The Mathematical Picture Language Project at Harvard

De Finetti Theorem for Braiding Parafermions

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Introduction

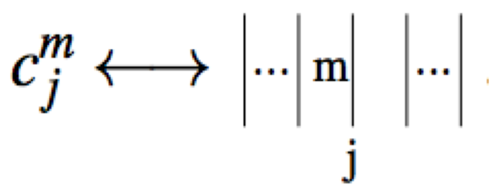
The famous de Finetti theorem in classical probability theory clarifies the relationship between permutation symmetry and the independence of a sequence of random variables. An infinite sequence of symmetric random variables can be written as a convex combination of an independent identically distributed sequence.

A related de Finetti theorem is crucial in statistical physics and quantum information for understanding the structure of permutation-symmetric states, and especially for the consideration of quantum entanglement of such states.

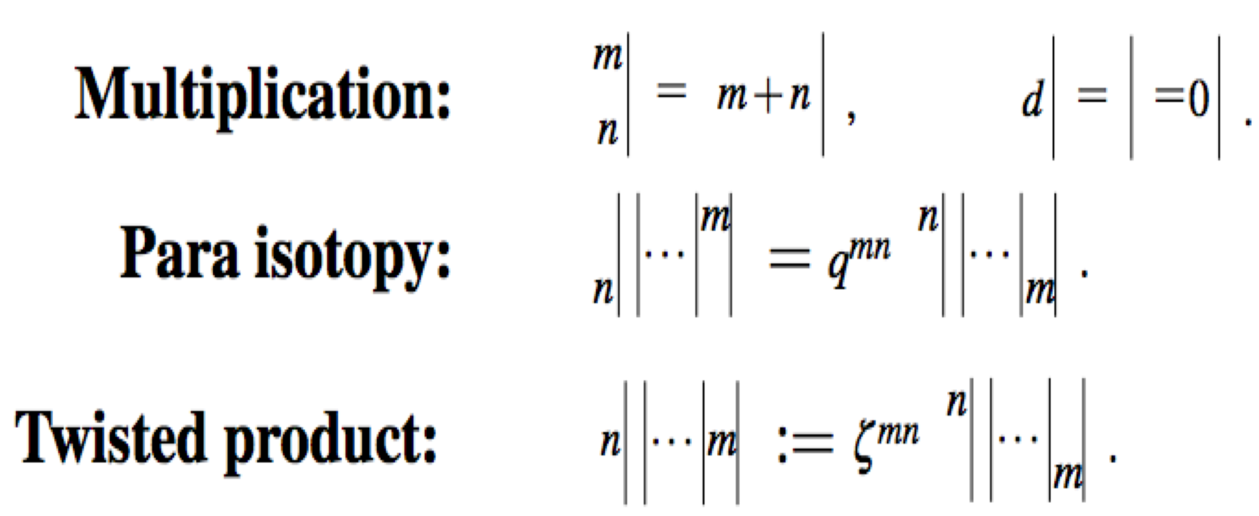
Here we establish a new result fitting naturally into the Mathematical Picture Language Project. We characterize states of the parafermion algebra that are invariant with respect to a natural action of the braid group.

Pictorial Language

In ref. [1,2] the first two authors propose a new picture language for the parafermion algebra, where the correspondence between parafermion operators and pictures is



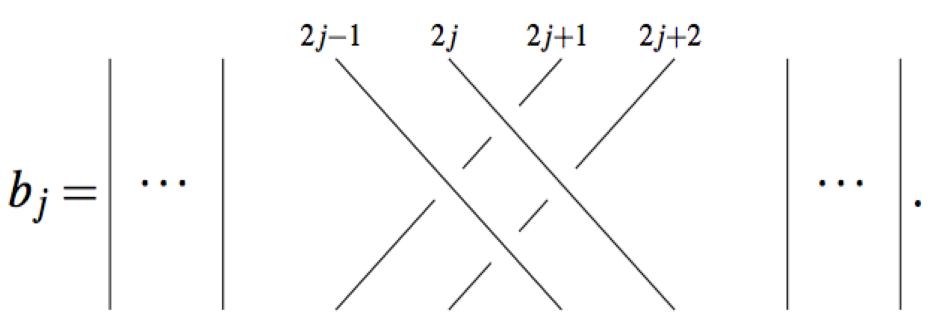
Moreover, it satisfies the following relationship



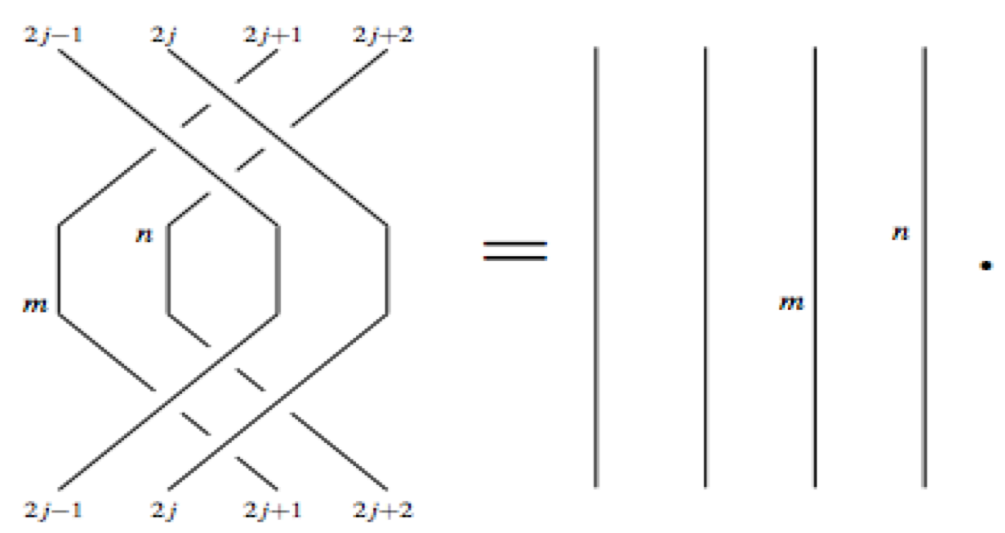
Here ζ is a chosen square root of q such that ζ^d²= q.

Four string braid

The following picture presents the (negative) four-string braid as a product of four two-string braids:



The diagram corresponding to the action of double braids on a pair of charged strings is a translation:



Main result

We prove a de Finetti-type result for parafermion algebras. In the case that the order d is square free, we show that the center of parafermion algebra is equal to the tail algebra of the parafermion algebra and that the tail algebra only consists of neutral elements.

In case that d is not square free, we show that the tail algebra does not equal the center of the parafermion algebra, and the tail algebra contains non-neutral elements. We characterize the tail algebra for extremal, braid-invariant states. The details are presented in ref. [3].

Reference

[1] Arthur Jaffe, Zhengwei Liu, and Alex Wozniakowski, Constructive simulation and topological design of protocols, New J. Phys. 19(6) (2017), 063016.
[2]] Arthur Jaffe, Zhengwei Liu, and Alex Wozniakowski, Holomorphic Software for Quantum Networks, Science China Mathematics, 61(4) (2018), 593–626.
[3] Kaifeng Bu, Arthur Jaffe, Zhengwei Liu and Jinsong Wu, arXiv:1805.05990

The Mathematical Picture Language Project at Harvard

The Mathematical Picture Language Project at Harvard is an effort to understand pictures in new ways. In particular we prove theorems about pictures and apply these results to different areas of mathematics and of physics. See https://mathpicture.fas.harvard.edu/
The photo shows the first meeting of our month-long working group during the Summer of 2018 in Beijing.