Lecture 42 of Adrian Ocneanu

Notes by the Harvard group

Lecture notes for 13 December 2017.

This course is about higher representation.We have fundamental representation.The details of this proof was given in August on Arthur's black board. Please look at figure 1.7.

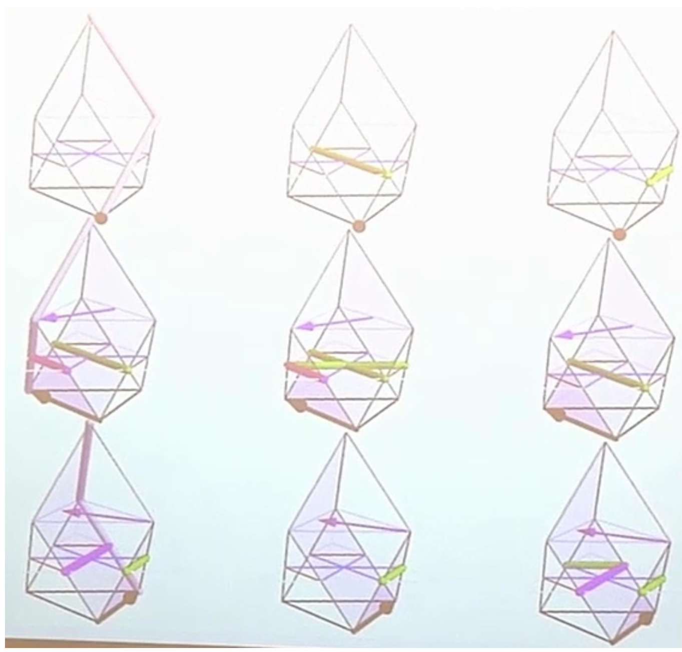

Figure 1:

What you see once again are some normalized coefficients outside,which are the multiplicity of these vectors.Next you can see is the higher theory we abtain from some thing which is on the boundary.The inside of a simplex here make the boundary into nothing which in quantum field theory coreponding to a complex number.

In particular,we have a simplex like this

This is sl_2 over (w.suljacent).Here we have $sl_n, n = 3$.Next we give an operation

$$E_{12}^{A,A'}.$$

For example

$$A = \{2, 3\}, A' = \{1\}, E_{12}^{A,A'} = E_{12}^{23,1}.$$

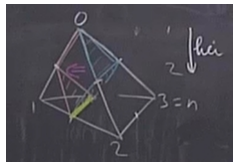

Figure 2:

We add top with height 1,the move is from 023,1 into 23,01.And the botton with height 2,the move is from 23,01 into 023,1.

For usual Gelfond Tsetlin,we have some path going like this Here is $sl(4)$ over $sl(2)$ and the act

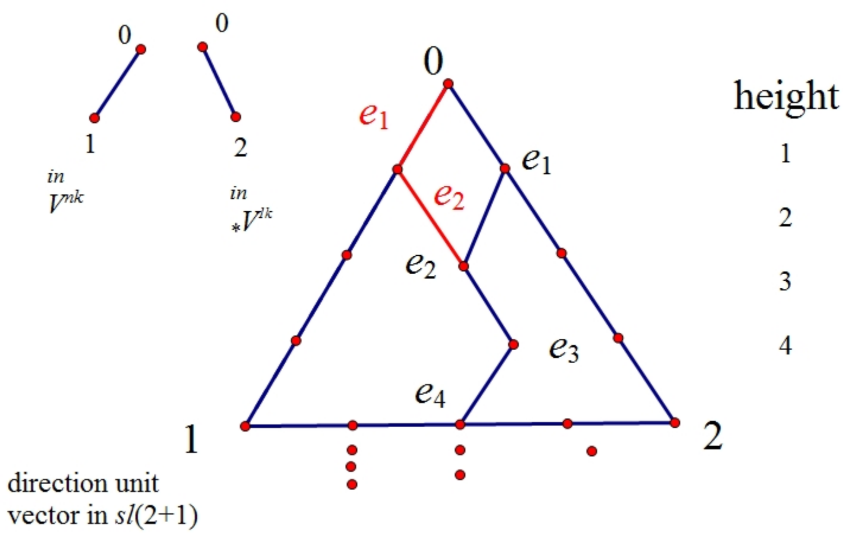

Figure 3:

of $sl(4)$ on $\mathbb{C}^4 = V^{\wedge 2}$.Hodge dual(*):

$$e_1 \wedge e_3(-)e_2 \wedge e_4,$$

where e_2 present and e_1 absent.E_{12} maps $e_2 \wedge e_4$ to $e_1 \wedge e_4$.Every blade sepatates $1, 2, ..., n$ into 2 subsets,which is called set composition.The operator $E_{12}^{A,A'}$ of height 1,2 maps

$$\begin{aligned} &height\ 1\ A, OA' \Rightarrow OA, A', \\ &height\ 2\ OA, A' \Rightarrow A, OA'. \end{aligned}$$

Sections

Please look at figure 1.10.It belongs to sl_2 over sl_2,where i and j in $\{1, 2, ..., n\}$,$i \neq j$. This is an induction to Gelfond-Tsetlin.

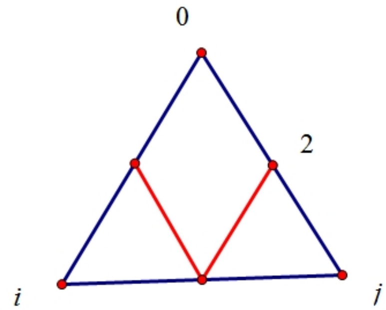

Figure 4:

Now we need the general case.Then we take the higher matrix.So let us see what happens in the general case now. Please look at figure 1.11. So when you look,this is why simplex exits in any

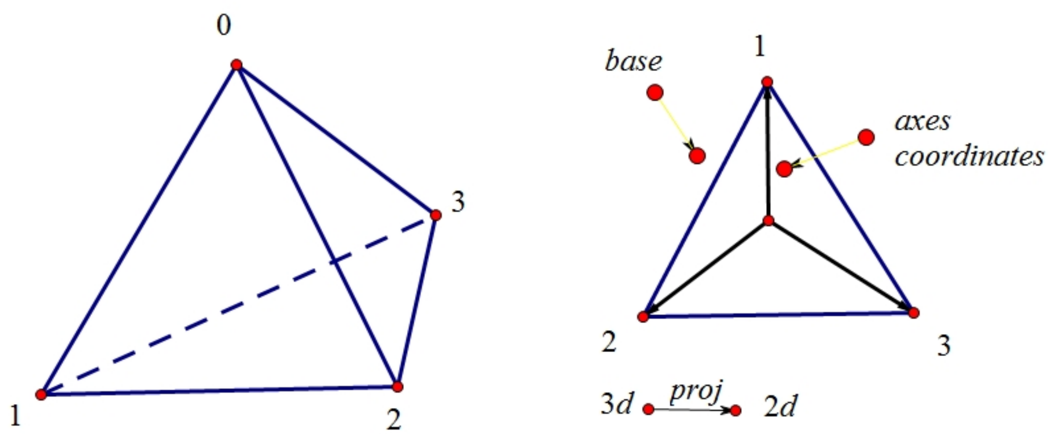

Figure 5:

dimention.

In any dimention,there are plus and minus unit vetor and simplex is the face of it.These are directions.Next we need fundamental intertwiner on the base. Please look at figure 1.12.

Let's check this in the usual case over sl_2.Please look at figure 1.13.

It is clear that if you have Hamiltonian path you have all the edges except one.That is the exact system on simple root.So this in my view should be at the very beginning of representation theory book.This part is known as well,the construction of simple root.Now we need to construct these blades and these center.

First,let me remind you the way to have coordinates in the simplex.Please look at figure 1.14.

This is for simplex,which we are going to use.Here now,if we go back down,let me write it again.We

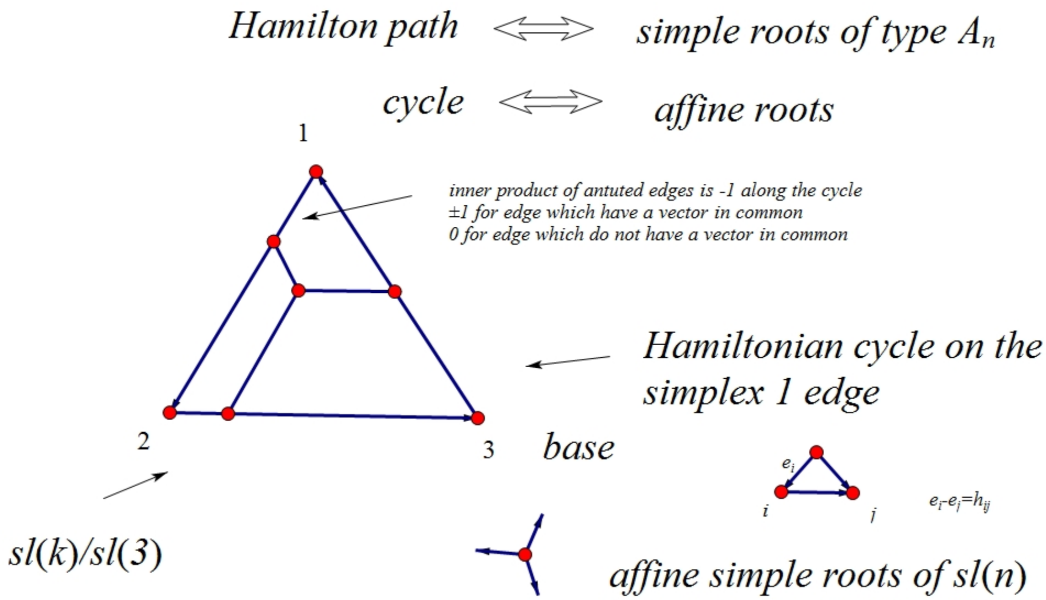

Figure 6:

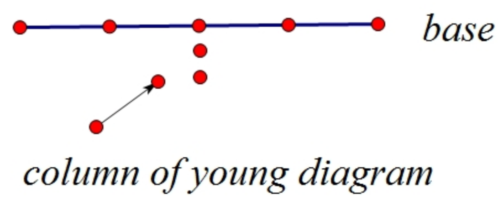

Figure 7:

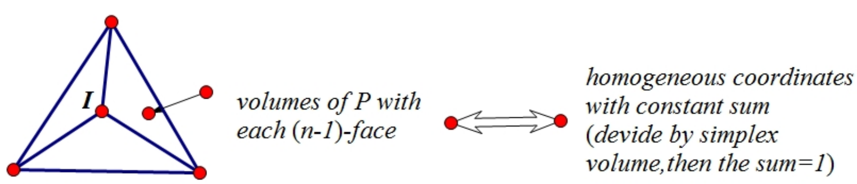

Figure 8:

continue with our blade here.The point is that for this blade,we have here some segments.The edge length is N.Please look at figure 1.15.

We have $\sum length(\rightarrow) = N$.On each edge $i \leftarrow j$ in the Hamiltonian cycle,we have an integer $\sum length = N$,giving coordinates of the center each length,giving the previous edge coordinates.The simplex could be a quotient with each vector labeled by a lump.

For instance,let's take one we have before,i.e.,sl_2/sl_n with $A \subset \{1, 2, ..., n\}, A'$.The number of such partitions of n into $k = 2$ parts Stirling $2(n, k)$.For example,Stirling $2(4, 2) = 7$.

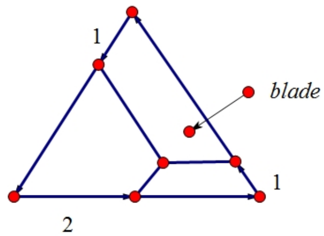

Figure 9:

A blade with codimention 1 is given by two trees.Please look at figure 1.16.

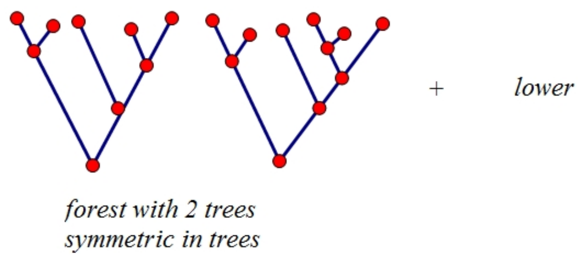

Figure 10:

Now we need a more think. Please look at figure 1.17.

This is called Hamiltonian cycle(affine roots).And all edges=all roots of type A_{n-1}.

Let us take an example here and this is 3 dimention complex viewed from an edge.We take the simple root like this. Please look at figure 1.18.

Please remember that I mentioned in the early part of the course,the clock.See figure 1.19.

So for the permutation of blade in simplex coordinates.And this is an affine clock.All handmove $N - 1$ dimentions, N coordinates with constant sum.If all the hands move then you get the whole space,the ambient space.This is root space of type $A_N - 1$.Next we need keep 2 hands fixed.So here the blades move clockwise without pathing each other. For the plate,see figure 1.20.You have hands at midnight fixed.

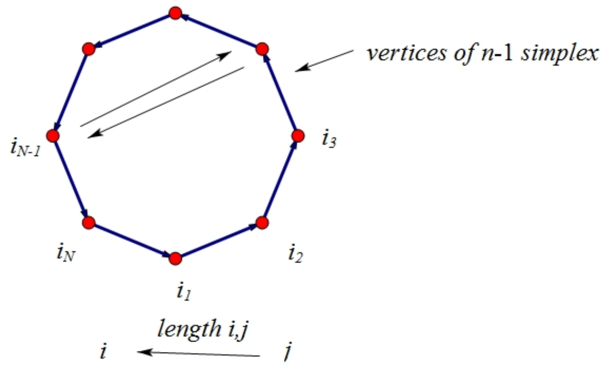

Figure 11:

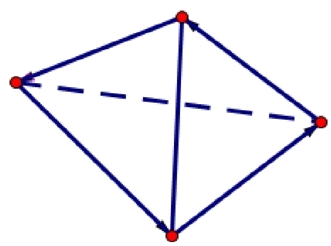

Figure 12:

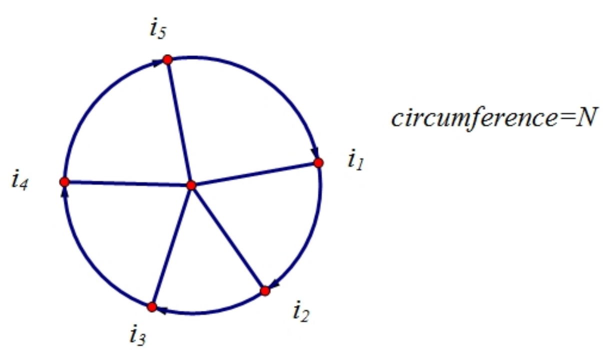

Figure 13:

So the equation here is

$$x_1 \geq s_1$$
$$x_1 + x_2 \geq s_1 + s_2$$
$$x_1 + x_2 + x_3 \geq s_1 + s_2 + s_3$$

6·

$$x_1 + x_2 + ... + x_n = N.$$

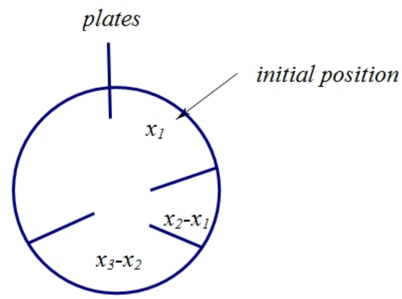

Figure 14:

Now we are going to construct the fin tree.And again this is not the general part.So the fin tree,let me take the conrect color which was on the botton of intertwiner if you remember.

Here you are going to have an intertwiner.See figure 1.21.

We are taking inside a blade which intersects the base on the given blade.Also in the speak of Gelfond-Tsetlin,we neglect horizontals parrallel to base.The node will go into one of the directions.You have three main directions.

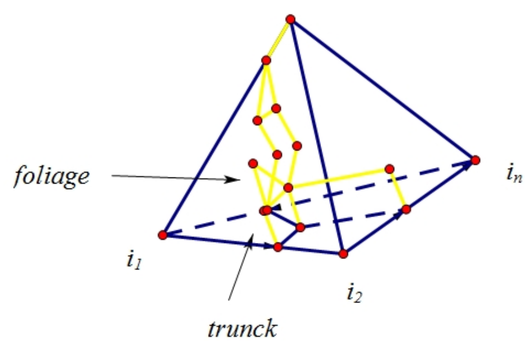

Figure 15:

And for the trunk,see figure 1.22.

So you have the multinomial

$$\frac{N!}{l_1!...l_n!}.$$

And the trunck posibilities=vectors in our generating inlp.So these are vetors of higher representation.

Next time we will take higher matrix.In this case,the higher matrix is 4 by 4.See figure 1.23.

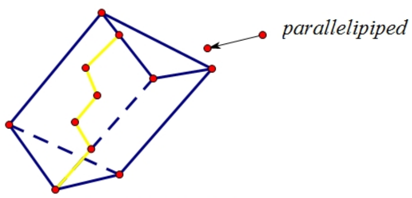

Figure 16:

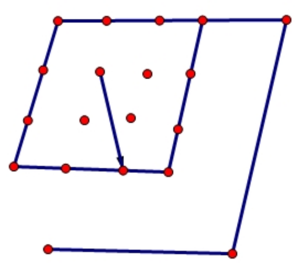

Figure 17: