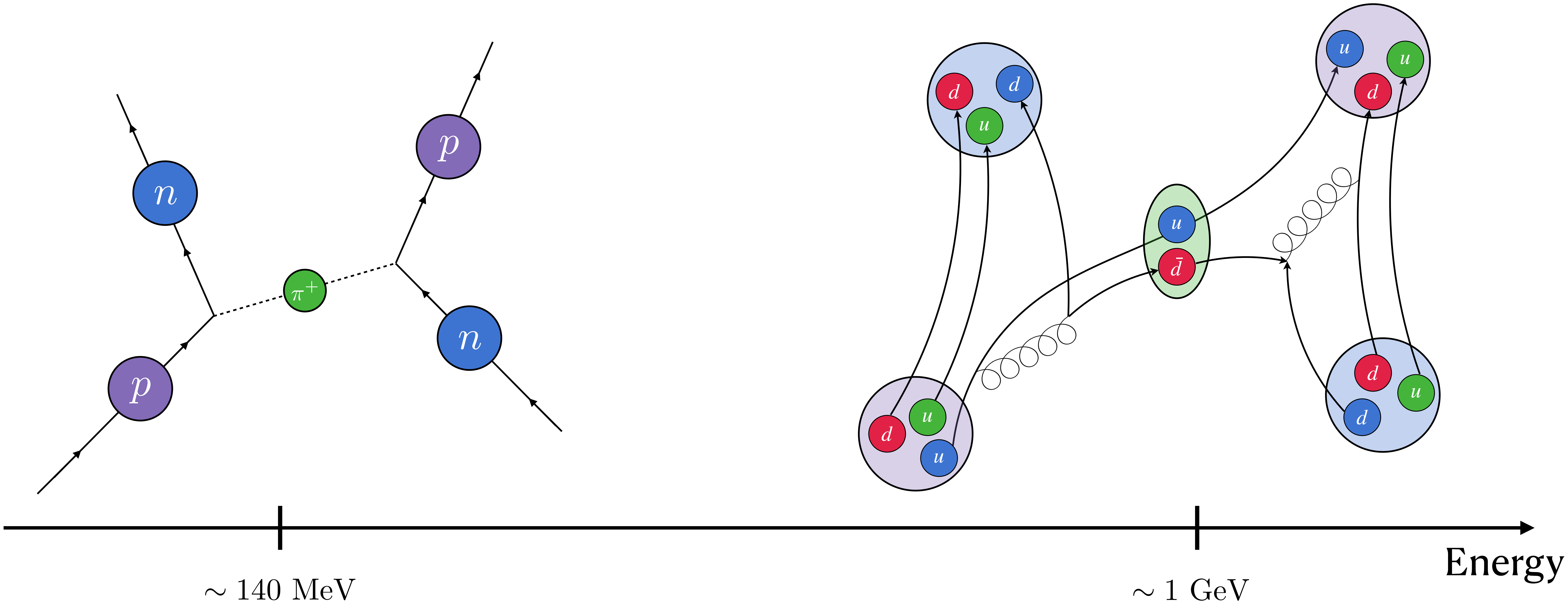
$\ldots \Big\}$

The continuum limit of the renormalized spectrum



https://doi.org/10.1103/9y1c-52sx, **Gustin** et. al

QFTs at Different Energy Scales

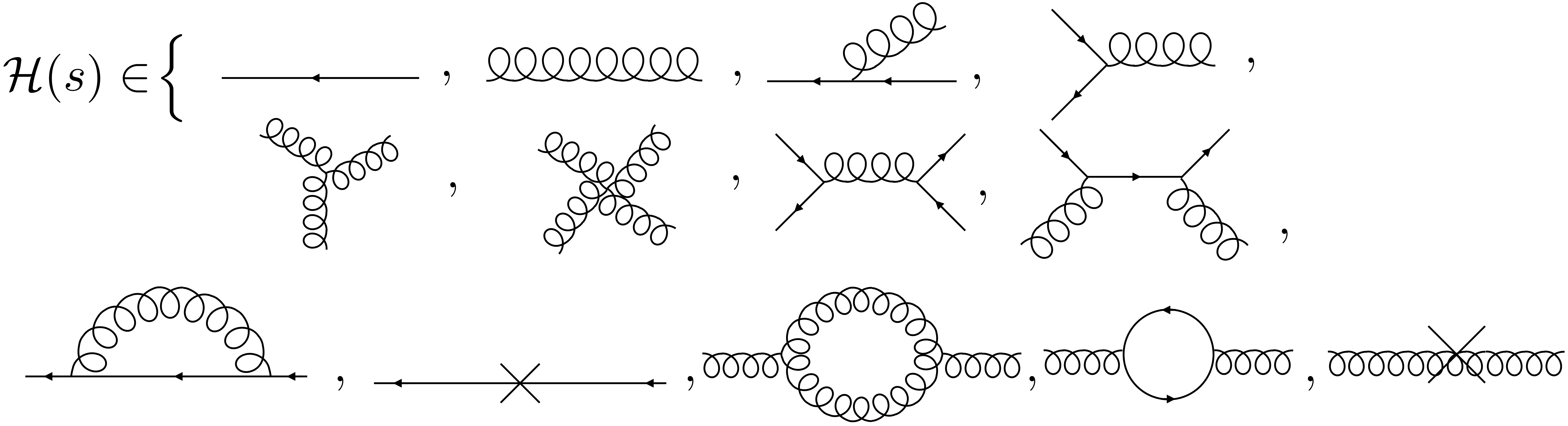


Adapted from *Metrology in Muon Count Methods*, Orduña

$\mathcal{O}(g^2)$ Renormalized QCD Hamiltonian

$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}_c\Big(i\gamma^\mu D_\mu - m_q\Big)\psi_c - \frac{1}{4}G^{\mu\nu}_a G^a_{\mu\nu}$$

$\mathcal{H}(s) \in \Big\{$



$\ldots \Big\}$

An end-to-end framework for quantum simulation of QFTs

Part I: $\mathcal{L} \xrightarrow[\text{Noether}]{\text{IF}\rightarrow\text{FF}} H \xrightarrow{\text{Regulate}} H(s_r) \xrightarrow{U(s)} H(s, s_r) \xrightarrow{X(s_r)} \mathcal{H}(s)$

Prelude Motivation

Part I Renormalized Quantum Field Theory Hamiltonians

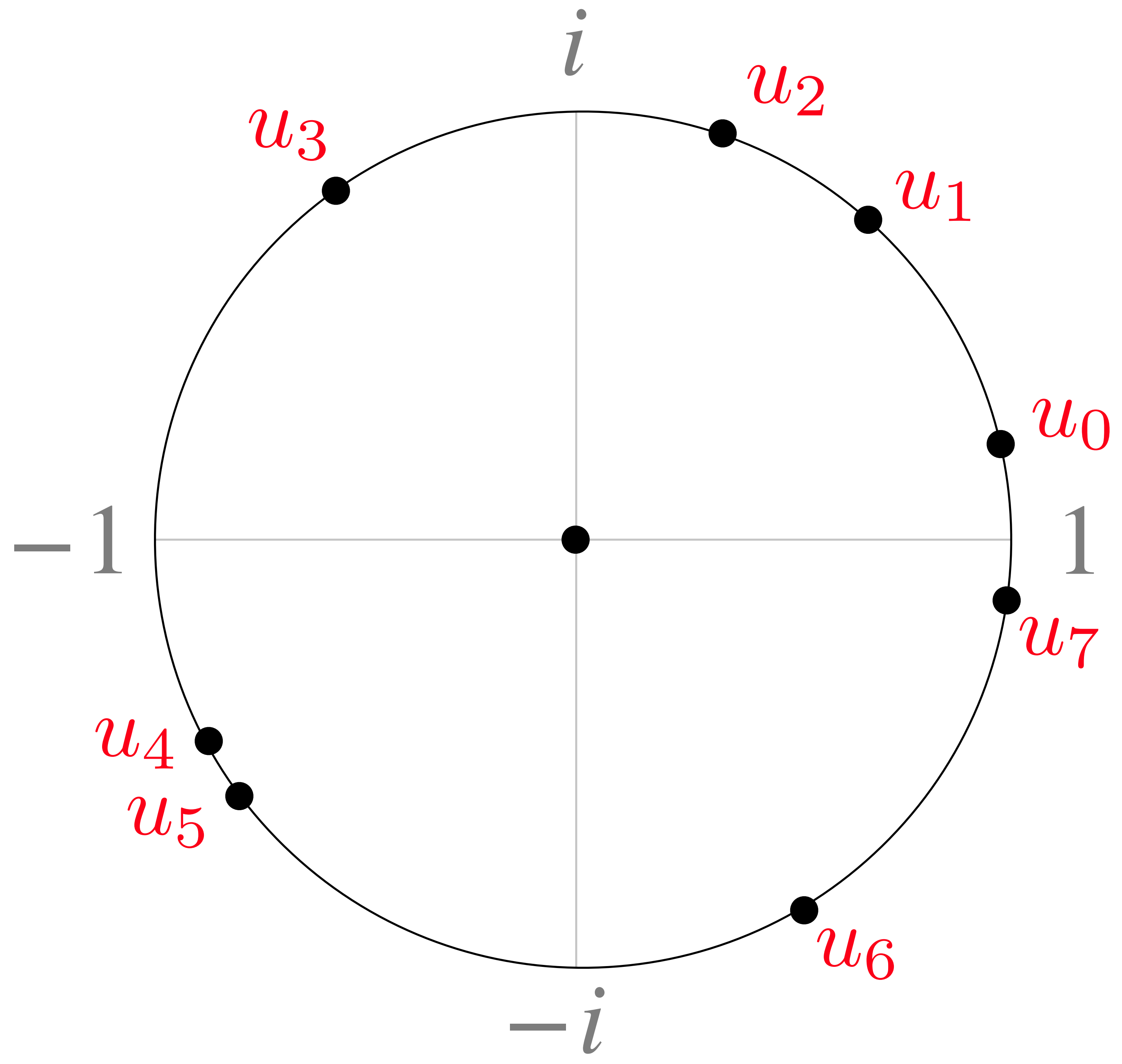
Part II **Quantum Simulation**

Part III Results

Part IV Conclusion

Quantum Simulation: Statics

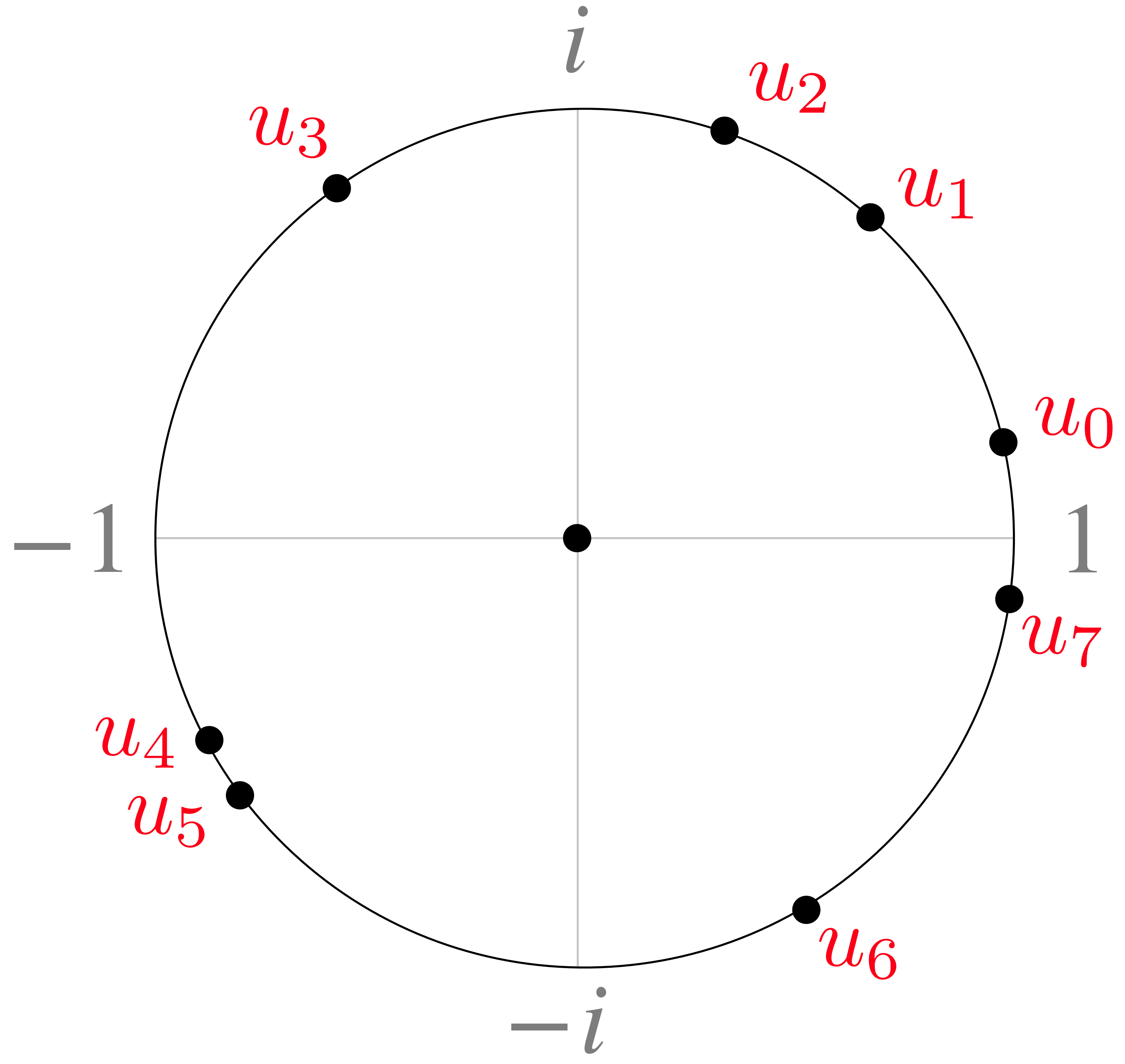
$$U\,|u\rangle = u\,|u\rangle \qquad |u| = 1$$



Quantum Simulation: Statics

$U|u\rangle = u\,|u\rangle \qquad |u| = 1$

$U|u\rangle = e^{i\pi\phi}\,|u\rangle \qquad \phi \in [-1, 1]$



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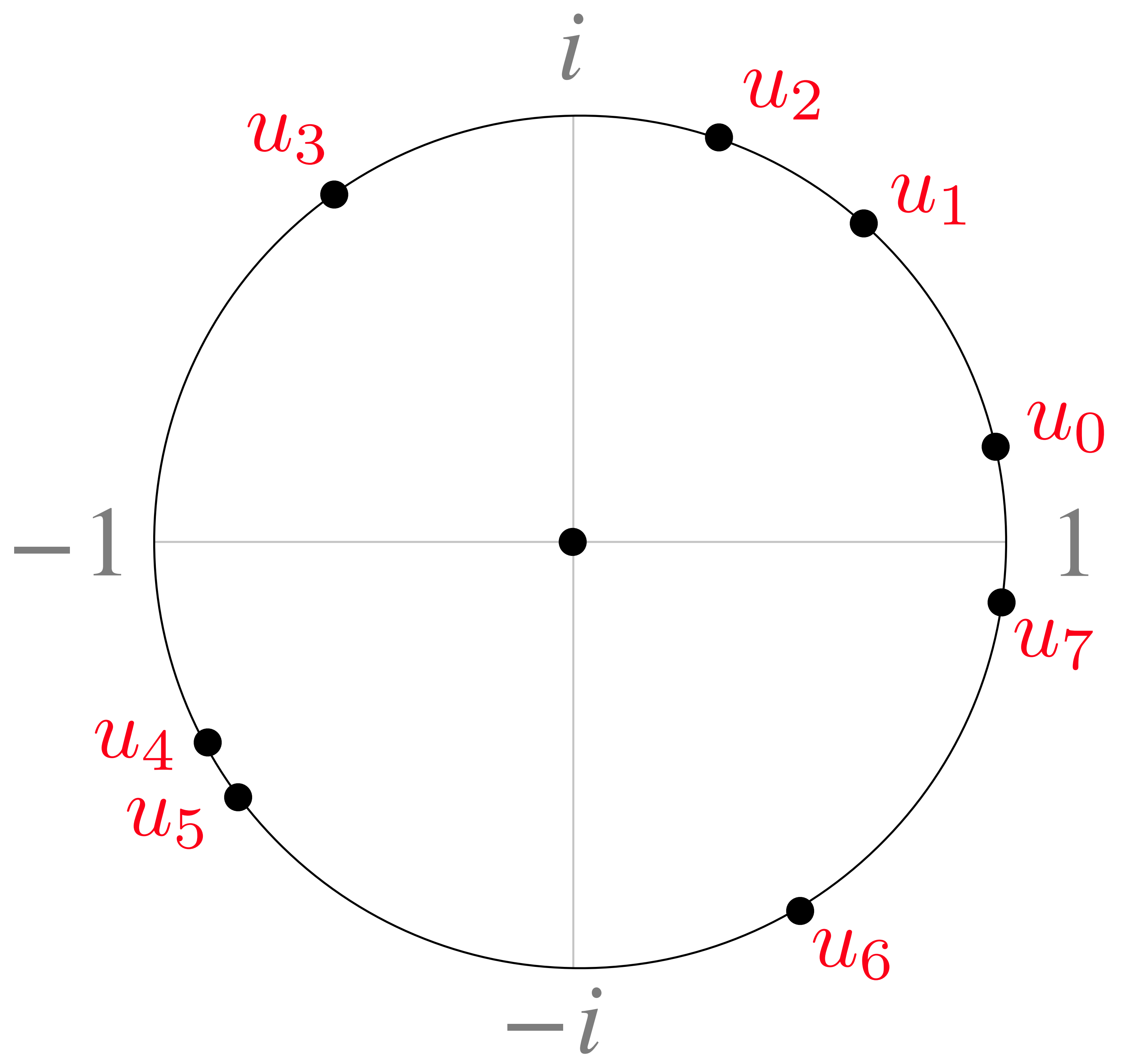
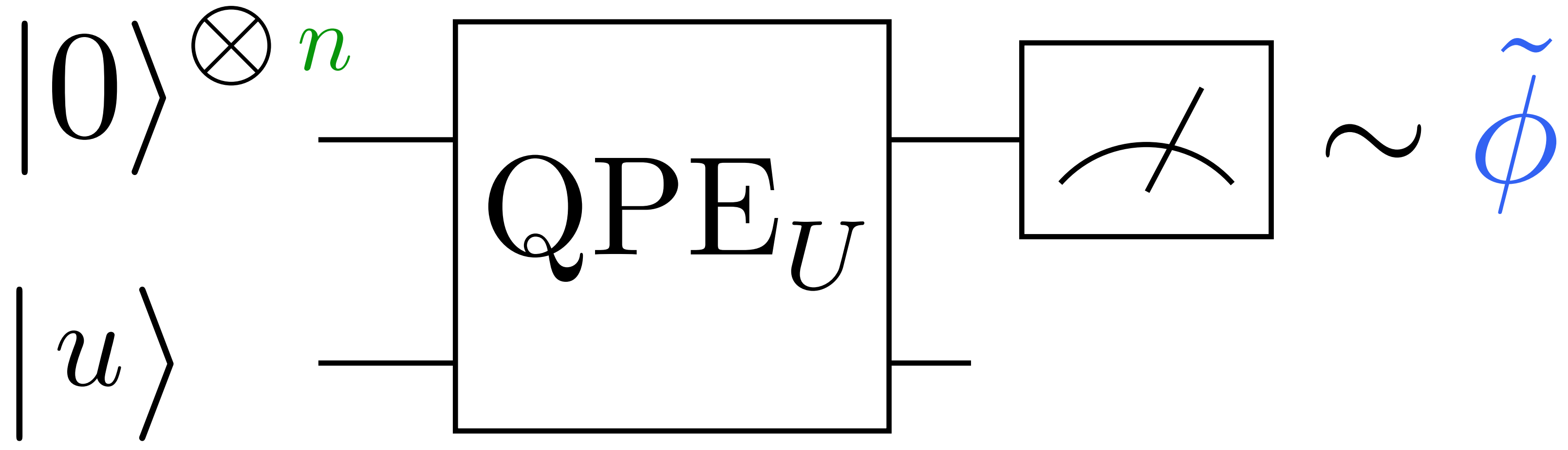


Quantum Phase Estimation:



$$\boxed{n \to \infty \implies |\phi - \tilde{\phi}| \to 0}$$

Hamiltonian Simulation

Goal   Estimate λ :

$$H|\lambda\rangle = \lambda|\lambda\rangle$$

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Solution Encode $H \rightarrow U_H$ such that

$$(a)\ U_H = U_n \ldots U_1 U_0$$

$$(b)\ \text{spectrum}(U_H) = f\Big(\text{spectrum}(H)\Big)$$

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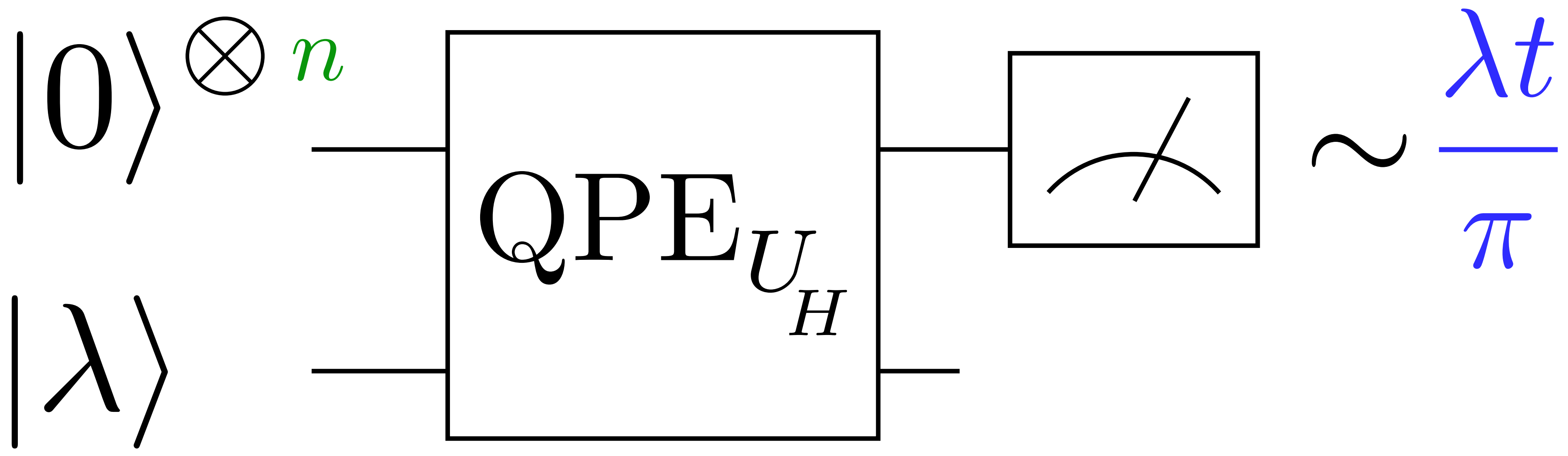
Example: Time evolution operator

$$U_H = e^{-iHt}$$

$$U_H|\lambda\rangle = e^{-i\lambda t}|\lambda\rangle$$



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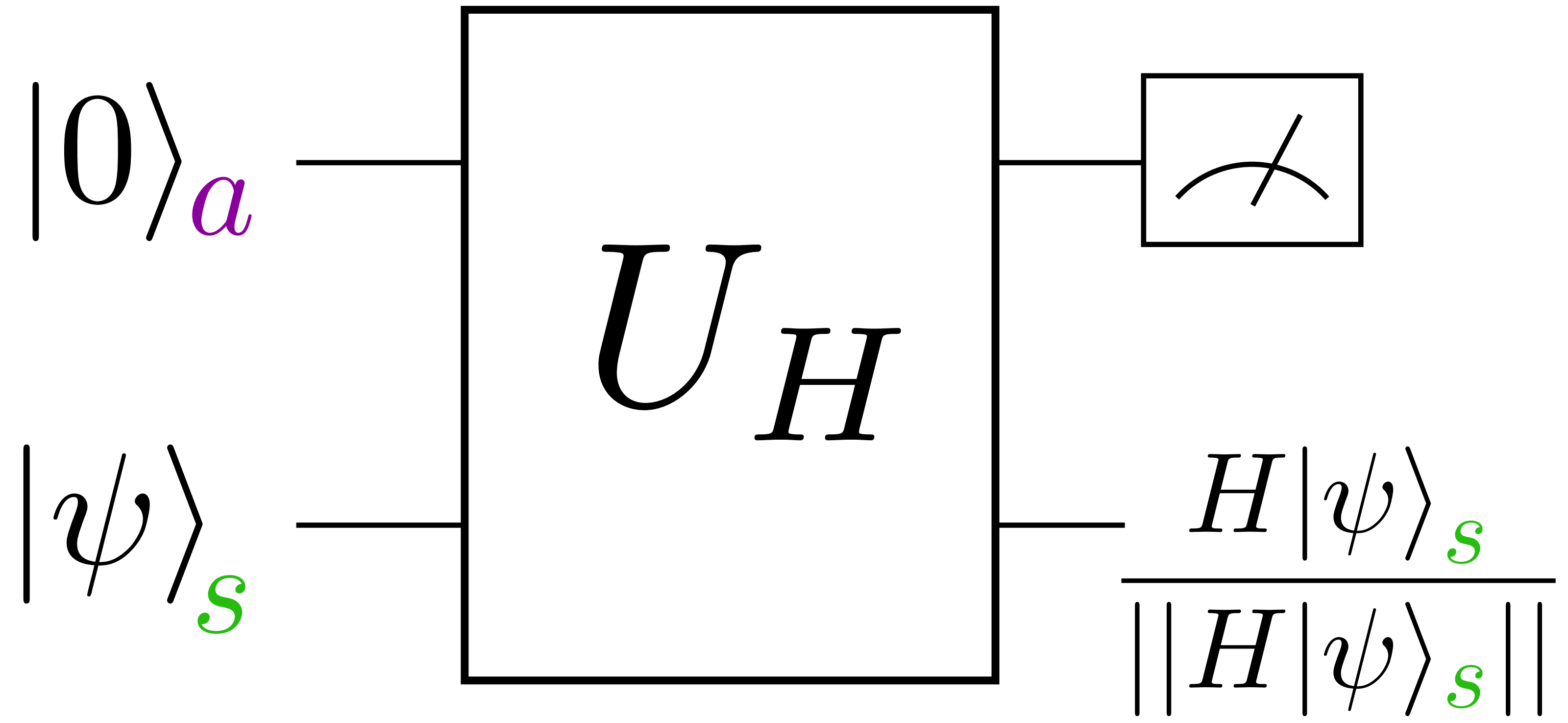
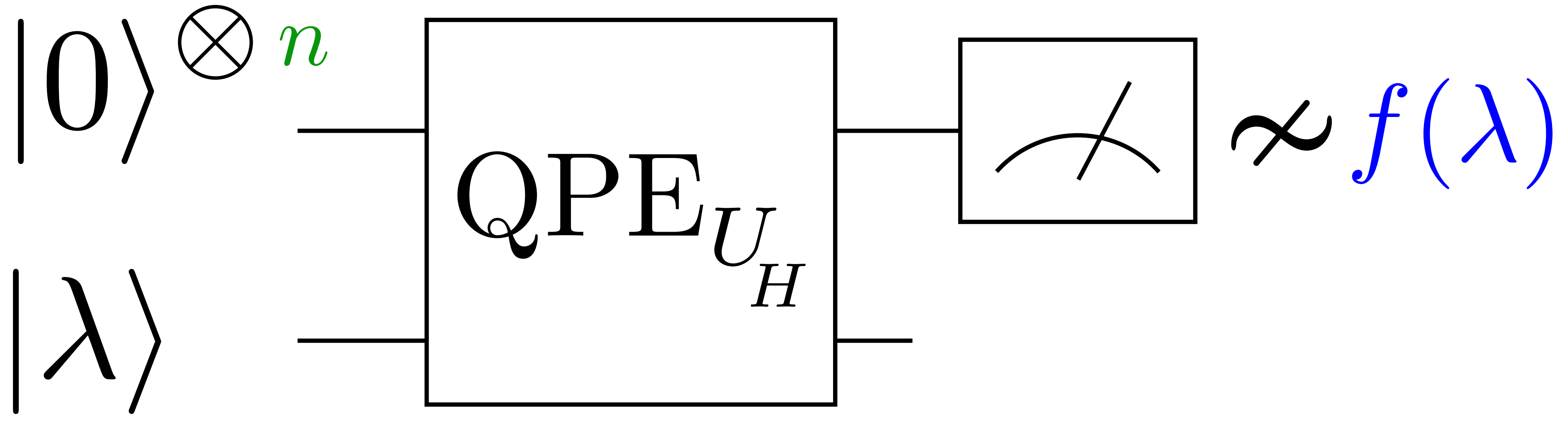
Example: Block Encoding (circa 2013)

$$H \to U_H = \begin{pmatrix} H/\alpha & \star \\ \star & \star \end{pmatrix}$$





Qubitization

$$\boxed{U_H^2 \overset{?}{=} \mathbb{I}}$$

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Yes

The Walk Operator

$$\mathcal{W}_H = U_H \Big(\underbrace{2|0\rangle_a \langle 0|_a - \mathbb{I}_a}_{\mathcal{R}_a} \Big)$$

$$\text{spectrum}(\mathcal{W}_H) = f\Big(\text{spectrum}(H)\Big)$$

Hamiltonian Simulation by Qubitization, Low and Chuang

Qubitization

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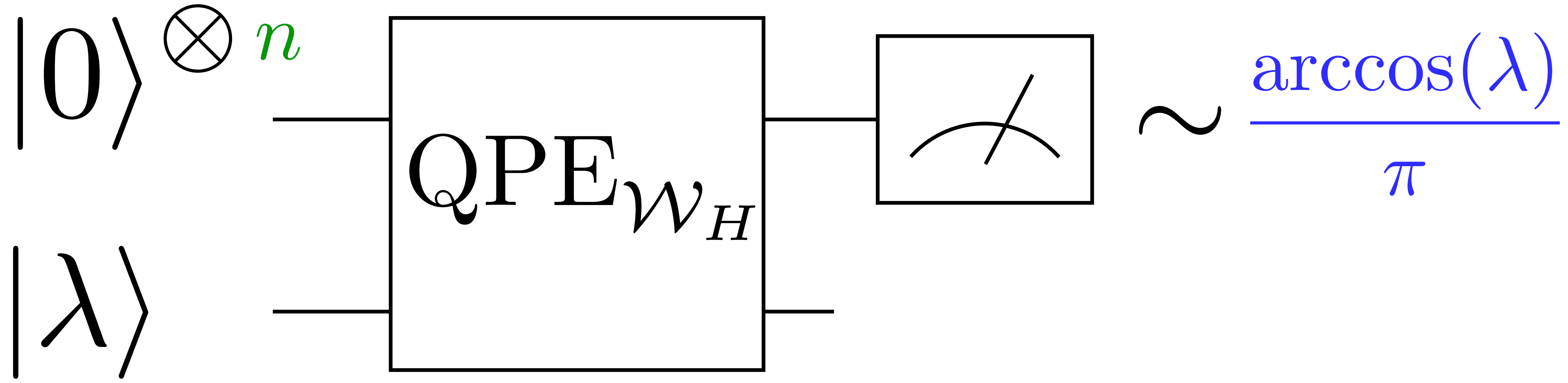
With minimal overhead can force

$$U_H^2 = \mathbb{I}$$



Block Encoding Hamiltonians

$$\boxed{H = \sum_{l=0}^{L-1} \alpha_l O_l}$$

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If $O_l^\dagger O_l = \mathbb{I}$

LCU: $$H = \sum_{l=0}^{L-1} \alpha_l \underbrace{U_l}_{O_l} \rightarrow \begin{pmatrix} \sum_{l=0}^{L-1} \frac{\alpha_l}{\alpha} U_l & \star \\ \star & \star \end{pmatrix} \equiv U_H$$

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If $O_l^\dagger O_l \neq \mathbb{I}$

Map to Pauli basis

$$O_l \rightarrow \sum_k P_{l,k}^{\otimes n}$$

- *Encoding Electronic Spectra in Quantum Circuits with Linear T Complexity.* Babbush, Gidney, Berry, Wiebe, et al.
- *Reducing molecular electronic Hamiltonian simulation cost for Linear Combination of Unitaries approaches.* Loaiza, Khan, Wiebe, Izmaylov
- *Resource-optimized fault-tolerant simulation of the Fermi-Hubbard model and high-temperature superconductor models.* Kan and Symons

$\dots$

Hamiltonian Simulation Using Linear Combinations of Unitary Operations. Childs & Wiebe

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. . .

Stay in second-quantized basis

$$U_{O_l} = \begin{pmatrix} O_l/\alpha_l & \star \\ \star & \star \end{pmatrix}$$

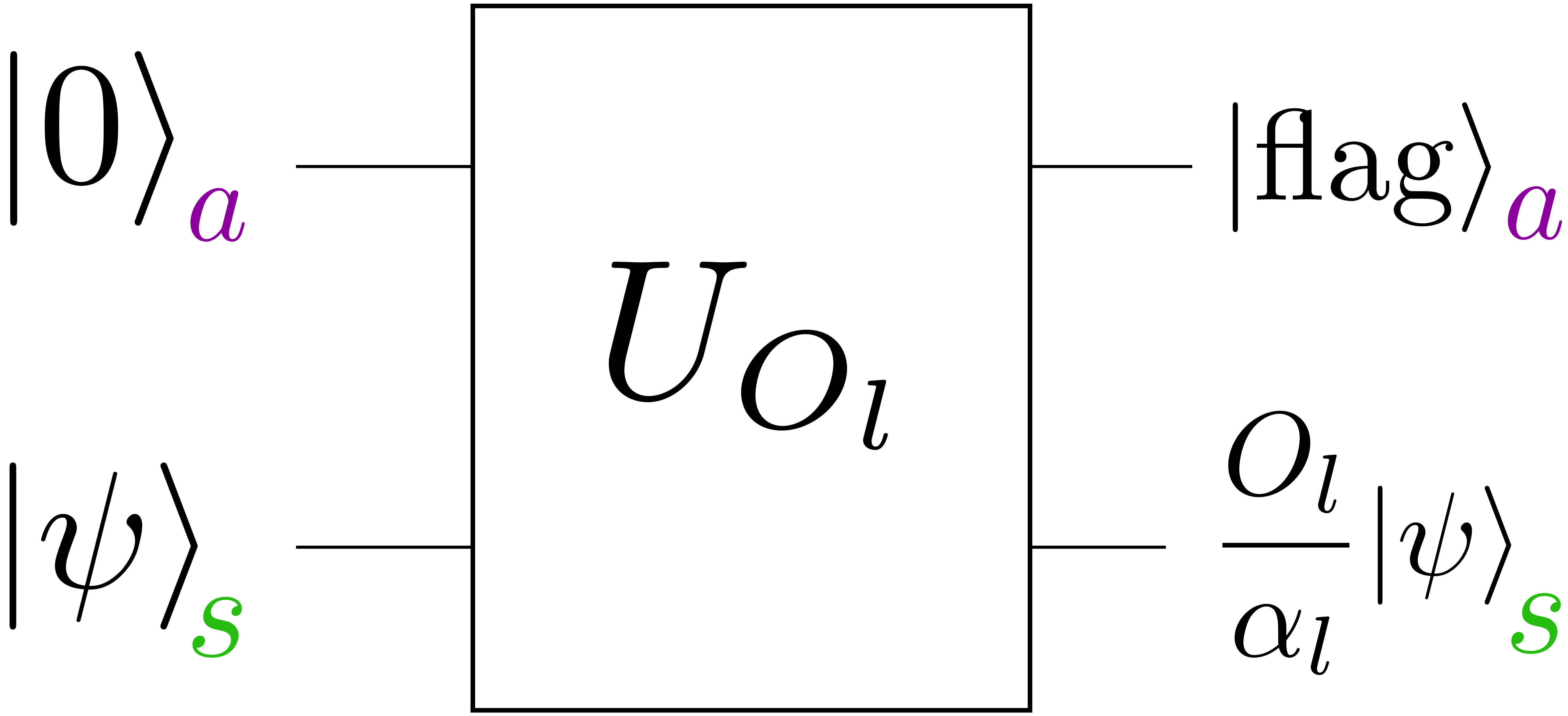
$$H = \sum_{l=0}^{L-1} \alpha_l O_l \rightarrow \tilde{H} = \sum_{l=0}^{L-1} \tilde{\alpha}_l U_{O_l}$$

LCU $U_{\tilde{H}}$ block-encodes H

Hamiltonian Simulation Using Linear Combinations of Unitary Operations. Childs & Wiebe
Quantum singular value transform and beyond: exponential improvements for quantum matrix arithmetic. Gilyén et al.

Ladder Operator Block Encoding (LOBE)

LOBE efficiently constructs $U_{O_l} = \begin{pmatrix} O_l/\alpha_l & \star \\ \star & \star \end{pmatrix}$ for O_l written in second quantization



https://doi.org/10.22331/q-2025-12-22-1953. Simon, **Gustin**, et. al

Prelude **Motivation**

Part I **Renormalized Quantum Field Theory Hamiltonians**

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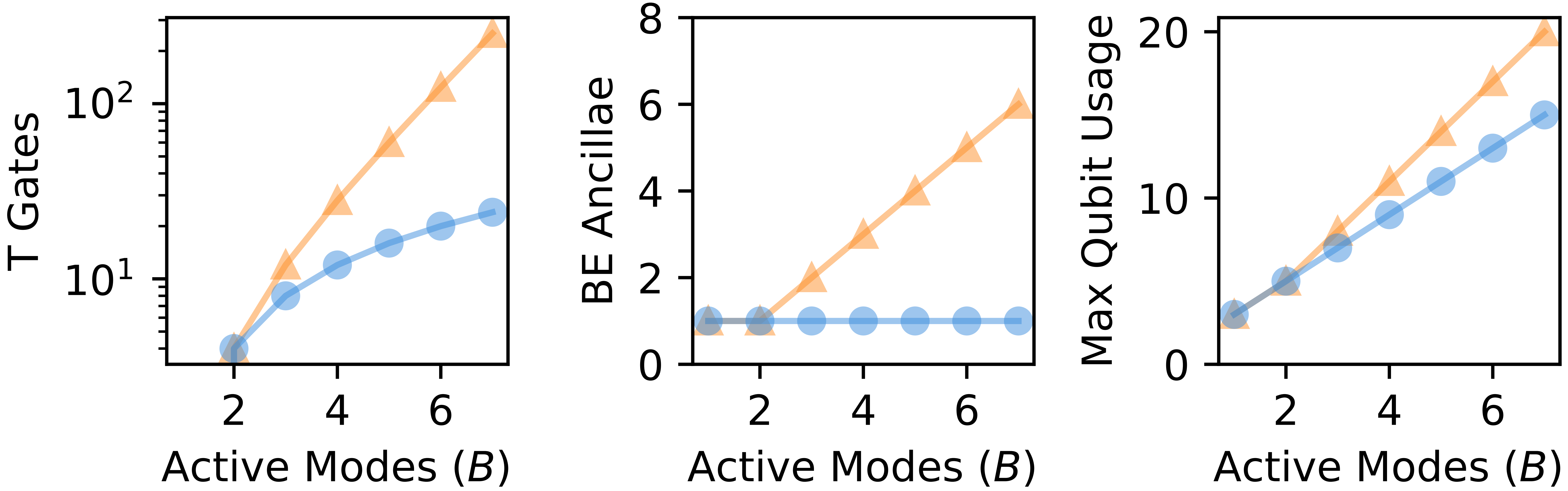
Comparative Performance of LOBE

$$O = b_0 b_1 \ldots b_B + h.c.$$



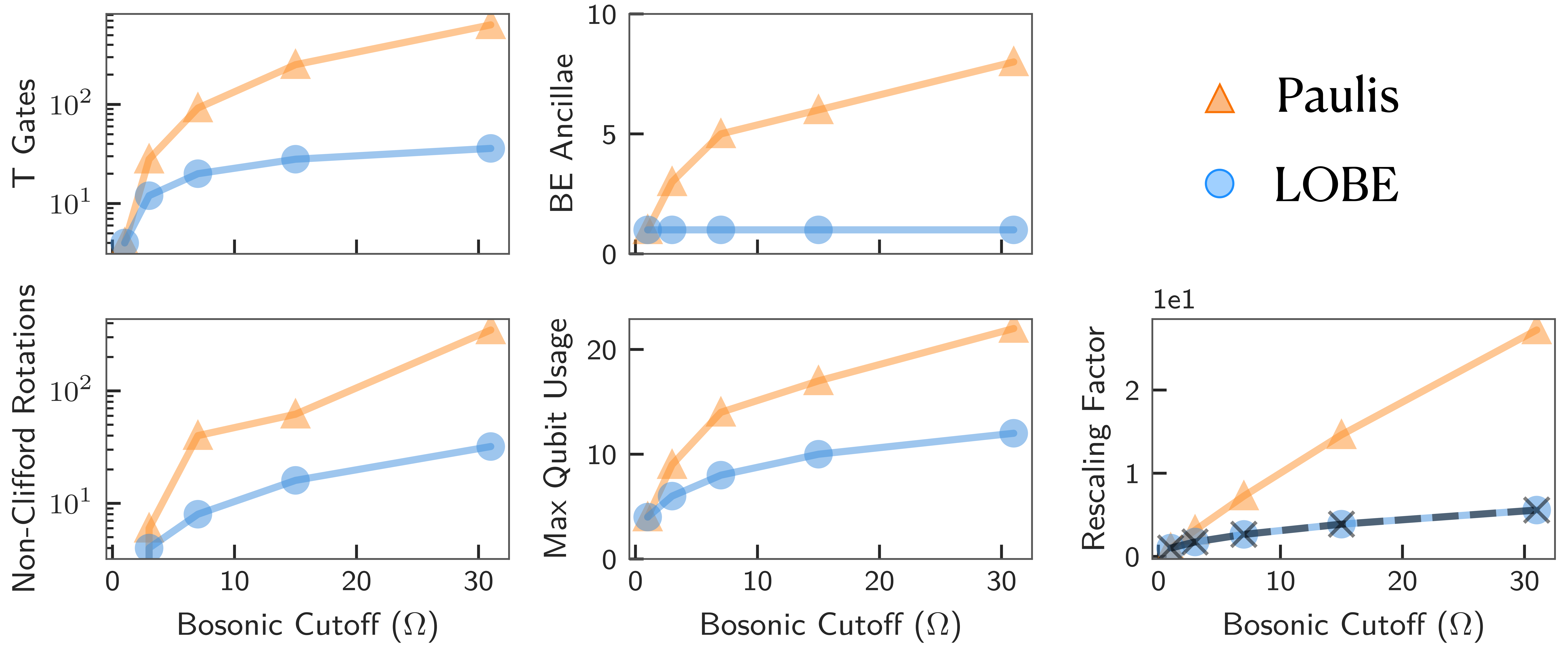
△ Paulis

○ LOBE

https://doi.org/10.22331/q-2025-12-22-1953. Simon, **Gustin**, et. al

Comparative Performance of LOBE

$$O = a_0$$



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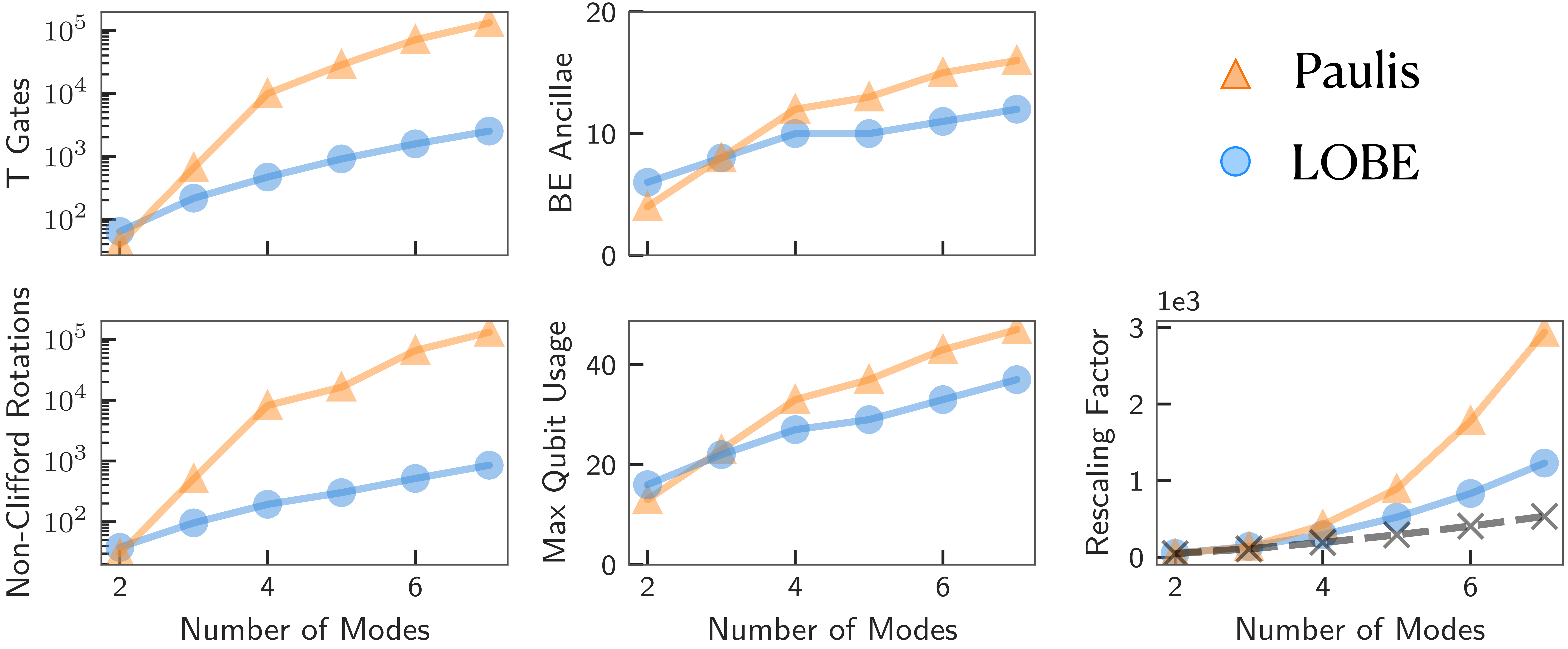
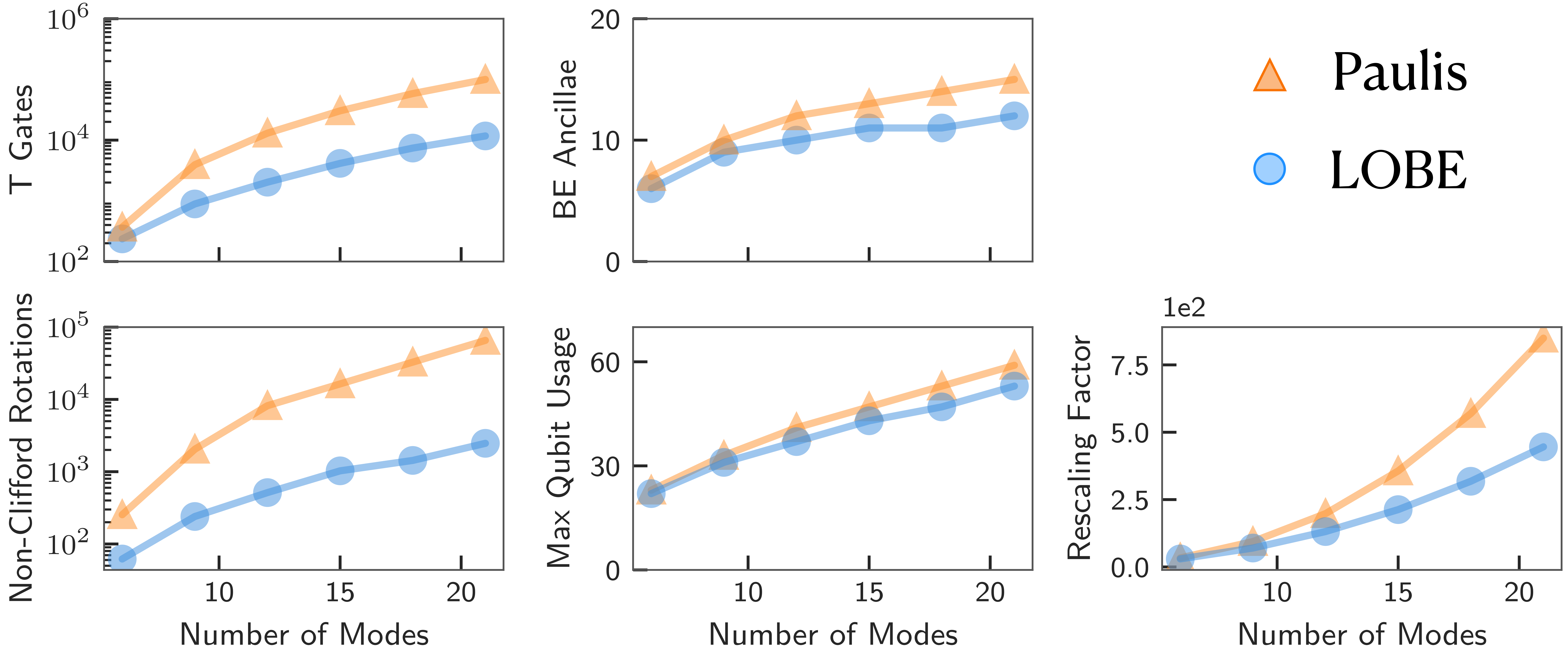
Comparative Performance of LOBE

ϕ^4 theory $\Omega = 3$



Comparative Performance of LOBE

Yukawa theory $\Omega = 3$



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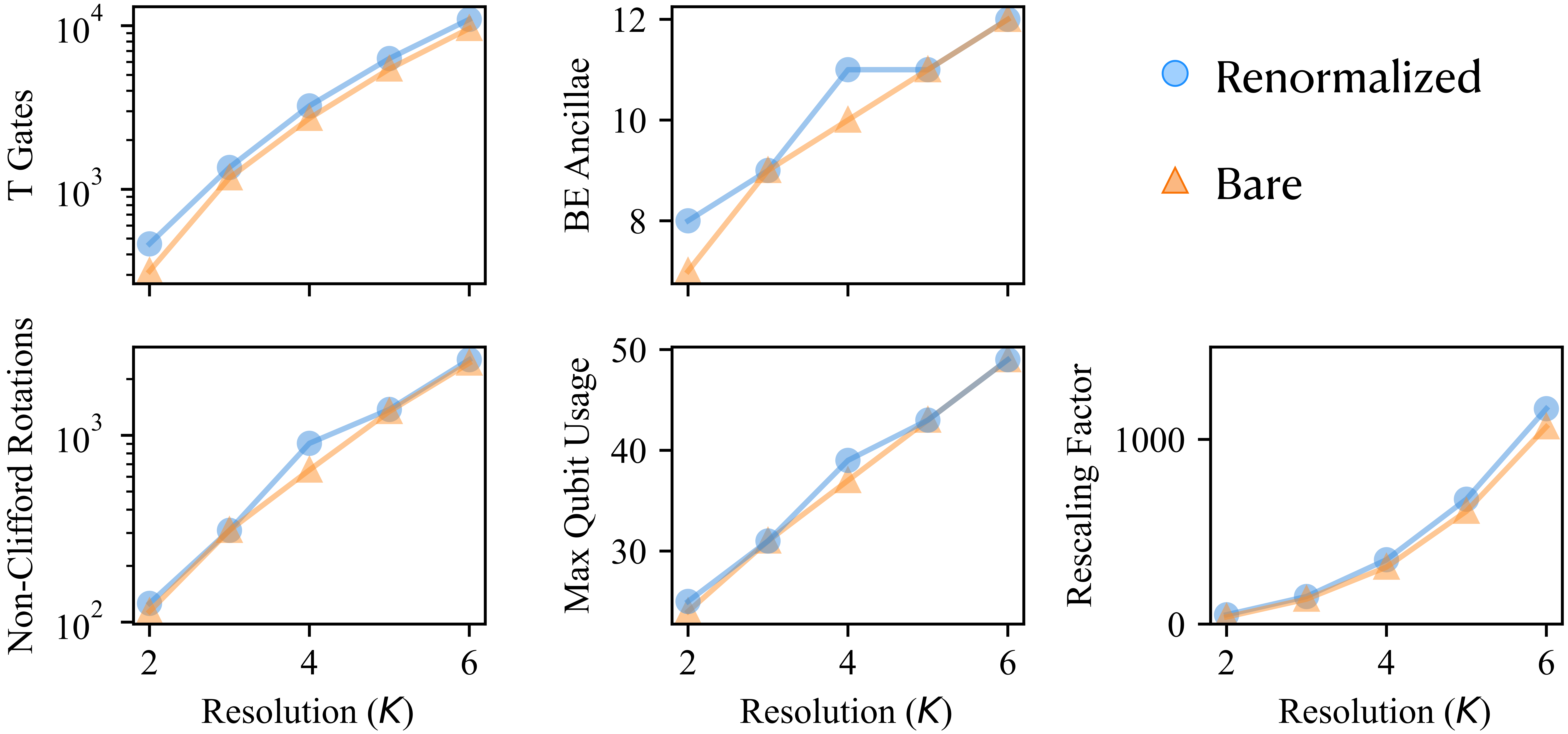
Does renormalization prohibitively increase the cost of quantum simulation?

Quantum Resource Estimates: Yukawa Theory



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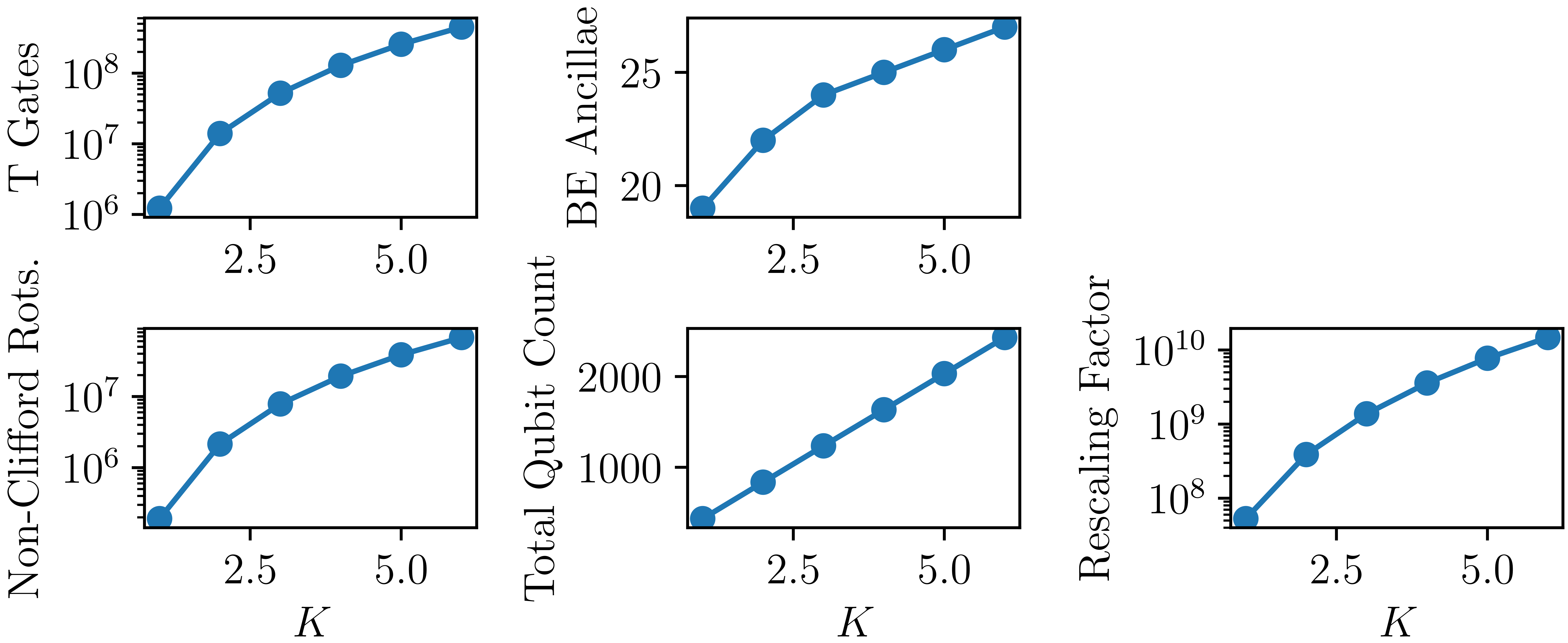
Quantum Resource Estimates: Renormalized QCD



Preliminary Results. **Gustin** et al.

Quantum Simulation of QCD Results in Context

	Number of T gates	Number of qubits

□ Time Evolution, 10^3 lattice (position space)

△ Eigenstate estimation, 10^3 lattice (momentum space)

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Gustin et. al (preliminary, pessimistic)△	$\mathcal{O}(10^{24})$	$\mathcal{O}(10^{5})$

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Prelude Motivation

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An end-to-end framework for quantum simulation of QFTs

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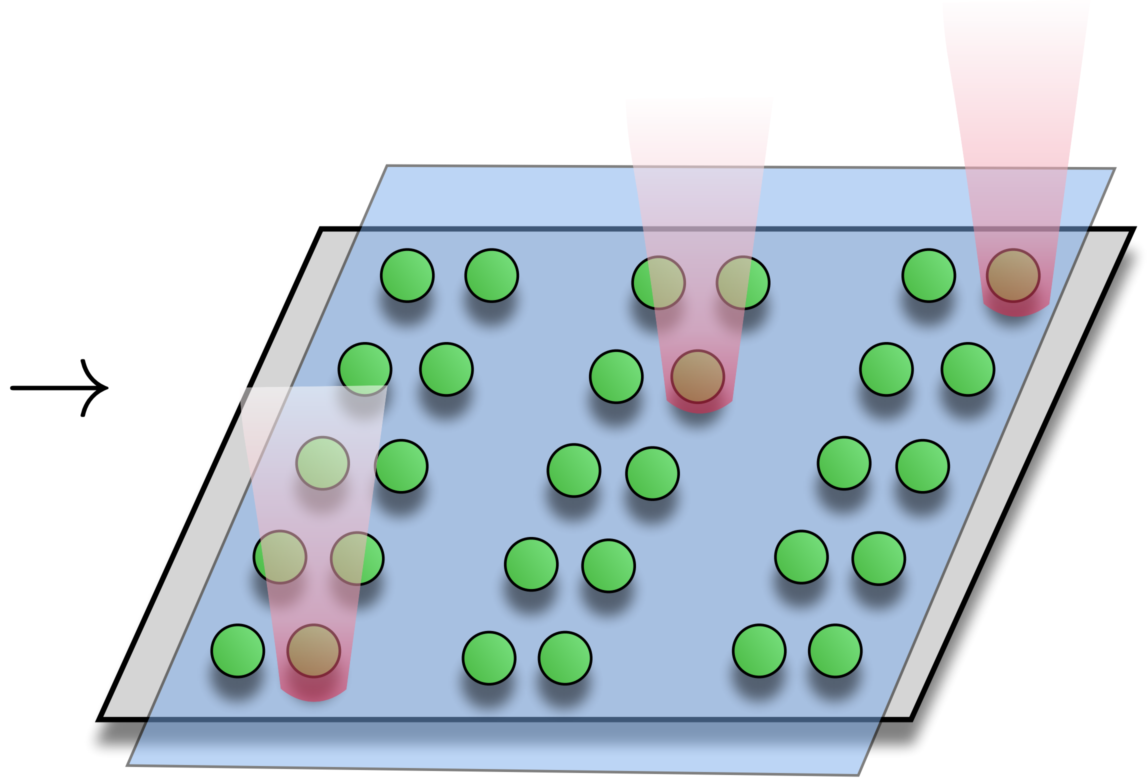
Part II: $\xrightarrow{LOBE} U_{\mathcal{H}(s)} \xrightarrow{\text{qubitization}} \mathcal{W}_{\mathcal{H}(s)} \to$



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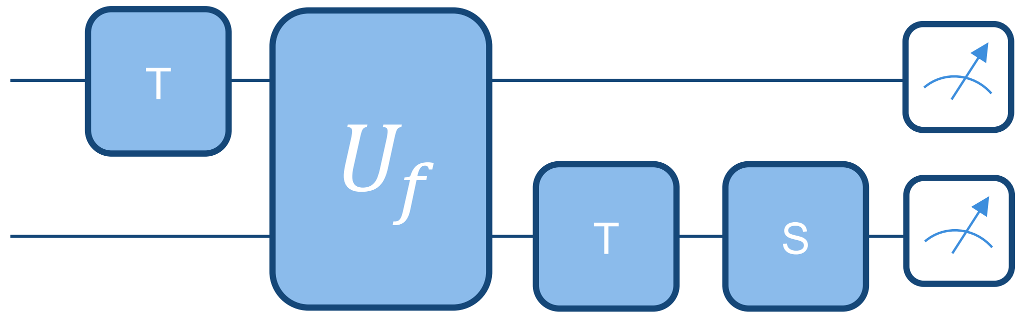
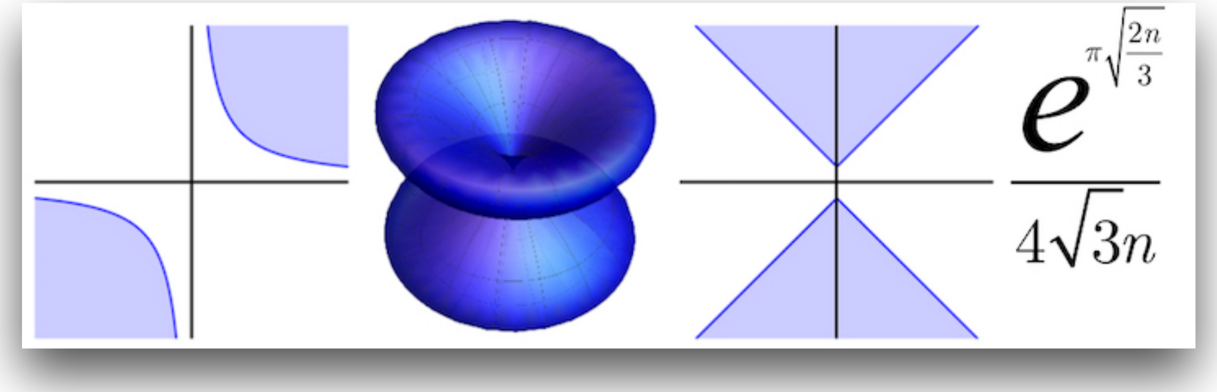
Open Particle

Tufts UNIVERSITY





Epilogue

Backup Slides

Quantum Computing

3 Parts:

Initialization

$$|0\rangle^{\otimes n} = \underbrace{|0\rangle \otimes \cdots \otimes |0\rangle}_{n}$$

Unitary evolution

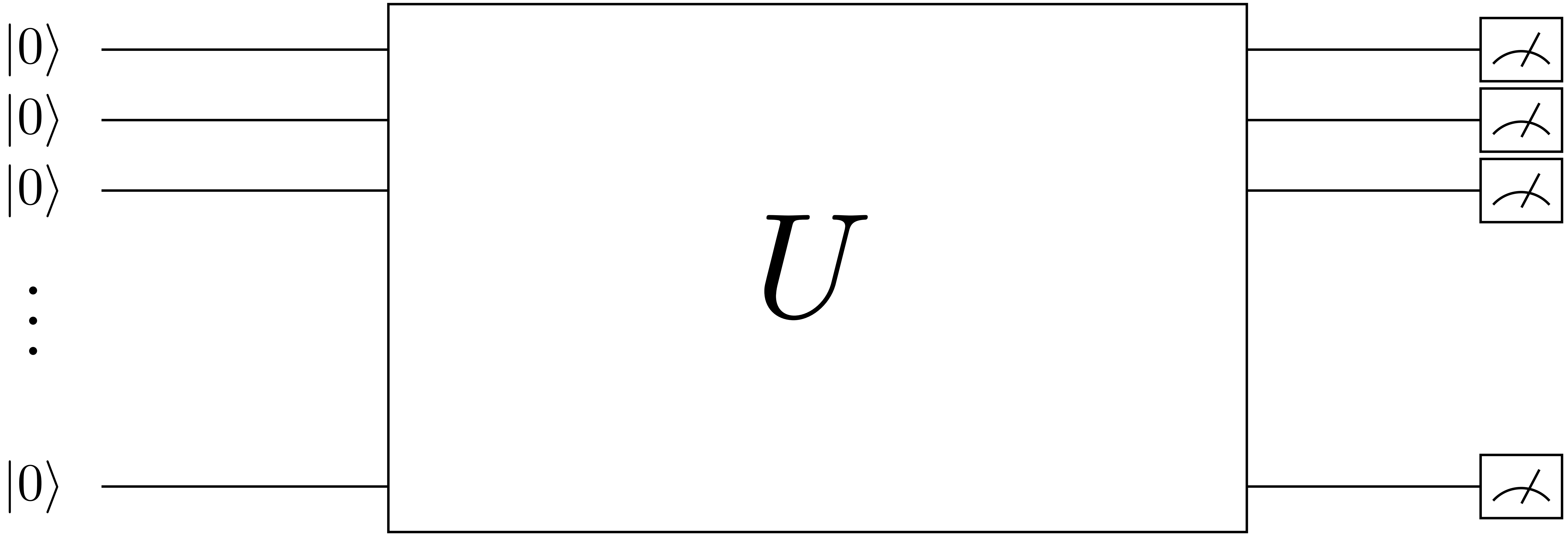
$$U = U_n \ldots U_1 U_0$$

$$U_i \in \text{universal gate set}$$

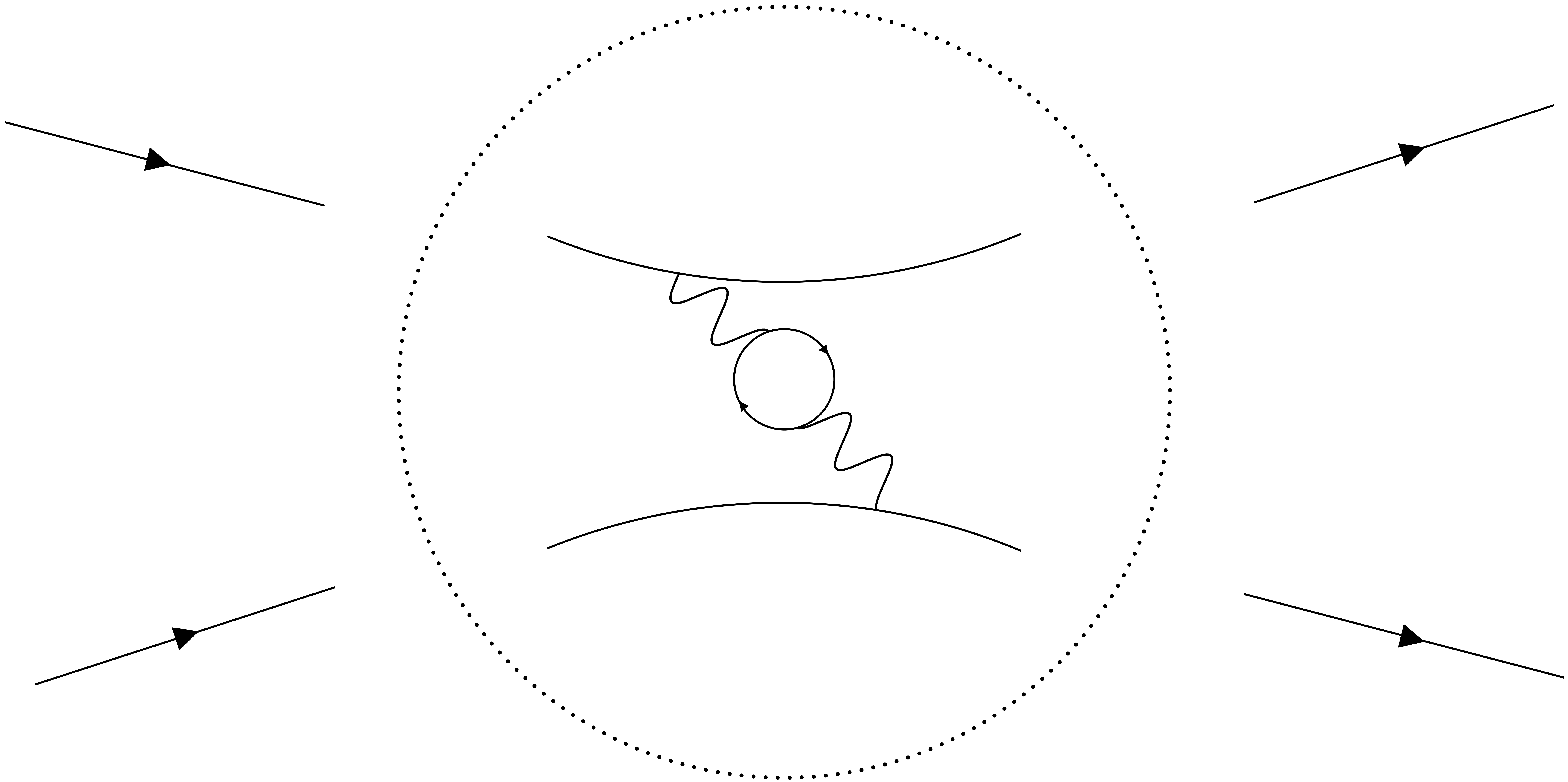
Measurement

$$|\psi\rangle = U|0\rangle^{\otimes n}$$

$$P(\phi) = \Big|\langle\phi|\psi\rangle\Big|^2$$



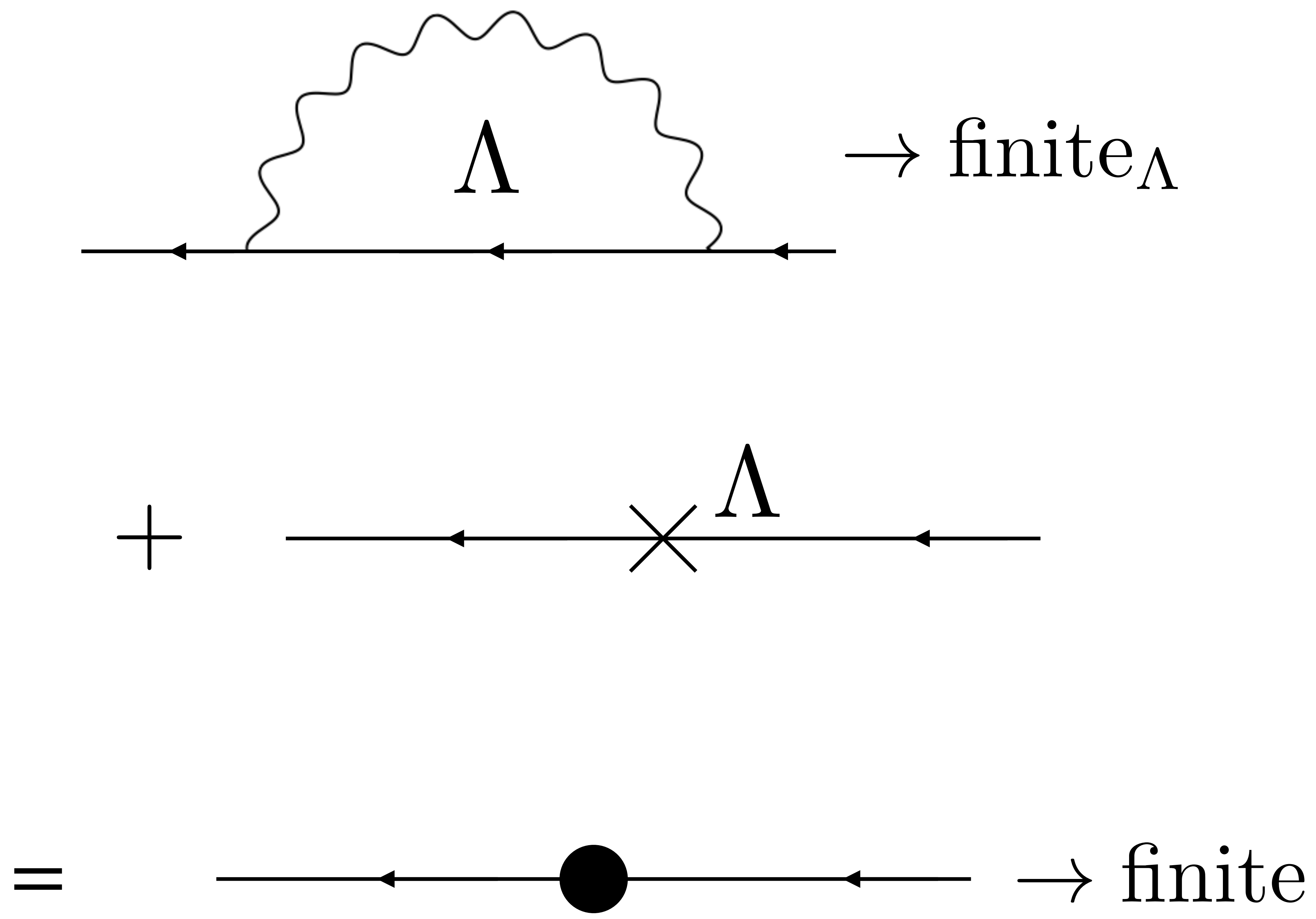
Divergences in Quantum Field Theory



The 4 Step Plan of Renormalization

Steps

1. Write down canonical theory
2. Regulate theory by imposing cutoffs
3. Fix counterterms
4. Observables converge as cutoff diverges



Trotterization

Find $U = U_n \dots U_1 U_0$ such that $\left|\left| U - e^{-iHt} \right|\right| \leq \epsilon$

$$H = \sum_{l=0}^{L-1} \alpha_l O_l \qquad \exp\left(- it \sum_{l=0}^{L-1} \alpha_l O_l \right) \neq \prod_{l=0}^{L-1} e^{-it\alpha_l O_l}$$

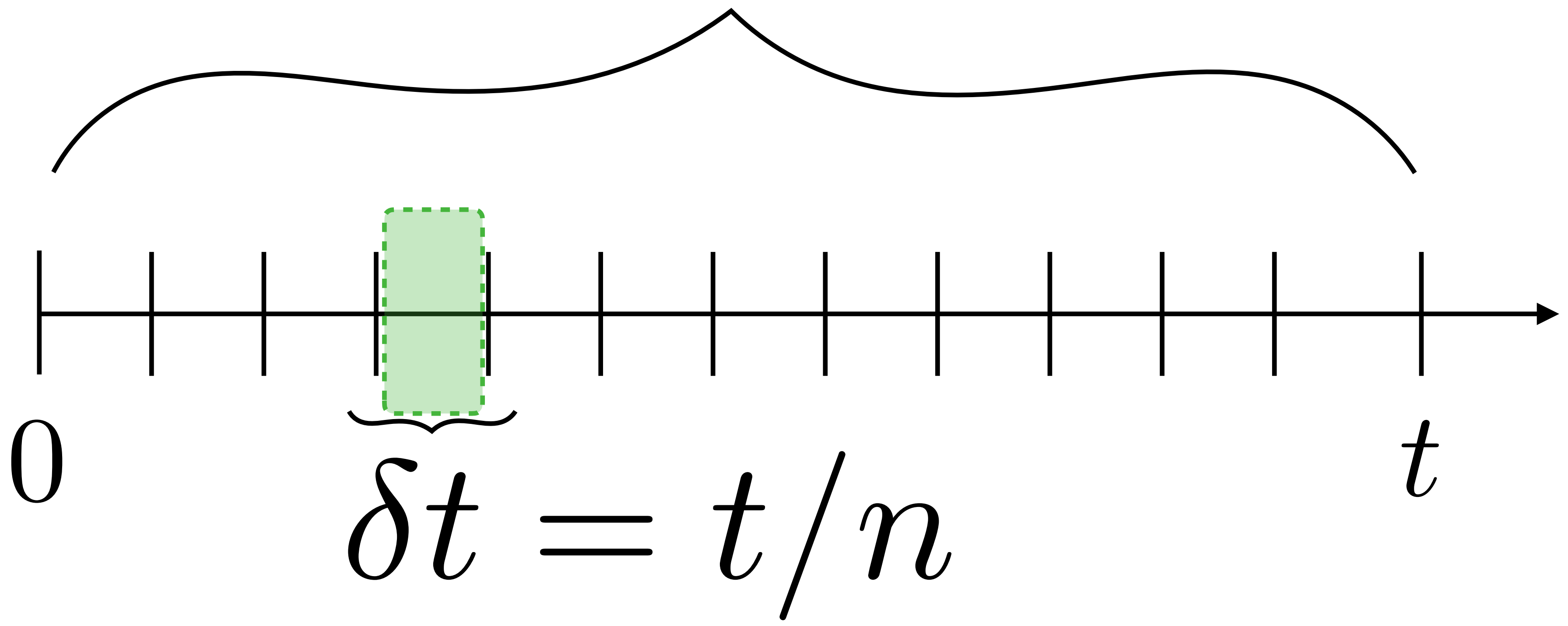
If $t \rightarrow \delta t \ll 1$ $$\exp\left(- i\delta t \sum_{l=0}^{L-1} \alpha_l O_l \right) \approx \prod_{l=0}^{L-1} e^{-i\delta t\alpha_l O_l}$$

$$\prod_{l=0}^{L-1} e^{-i\delta t\alpha_l O_l} = e^{-i\delta t\alpha_0 O_0} \; e^{-i\delta t\alpha_1 O_1} \; \dots \; e^{-i\delta t\alpha_{L-1} O_{L-1}}$$

Trotterization $t > \delta t$

$$e^{-iHt} = \left(e^{-iHt/n}\right)^n$$

$$\approx \left(\prod_{l=0}^{L-1} e^{-i\alpha_l O_l t/n}\right)^n$$

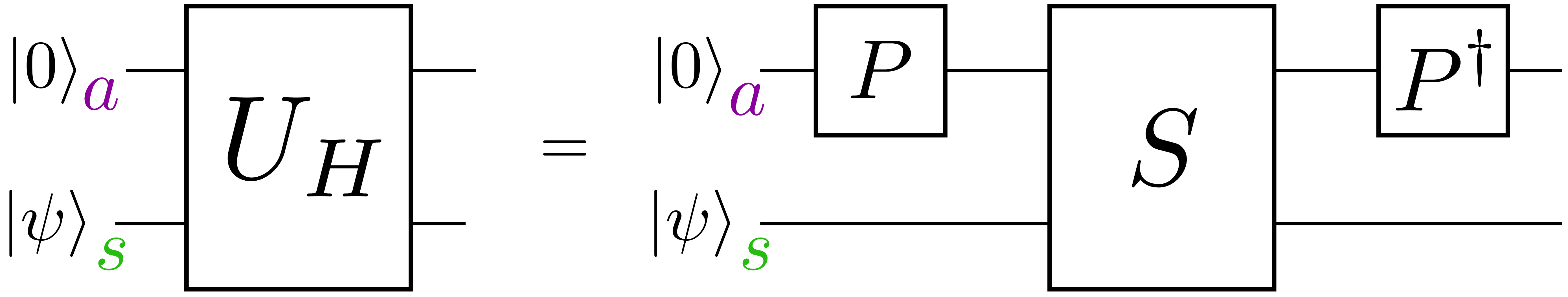


$$\left(\prod_{l=0}^{L-1} e^{-i\alpha_l O_l t/n}\right) = e^{-i\alpha_0 O_0 t/n} \; e^{-i\alpha_1 O_1 t/n} \; \cdots \; e^{-i\alpha_{L-1} O_{L-1} t/n}$$

On the Product of semi-groups of operators, Trotter

Linear Combination of Unitaries

$$H = \sum_l \alpha_l U_l$$



- PREPARE (P) unitary stores coefficients (α_l) in an ancilla register

$$P|0\rangle_a = \sum_l^L \sqrt{\frac{\alpha_l}{\alpha}}|l\rangle_a$$

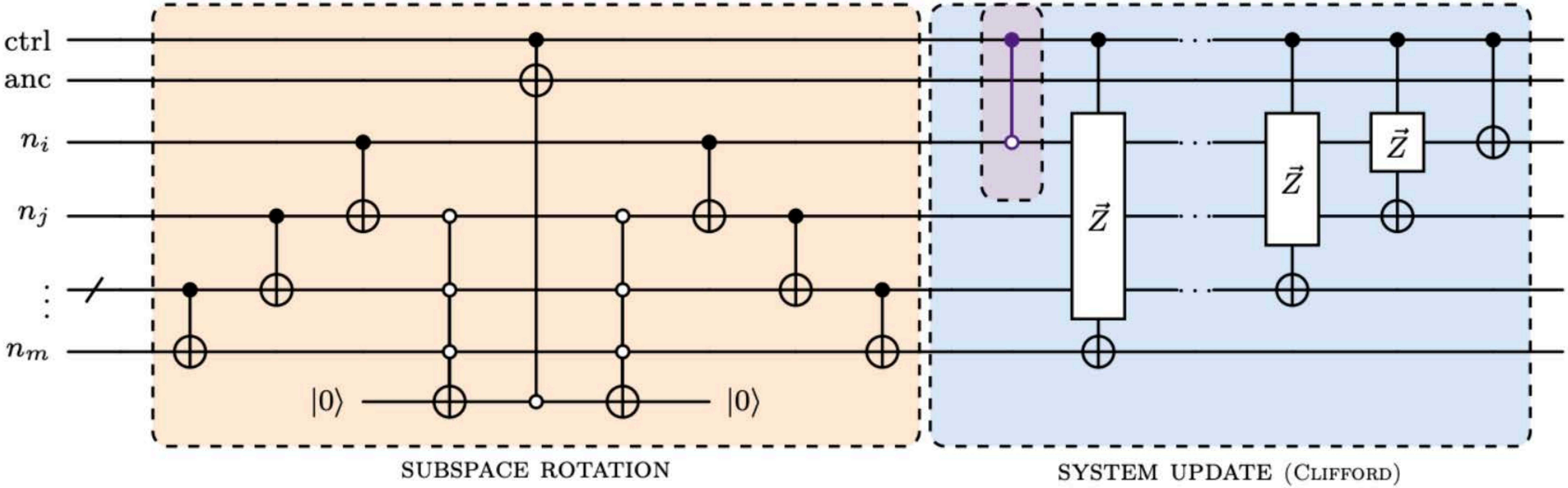
- SELECT (S) unitary indexes over U_l's

$$S = \sum_{l=0}^{L-1} |l\rangle_a \langle l|_a \otimes (U_l)_s$$

Hamiltonian Simulation Using Linear Combinations of Unitary Operations. Childs & Wiebe

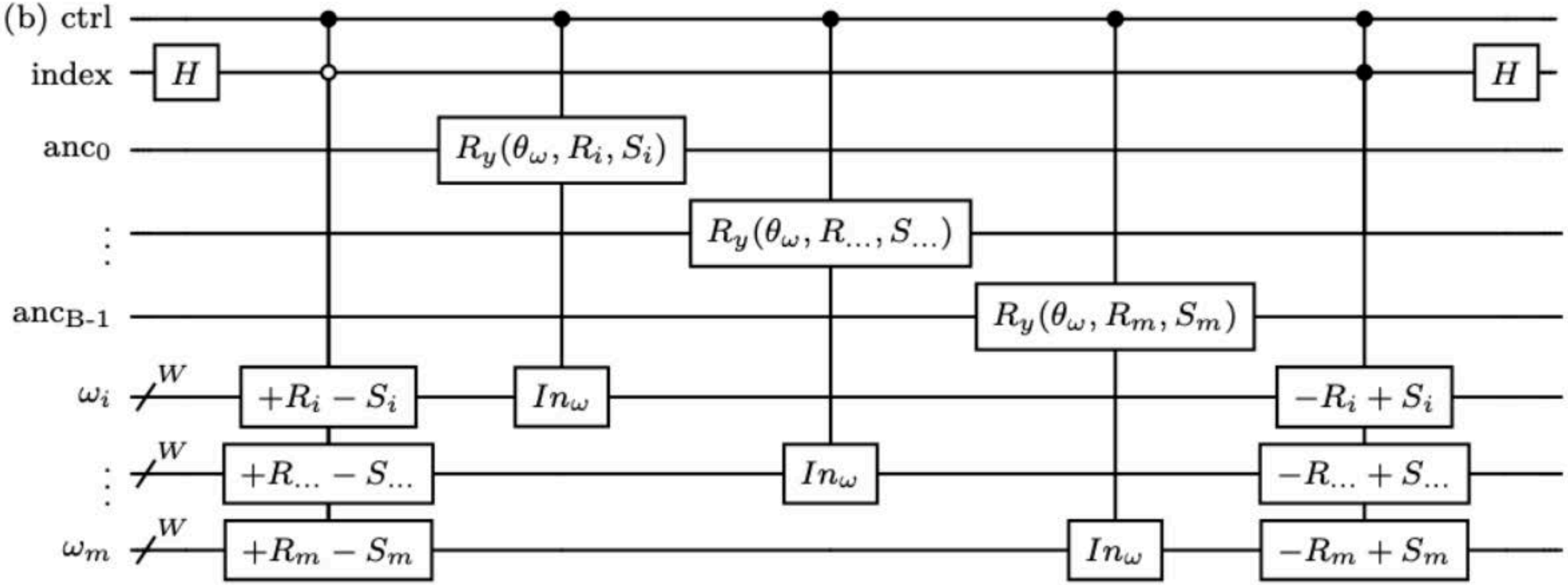
Fermionic LOBE Unitaries

$$O = b_i b_j \ldots b_m + b_m^\dagger \ldots b_j^\dagger b_i^\dagger$$



Generalized Bosonic LOBE Unitaries

$$O = (a_i^\dagger)^{R_i}(a_i)^{S_i} \dots (a_m^\dagger)^{R_m}(a_m)^{S_m} + h.c.$$



Why do we use the walk operator?

$$U_H^2 = \mathbb{I}$$

$$U_H|0\rangle_a|\lambda\rangle_s = \bar{\lambda}|0\rangle_a|\lambda\rangle_s + \sqrt{1-|\bar{\lambda}|^2}|\perp\rangle_{a,s}$$

$$U_H|\perp\rangle_{a,s} = \sqrt{1-|\bar{\lambda}|^2}|0\rangle_a|\lambda\rangle_s - \bar{\lambda}|\perp\rangle_{a,s}$$

$$U_H = \begin{pmatrix} \bar{\lambda} & \sqrt{1-|\bar{\lambda}|^2} \\ \sqrt{1-|\bar{\lambda}|^2} & -\bar{\lambda} \end{pmatrix}$$

U_H has eigenvalues ± 1

Spectral information is lost!

$$\mathcal{W}_H = U_H \Big(\underbrace{2|0\rangle_a \langle 0|_a - \mathbb{I}_a}_{\mathcal{R}_a} \Big)$$

$$\mathcal{W}_H|0\rangle_a|\lambda\rangle_s = \bar{\lambda}|0\rangle_a|\lambda\rangle_s + \sqrt{1-|\bar{\lambda}|^2}|\perp\rangle_{a,s}$$

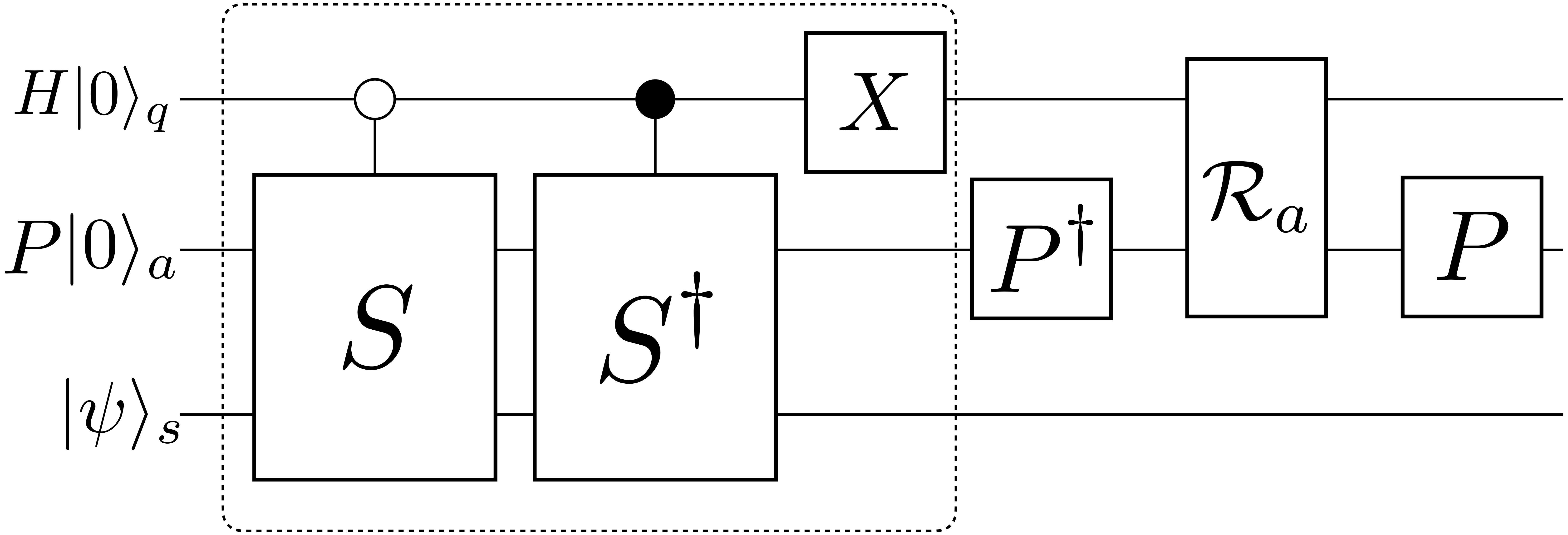
$$\mathcal{W}_H|\perp\rangle_{a,s} = -\sqrt{1-|\bar{\lambda}|^2}|0\rangle_a|\lambda\rangle_s + \bar{\lambda}|\perp\rangle_{a,s}$$

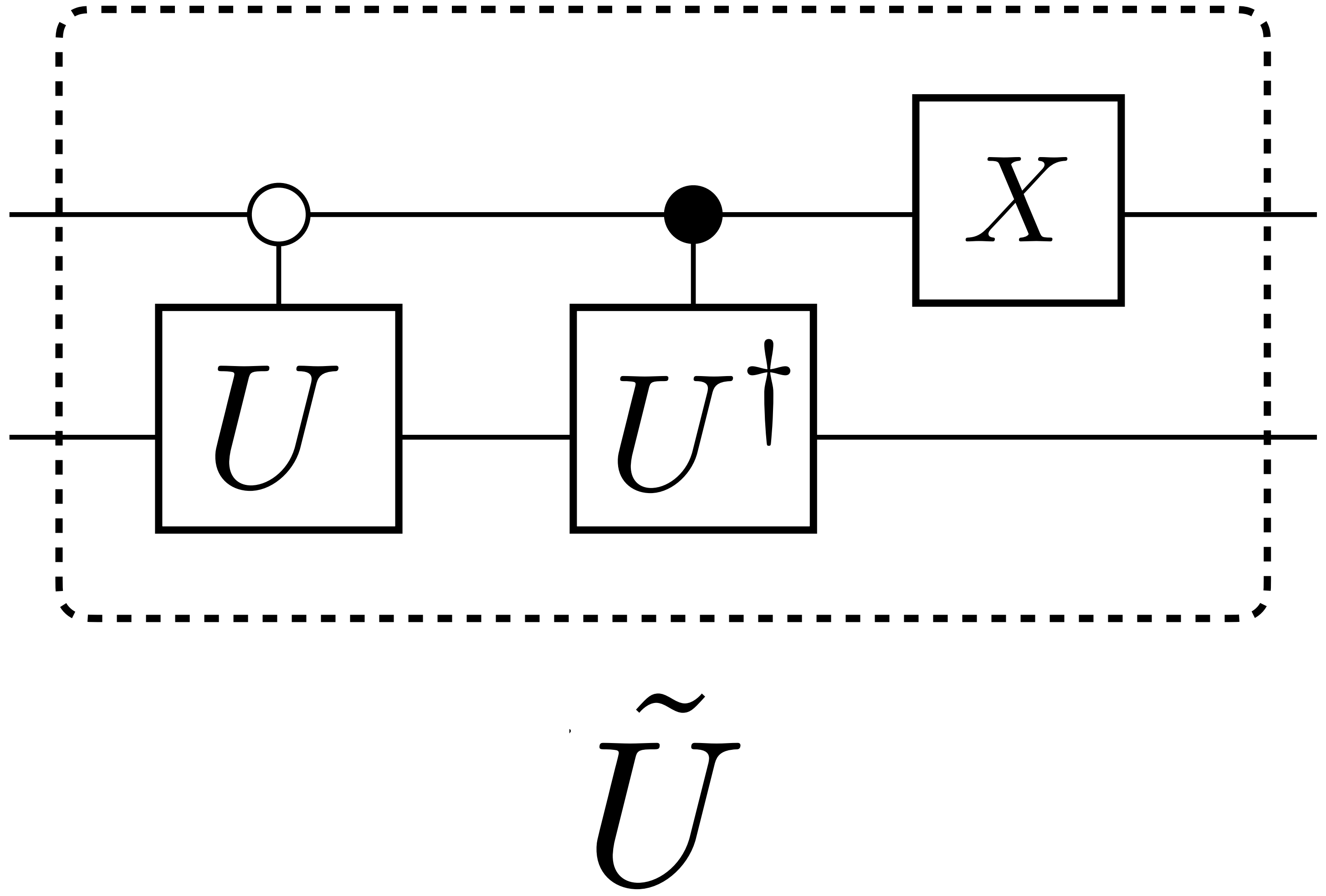
$$\mathcal{W}_H = \begin{pmatrix} \bar{\lambda} & -\sqrt{1-|\bar{\lambda}|^2} \\ \sqrt{1-|\bar{\lambda}|^2} & \bar{\lambda} \end{pmatrix}$$

has eigenvalues $\exp \pm i \arccos(\bar{\lambda})$

Existence of Qubitization

Lemma 10 (Existence of Qubitization). For all signal unitaries S that implement the Hermitian signal operator $\langle 0 | P^\dagger S P | 0 \rangle = H$, there exists a quantum circuit S', using one more qubit, that queries controlled-S and controlled-$S^\dagger$ once to implement the same signal operator. Moreover, S' satisfies the conditions Eq. (17)



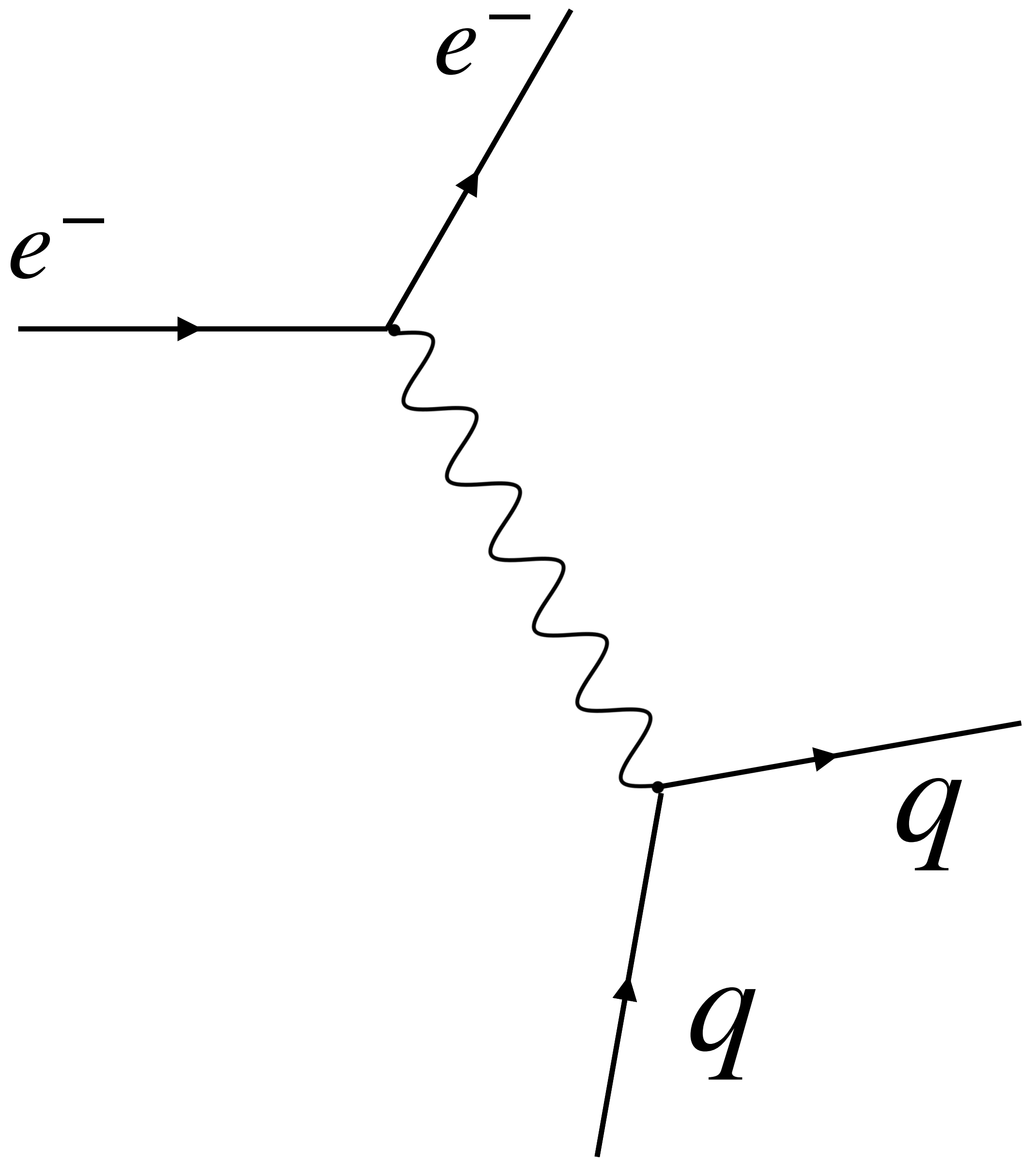


$$\boxed{(\tilde{U})^2 = \mathbb{I}}$$

The Factorization Theorem

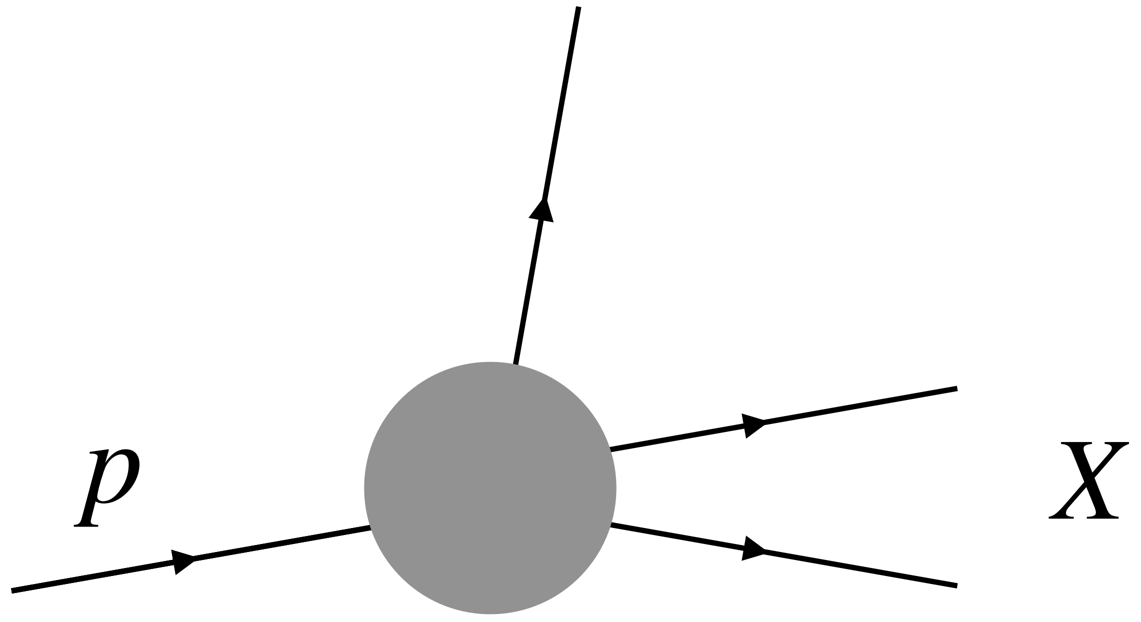
- We can separate the hard and soft part of the interaction
  - Hard: Perturbatively calculable
  - Soft: Not calculable in pQCD. Must resort to experimental data or models

Hard part:



To understand the soft part of DIS, and thus the total cross-section, we must understand nature of bound states of QCD

Soft part:



Conserved currents and Noether's Theorem

$\mathcal{L}(\phi, \partial\phi)$ invariant under $\phi \to e^{-i\alpha}\phi$

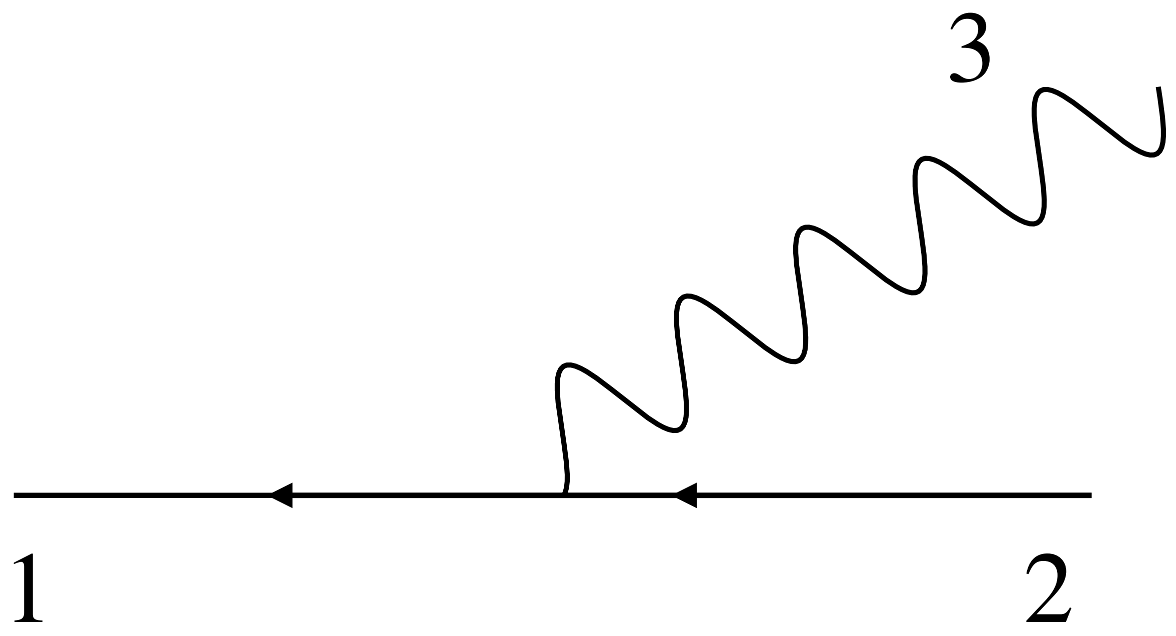
Noether's current: $J_\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_n)} \frac{\delta \phi_n}{\delta \alpha}$ is conserved $\partial^\mu J_\mu = 0$

$$Q = \int d^3x J_0$$

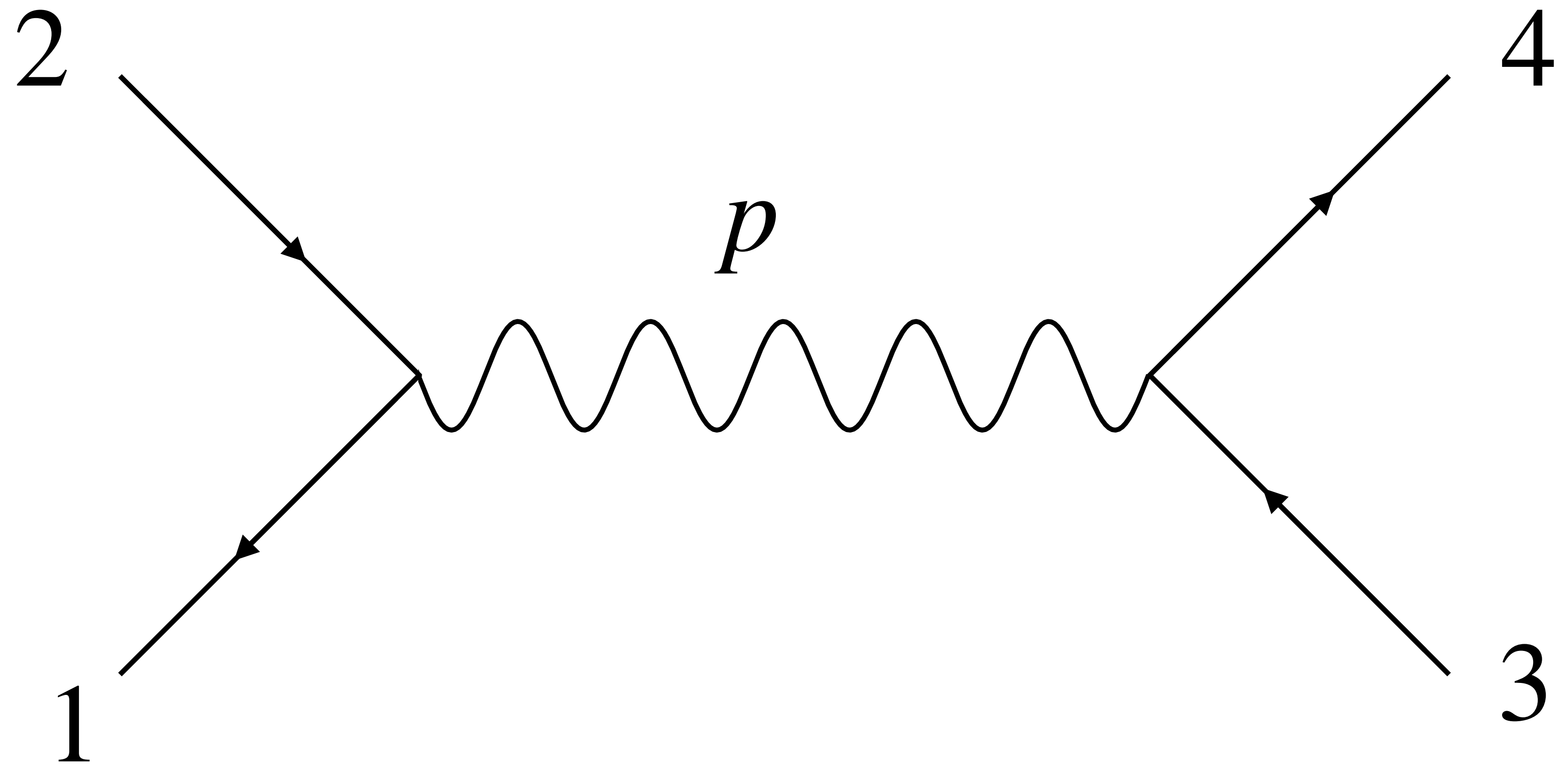
$$\partial^t Q = \int d^3x \partial^t J_0 = \int d^3x \vec{\nabla} \cdot \vec{J} \underset{\uparrow}{=} 0$$

Vanishes at boundary

Effective Vertices

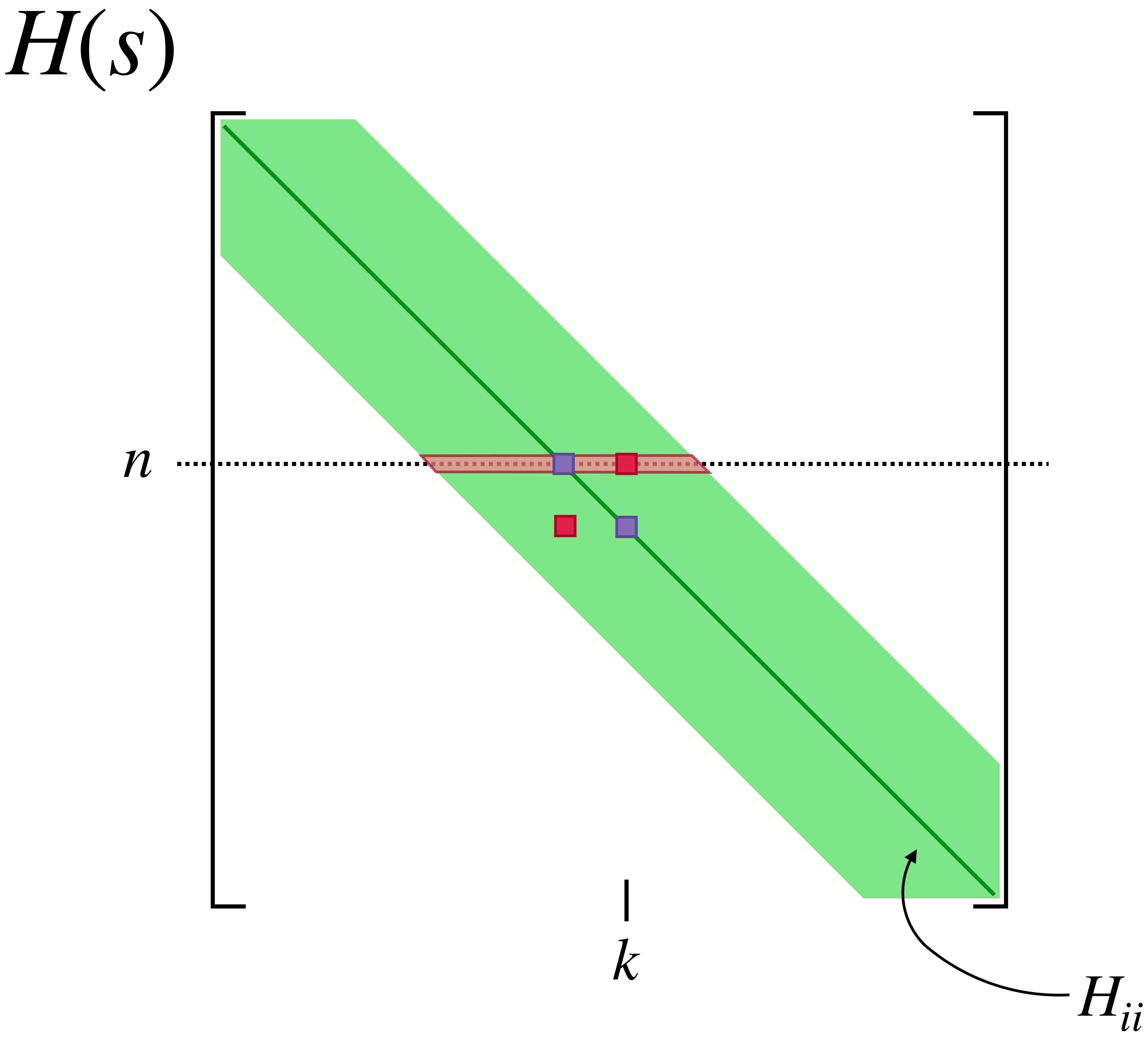


$$c_{123}(s) \sim \bar{u}_1 \gamma^\mu u_2 \epsilon_{\sigma\mu} e^{-s(p_1^- - p_2^- - p_3^-)^2} \, e^{-s_r(p_1^- - p_2^- - p_3^-)^2}$$



$$\begin{aligned} c_{1234}(s) \sim \bar{u}_1 \gamma^\mu u_2 \frac{d_{\mu\nu}(p)}{p^+} \bar{u}_3 \gamma^\nu u_4 \, & e^{-s_r(p_1^- + p_2^- - p_3^- - p_4^-)^2} \\ & \times \frac{1}{2} \left(\frac{1}{p_1^- + p_2^- - p^-} - \frac{1}{p_3^- + p_4^- - p^-} \right) \\ & \times \Big(e^{-s(p_1^- + p_2^- - p^-)^2} e^{-s(p_3^- + p_4^- - p^-)^2} \\ & \qquad - e^{-s(p_1^- + p_2^- - p_3^- - p_4^-)^2} \Big) \end{aligned}$$

Why is a band-diagonal $H(s)$ preferred?



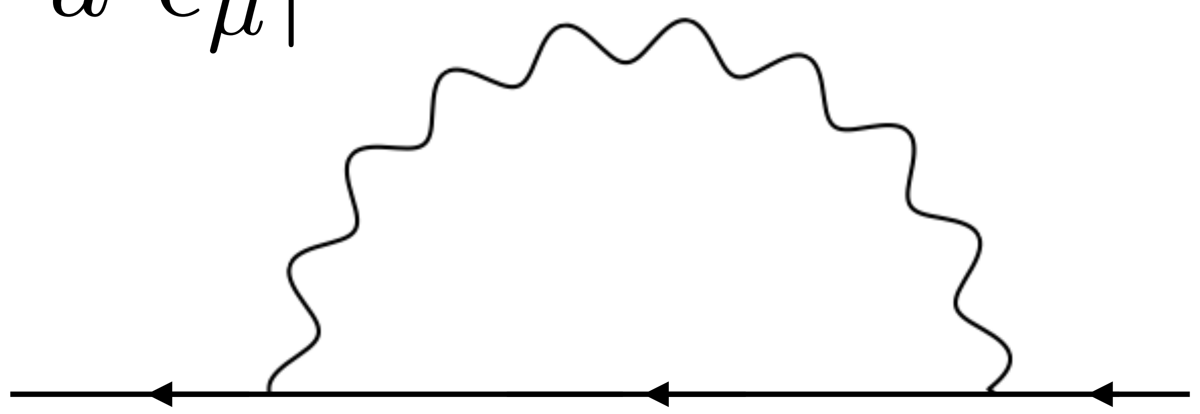
$$E_n^{(2)} = \sum_{k \neq n} \frac{H_{nk} H_{kn}}{E_n^{(0)} - E_k^{(0)}}$$

For canonical H, no guarantee this sum converges:

e.g. take $h_{ij} = 1$ $\sum_{n=1}^{\infty} \frac{1}{n} = \infty$

$$H_{nk}^{(1)}(s) H_{kn}^{(1)}(s) \sim |\bar{u}\gamma^{\mu} u \; \epsilon_{\mu}|^2$$

$\sim$



SRG Flow

$$\frac{dH(s)}{ds} = \Bigg[\underbrace{-U^\dagger(s)\frac{dU(s)}{ds}}_{\mathcal{G}(s)}, H(s)\Bigg]$$

$$U(s) = T_s \exp\left(\int_0^s ds' \mathcal{G}(s')\right)$$

$\mathcal{G}(s)$ "generator" $\mathcal{G}_{ij}(s) = -\mathcal{G}^*_{ji}(s)$

Generators:

Mielke: $\mathcal{G}(s) = M \odot H(s)$

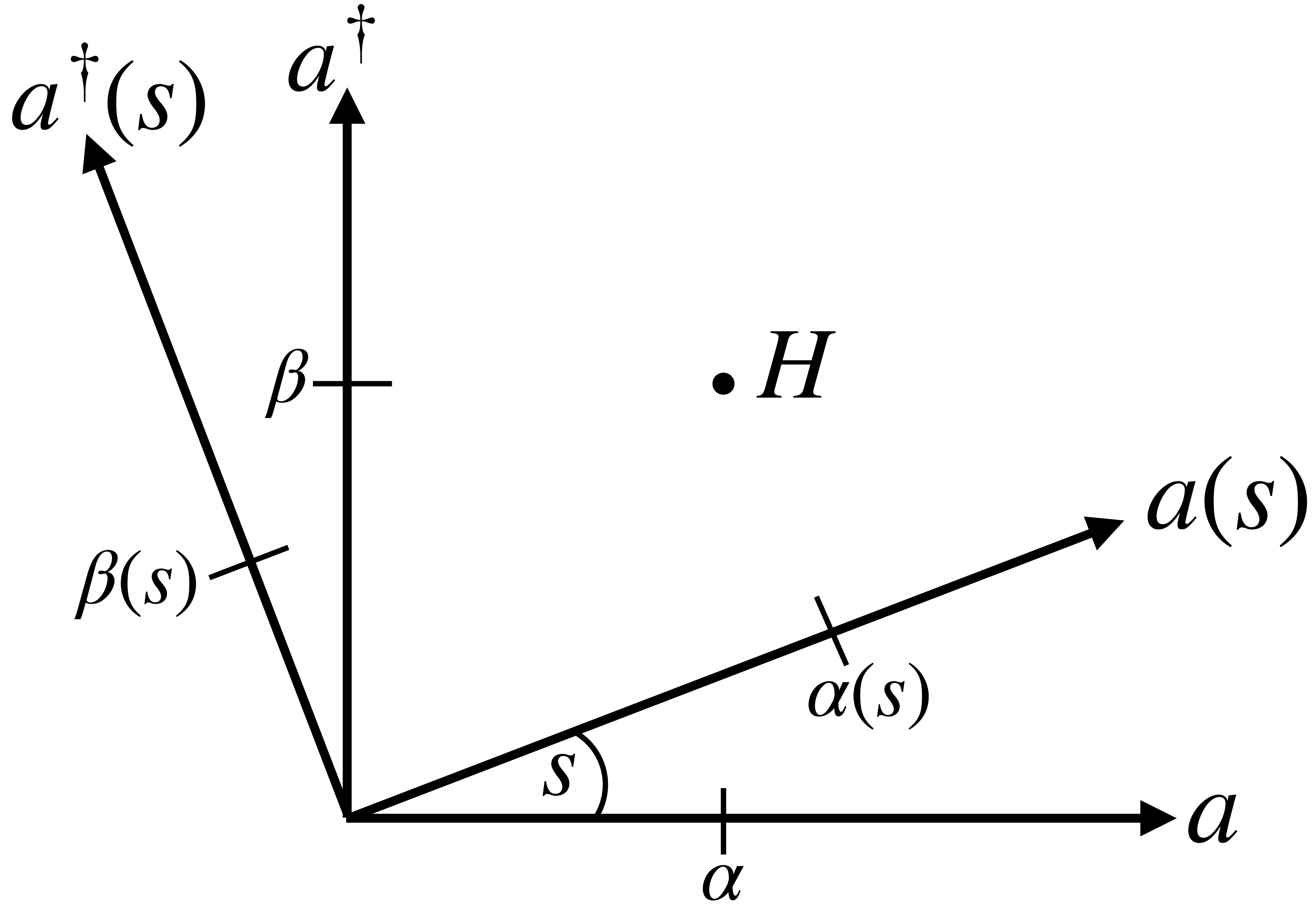
Głazek-Wilson: $\mathcal{G}(s) = \Big[H_0, H(s)\Big]$

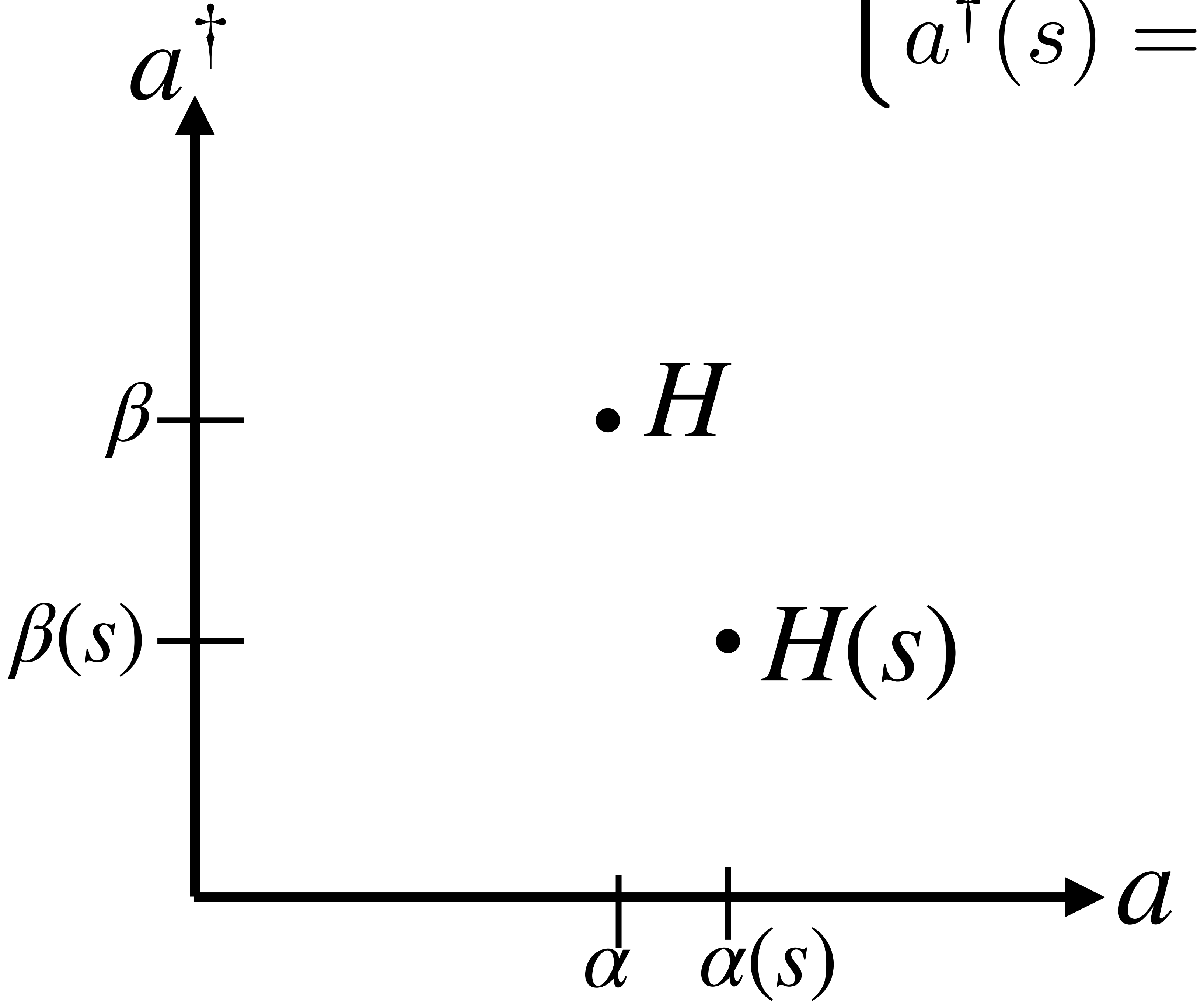
Wegner: $\mathcal{G}(s) = \Big[\mathrm{diag}\Big(H(s)\Big), H(s)\Big]$

Block-diagonal: $\mathcal{G}(s) = PH(s)P + QH(s)Q$

Unitary transformation of H

$$\begin{cases} a(s) = U(s)aU^\dagger(s) \\ a^\dagger(s) = U(s)a^\dagger U^\dagger(s) \end{cases}$$





$$H = \alpha a + \beta a^\dagger$$

$$H = \alpha(s)a(s) + \beta(s)a^\dagger(s)$$

$$\begin{aligned} H(s) &= \alpha(s)a + \beta(s)a^\dagger \\ &= U(s)^\dagger H U(s) \end{aligned}$$

Mapping Fermions/Bosons to Qubits

Fock

State

$$|n\rangle = \bigotimes_i |n_i\rangle$$

Fermions: $n_i \in \{0, 1\}$

Bosons: $n_i \in \{0, 1, 2, \dots, \Omega\}$

Operator

Fermions:
$$\begin{cases} b^\dagger|0\rangle = |1\rangle \\ b|1\rangle = |0\rangle \end{cases} \quad \left\{b, b^\dagger\right\} = \mathbb{I}$$

Bosons:
$$\begin{cases} a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle \\ a|n\rangle = \sqrt{n}|n-1\rangle \end{cases}$$
$$\left[a, a^\dagger\right] = \mathbb{I}$$

$$\xrightarrow[\text{Standard Binary}]{\text{Jordan Wigner}}$$

Qubit

Fermions: $n_i \rightarrow q_i \in \{0, 1\}$

Bosons: $n_i \rightarrow \lceil \log_2(\Omega + 1) \rceil_i$ qubits

Fermions: $b_i \rightarrow \bigotimes_{j<i} Z_j \frac{1}{2}\Big(X_i + iY_i\Big)$

$-$ for $b_i^\dagger$

Bosons:
$$a^\dagger = \sum_{s=0}^{\Omega-1} \sqrt{s+1}|s+1\rangle\langle s|$$
$$a = \sum_{s=0}^{\Omega-1} \sqrt{s+1}|s\rangle\langle s+1|$$