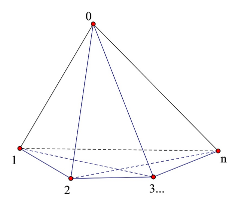
Lecture 41 of Adrian Ocneanu

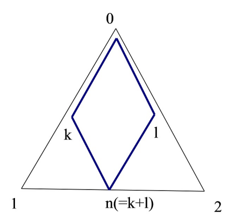
Notes by the Harvard group

Lecture notes for 8 December 2017.
Today we will define tunnels.

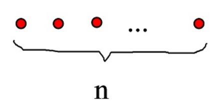
We work for the higher sl(2) over sl(n). This is a pyramid like this:



Remember in sl(2) case, this is the representation of σ_n of sl(2), a vector $e_1^k e_2^l$ where n is the



degree we took a point with multiplicity $n = k + l$. This is interpreted as the Young diagram which

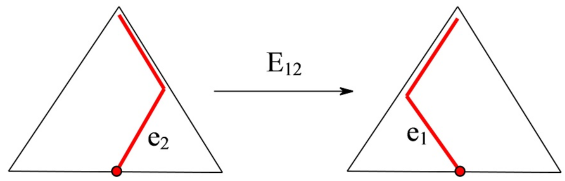


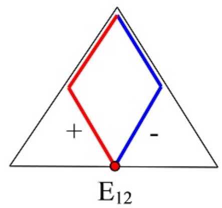
is growing into the blades above. Moreover

$$e_1^k e_2^l \xrightarrow{E_{12}} coef\ E_1^{k+1} e_2^{l-1}$$

Graphically, E_{12} acts as: We can thus represent the tunnel:

Assume now we have an intertwiner on the base: where a, b, c are multiplicities. So the bottom is the intertwiner and a blade like this can grow either into a triangle or a square: Now the triangle is a degenerate blade with given boundary at the base. At the bottom, the blade is $1, 23$. It is a partition of coordinates into 2 nonempty parts. and So it means that the way to grow it is to put in turn a





zero in each of the parts. This is how a blade grows in a 2 by 2 simplex, you take the top coordinate and you add it into either box.

Graphically, cut a section of the simplex: This is how you describe by section the blades which are inside the 2 by 2 period, they are in the 3 by 3 plane. (19:30)

We can define an operator $E_{1,23}$ graphically on this section:

$$E_{1,23} \;:\; 01, 23 \;\rightarrow\; 1, 023$$

In general we have some operator $E_{A,A'}$, where $A \subset \{1, 2, ..., n\}$, $A' = A^c$. The best way to see this is with sections. The operator $E_{1,23}$ acts on the 3 sections like: If you have an operator $E_{A,A'}$, on the section $, i, j$, it acts like This is what we call a tunnel for $sl(2)$ over $sl(n)$. This means you are $n-2$ dimenions higher than the usual Gelfand-Tsetlin.

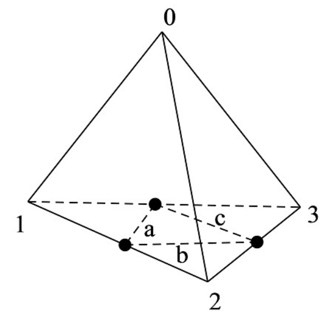
(See answer to question at 32:42.)

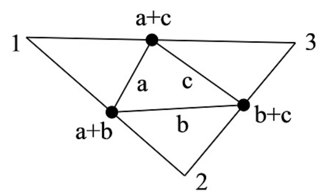
$$six\ j \; = \sum \pm \frac{(\text{number of blades} + 1)!}{\prod(\text{multiplicity of blades})!}$$

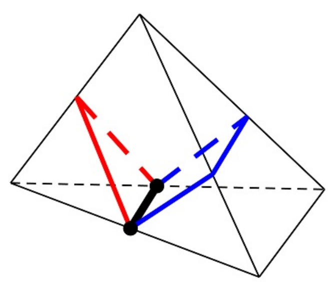
Let me show them to you on the simplex here: the blades expand the box in the 3 directions. The triangles give the length of the prongs. (See video 51:05.)

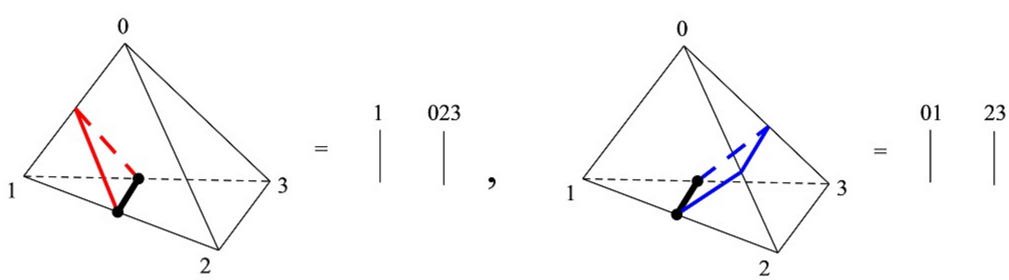
We will call our freedom of choice *gauge*. So the gauge is exactly 4 triangles going to 3 squares.

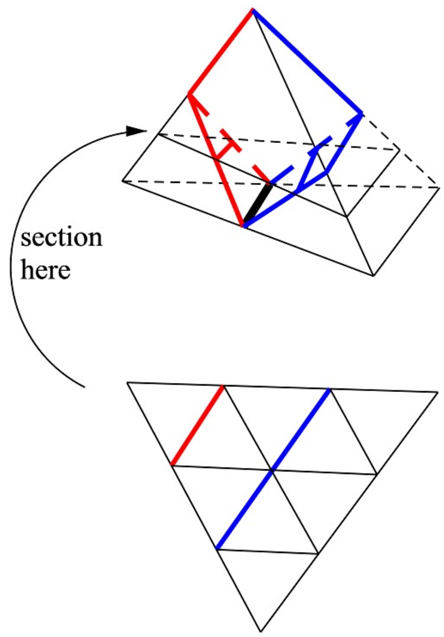
Here the box can grow by 1 each direction. This way the prongs will get smaller by 1. You can keep doing this until either a box or a prong goes to 0. This is the *breathing*. (See video 54:30 for breathing and curvature)

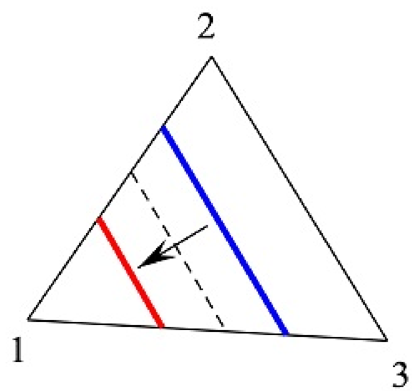

0
1
3
a
c
b
2

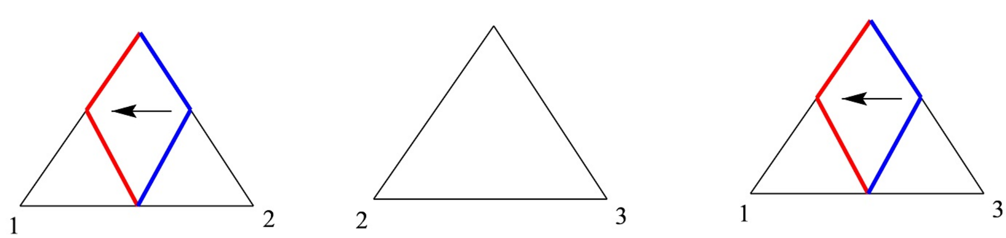

a+c
1
3
a
c
b
a+b
b+c
2

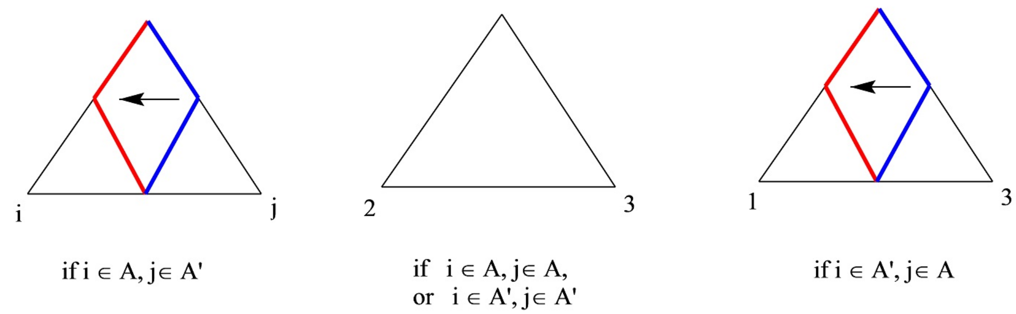



0
1
3
2
=
1
023
,
0
1
3
2
=
01
23

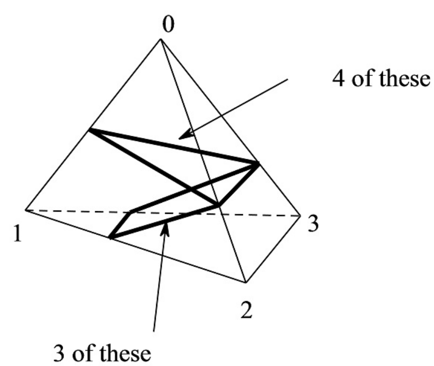

section
here


2
1
3

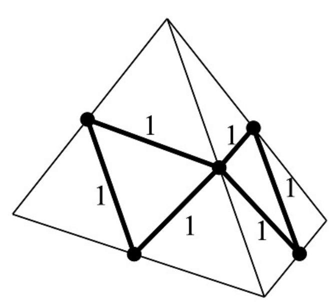

1
2
2
3
1
3



i
j
if i ∈ A, j∈ A'
2
3
if i ∈ A, j∈ A,
or i ∈ A', j∈ A'
1
3
if i ∈ A', j∈ A



0
4 of these
1
3
2
3 of these



1
1
1
1
1
1