Resource theory of quantum scrambling

arXiv:2208.10477

Mathematical Picture Language Seminar
November 15, 2022

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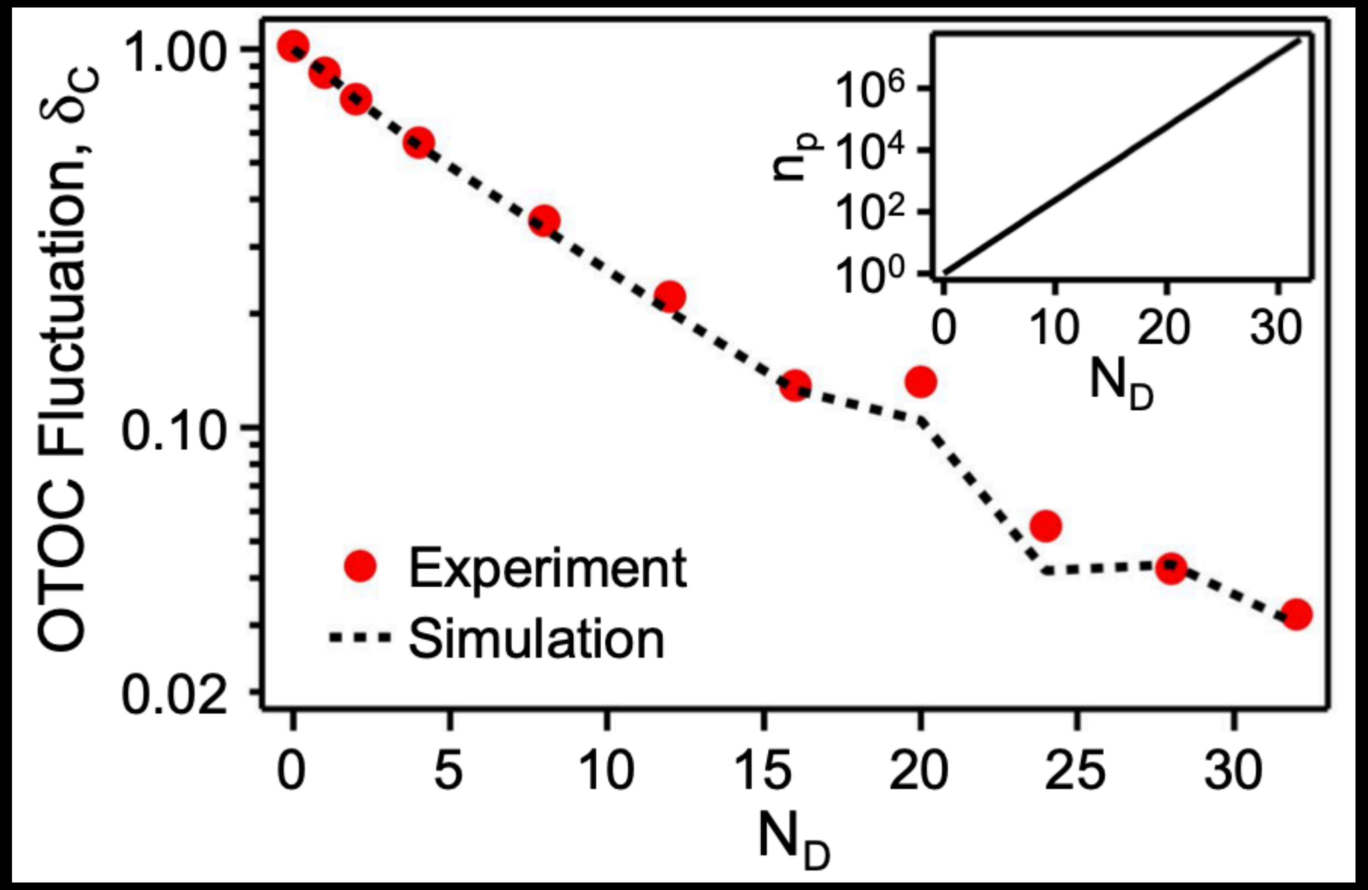
Main results

Develop a theory to define and measure scrambling

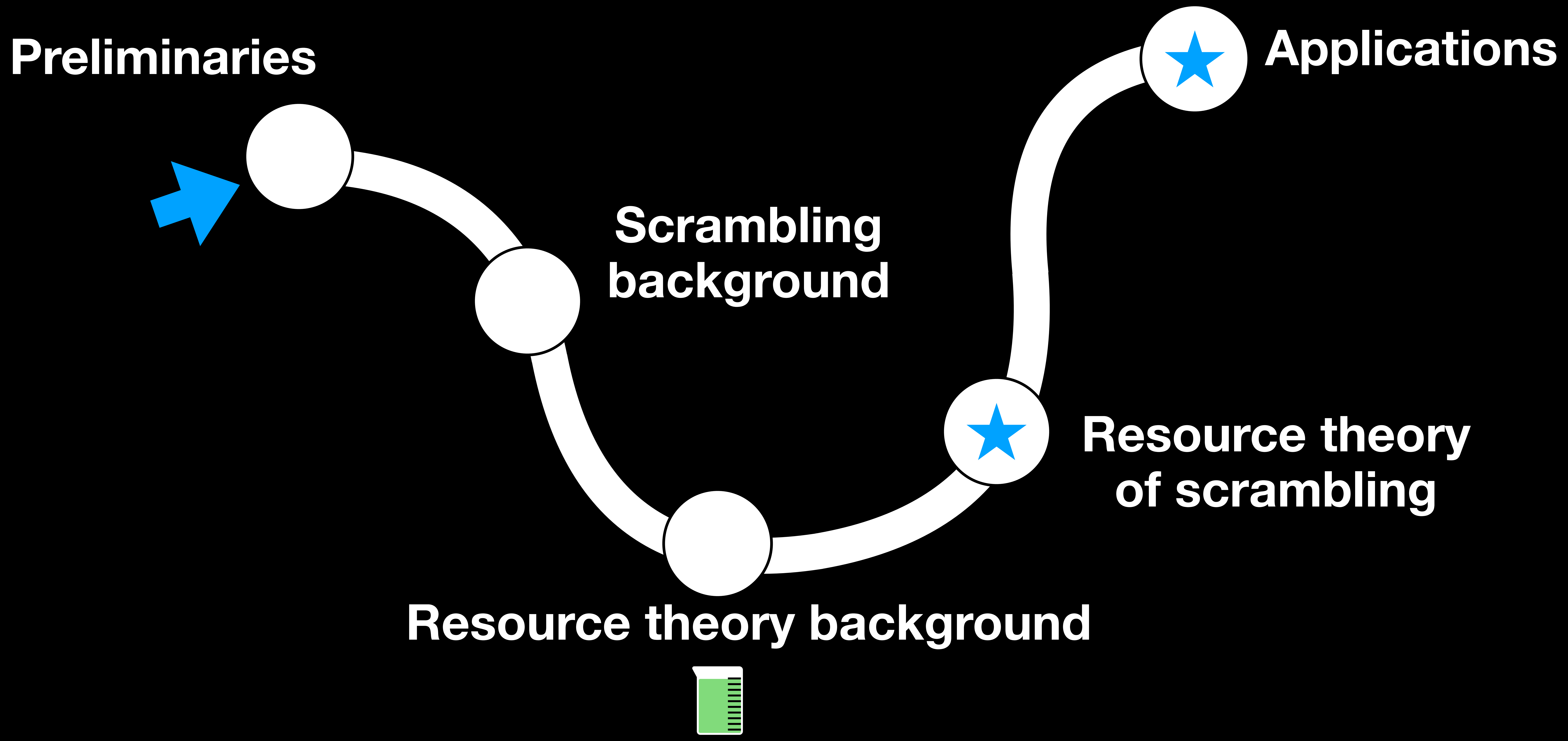
Use this

To explain this



X. Mi et al., Science 374, 1479 (2021).

Where we're going



Preliminaries

Pauli matrices

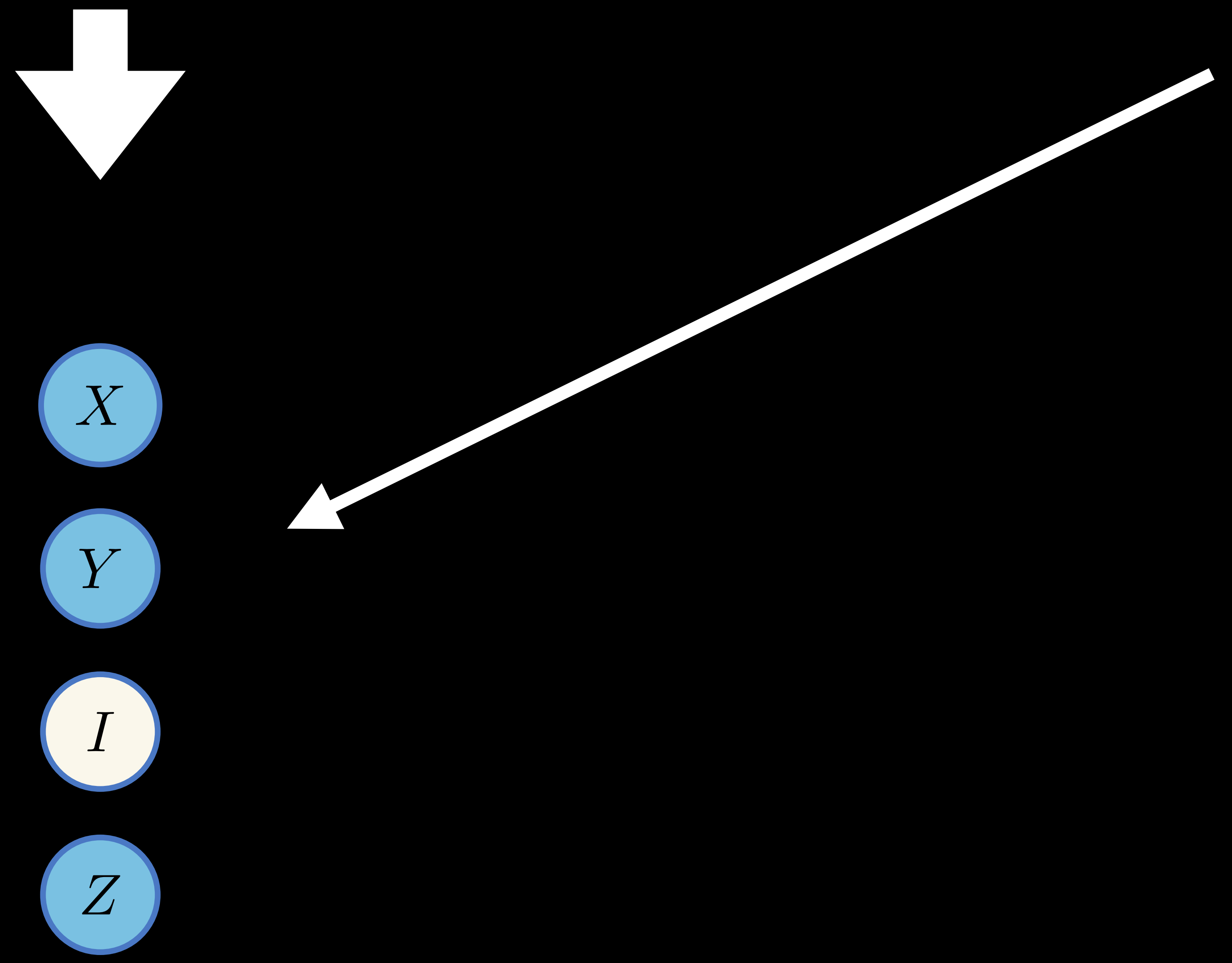
$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Pauli string

$$P = X \otimes Y \otimes I \otimes Z$$



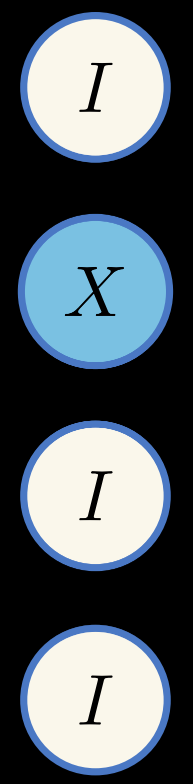
Pauli weight

# of blue dots in string

$$W(P) = 3$$

Preliminaries

Local Pauli string



$$W = 1$$

Preliminaries

Pauli group

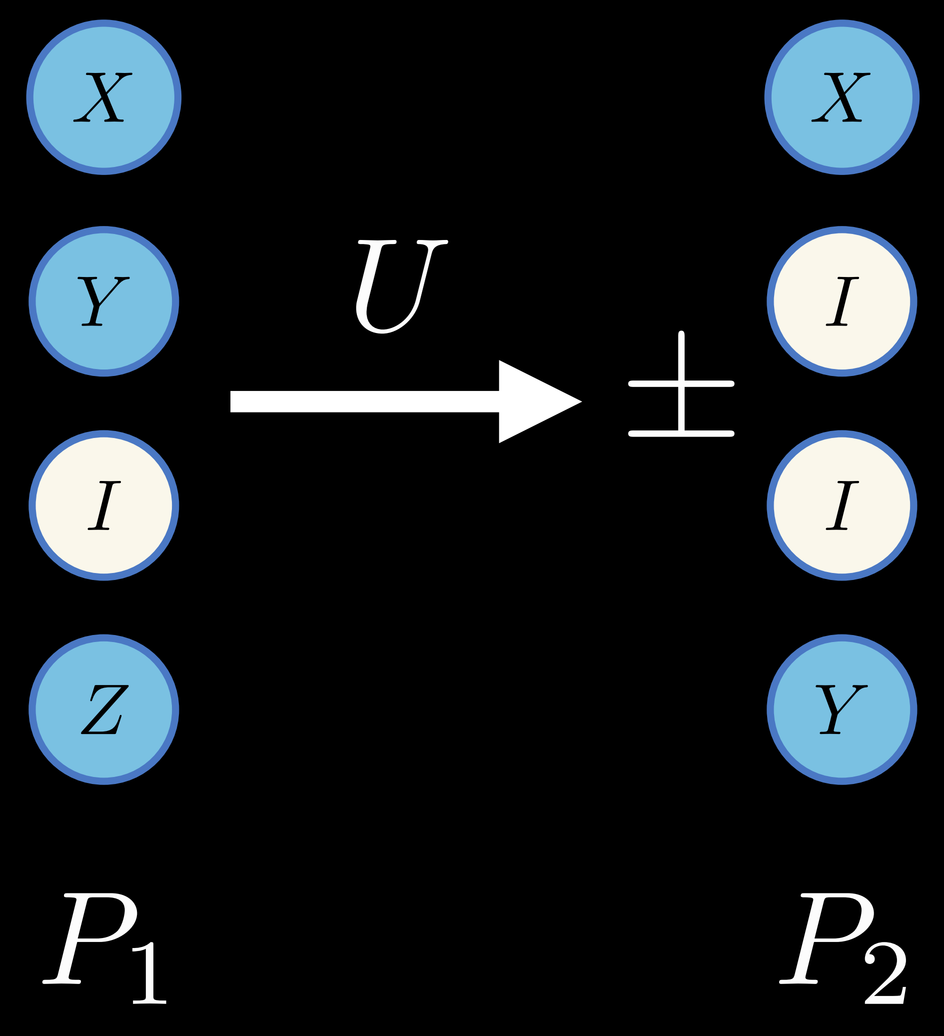
(on n-qubits)

$$\mathcal{P}_n = \{\text{All Pauli Strings}\}$$

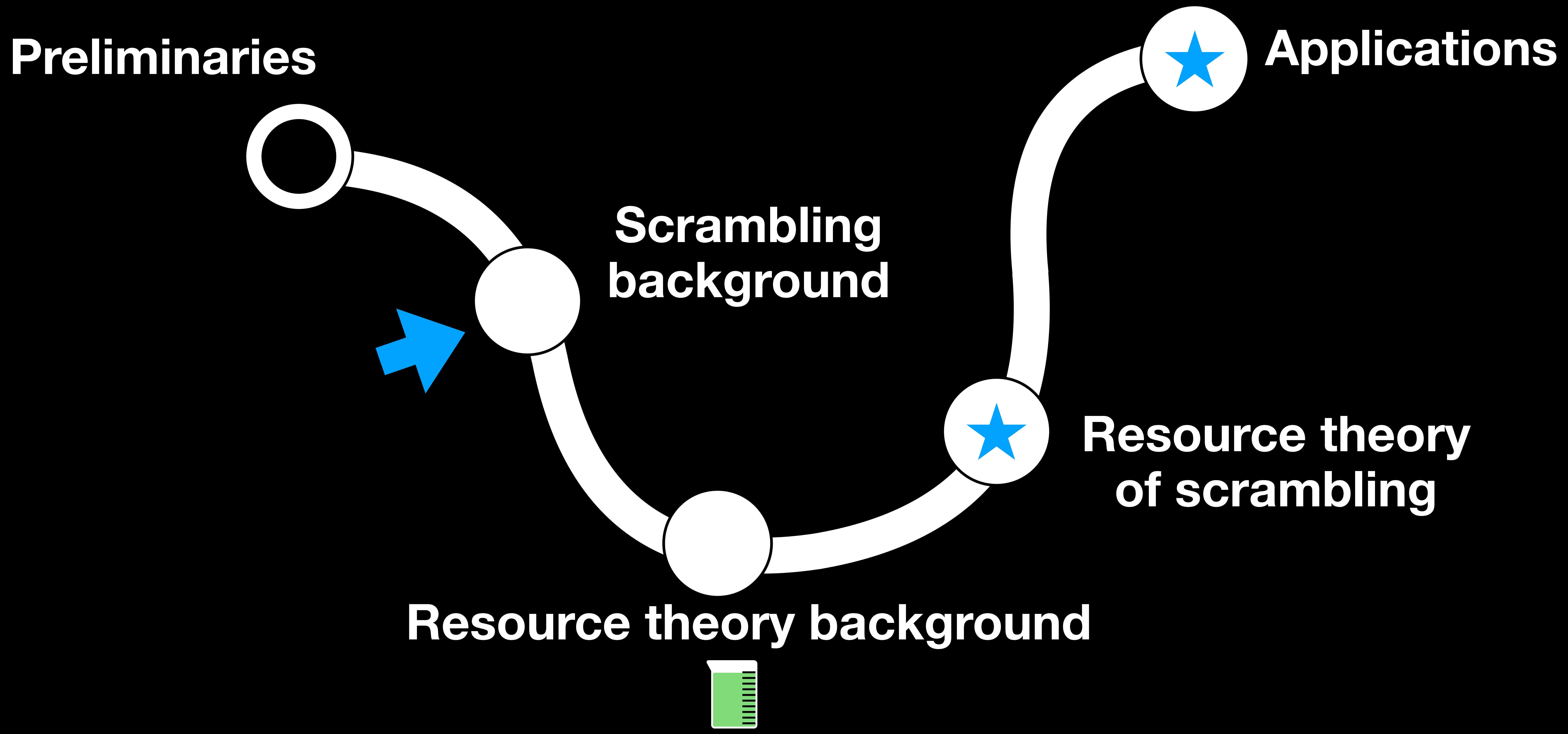
Clifford unitaries

Map Pauli strings to Pauli strings

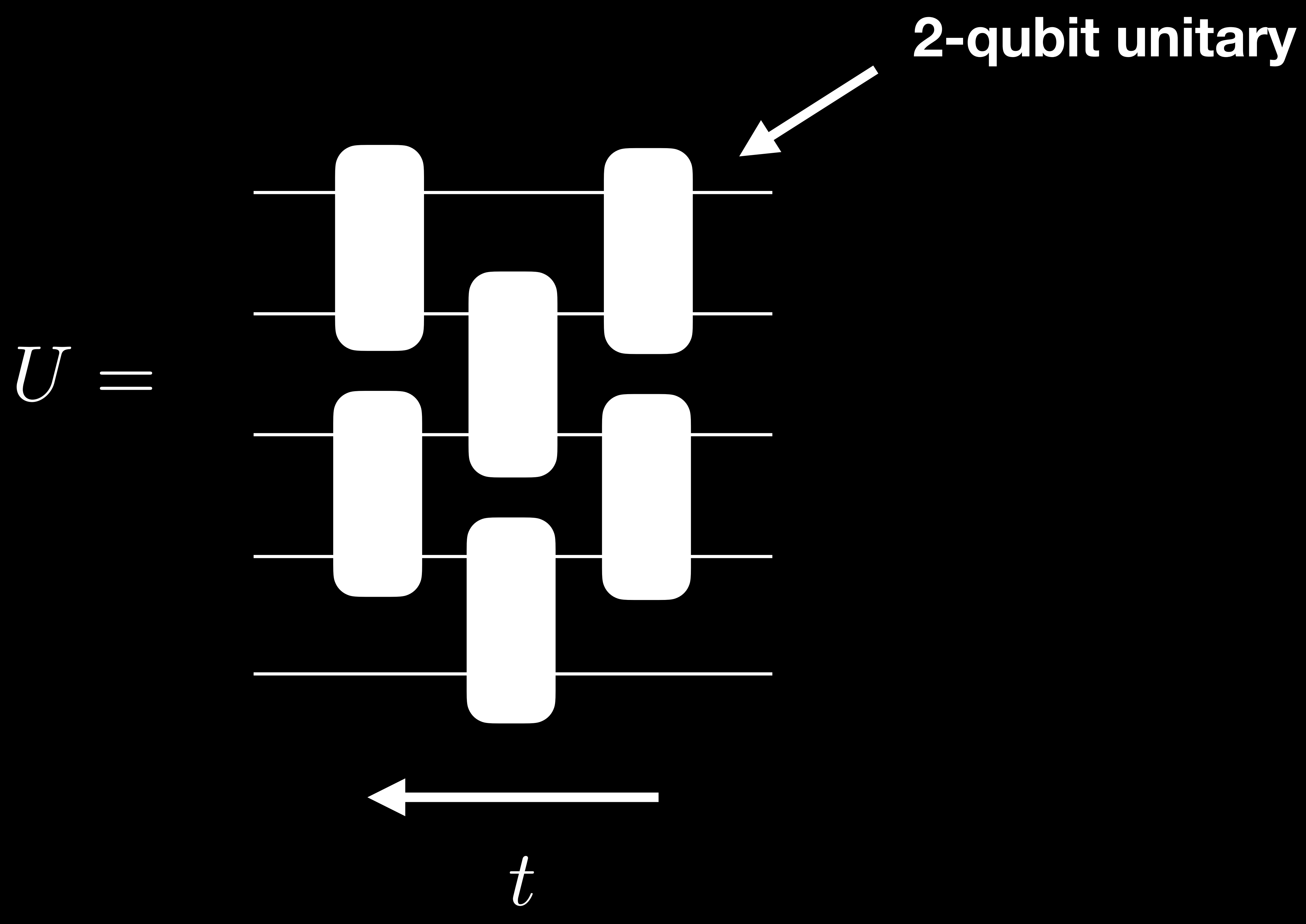
$$U^\dagger P_1 U = \pm P_2$$



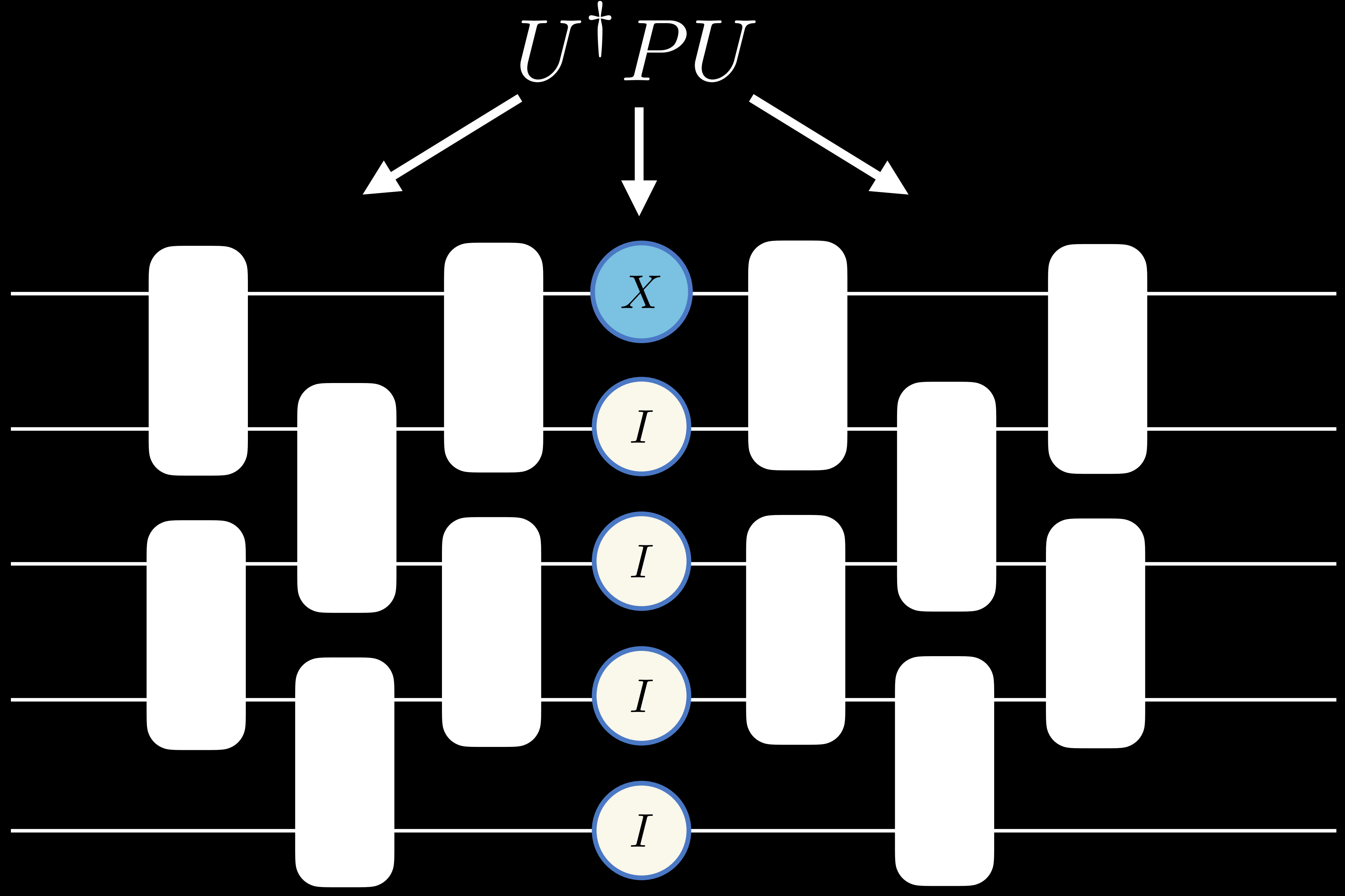
Where we’re going



Random unitary

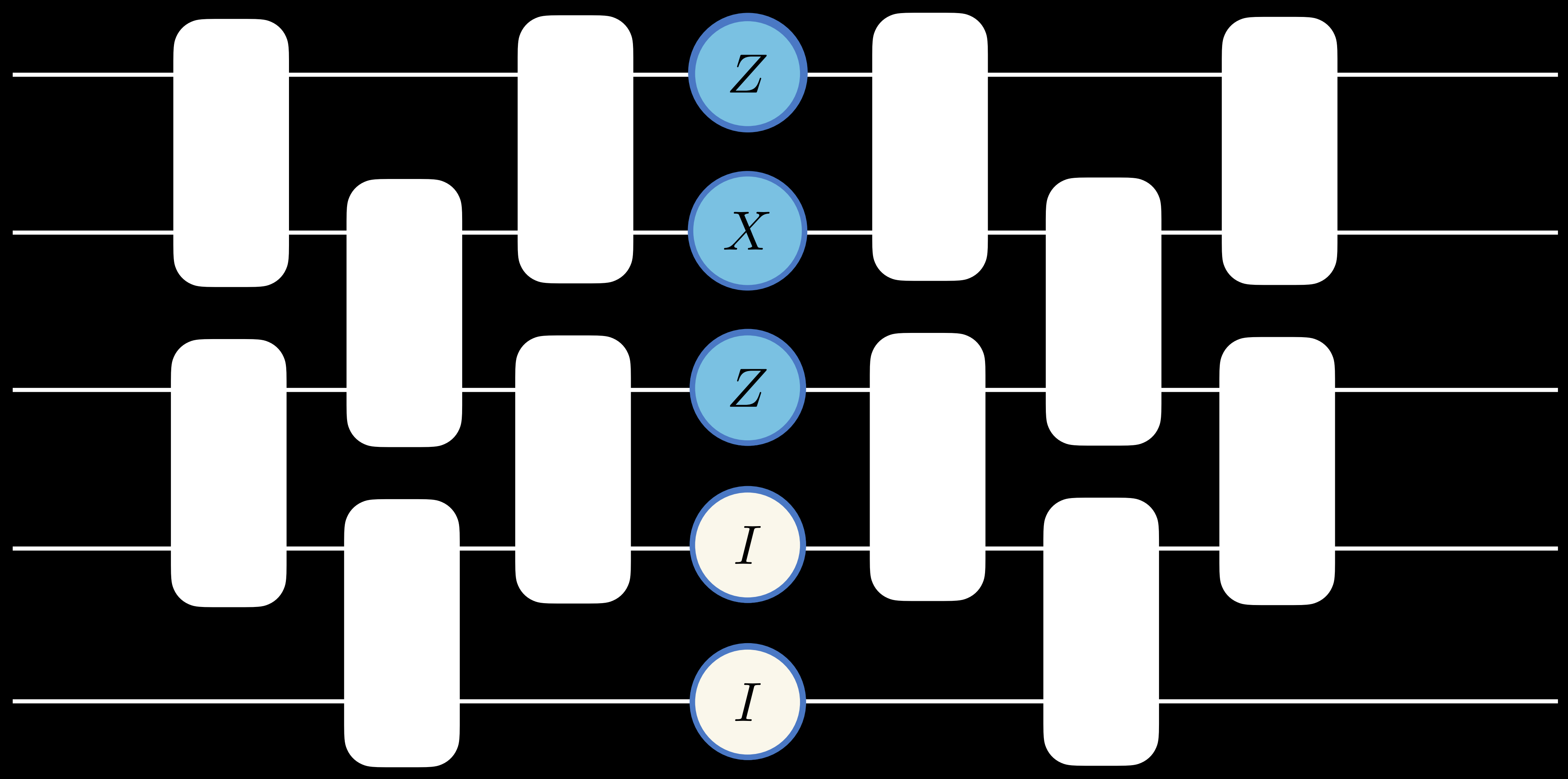


How does U spread local information?



How does U spread local information?

$$U^\dagger P U$$



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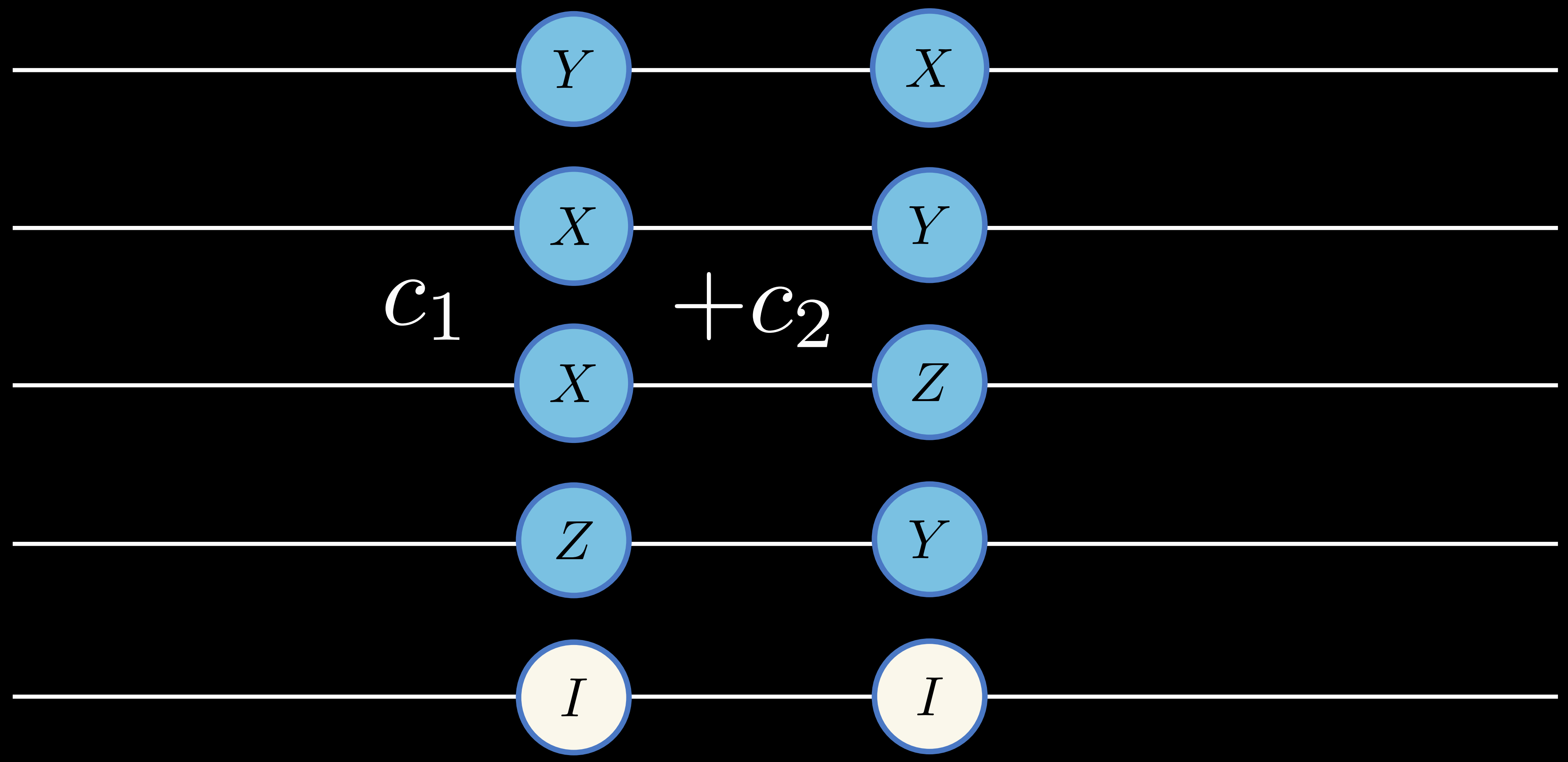
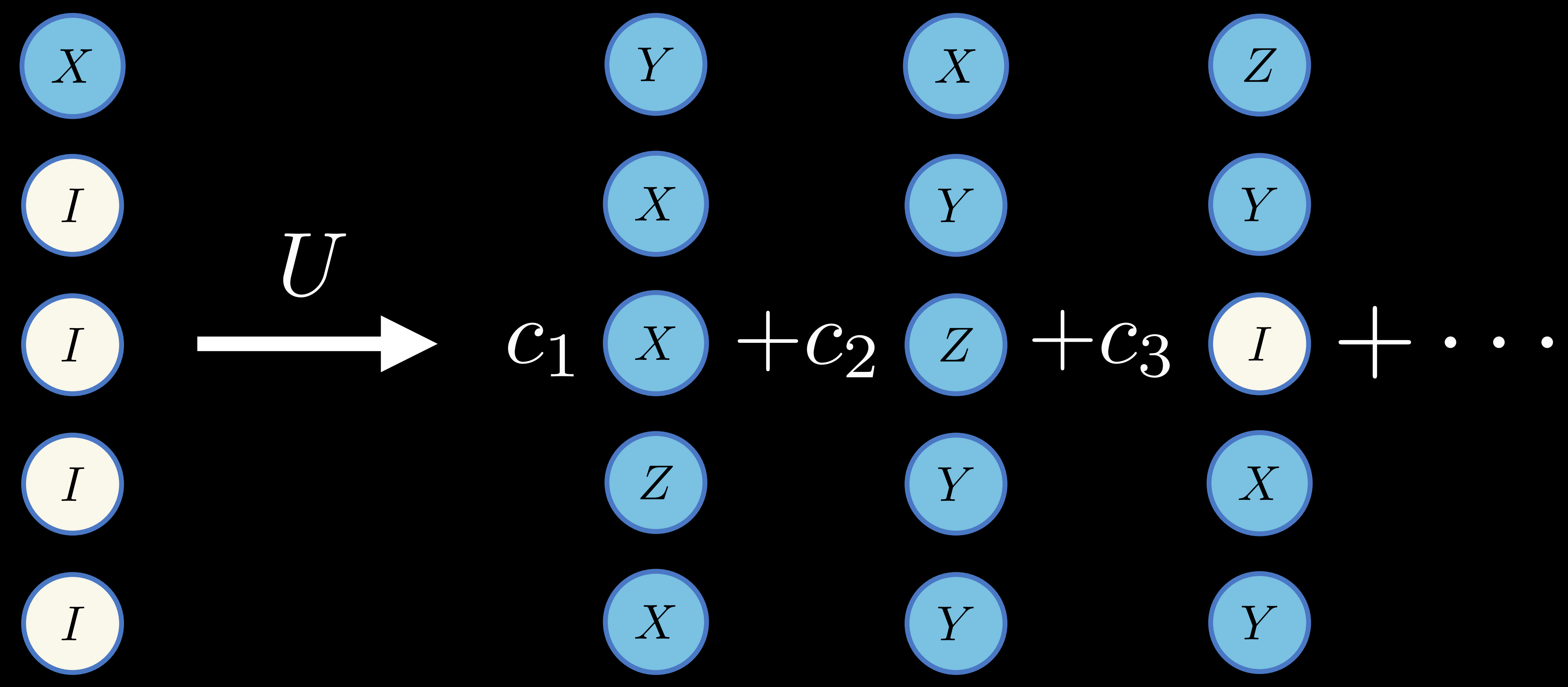
$$U^\dagger P U$$



Scrambling unitaries do the following.



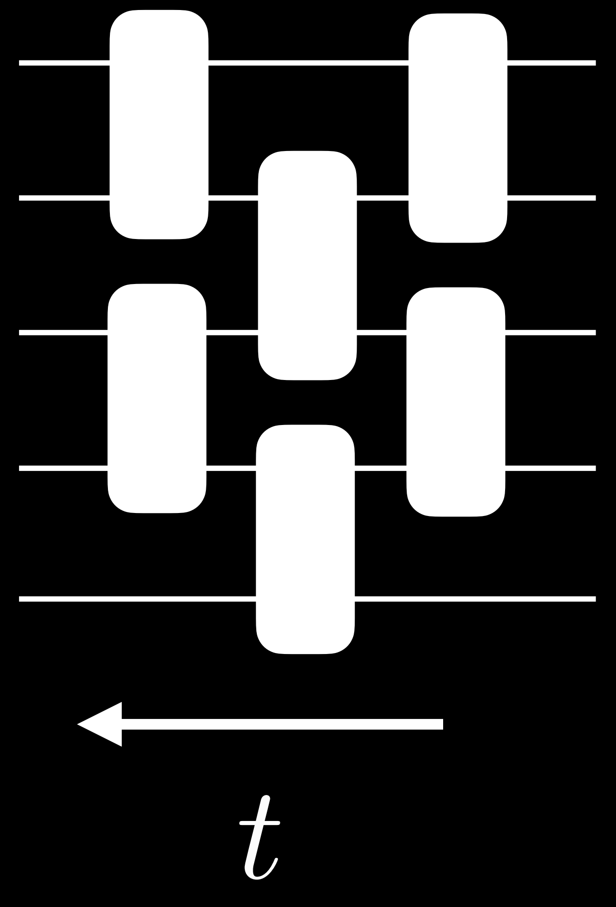
1. Increase # of strings
2. Increase # of blue dots

Intuitively: scrambling describes spread of local information

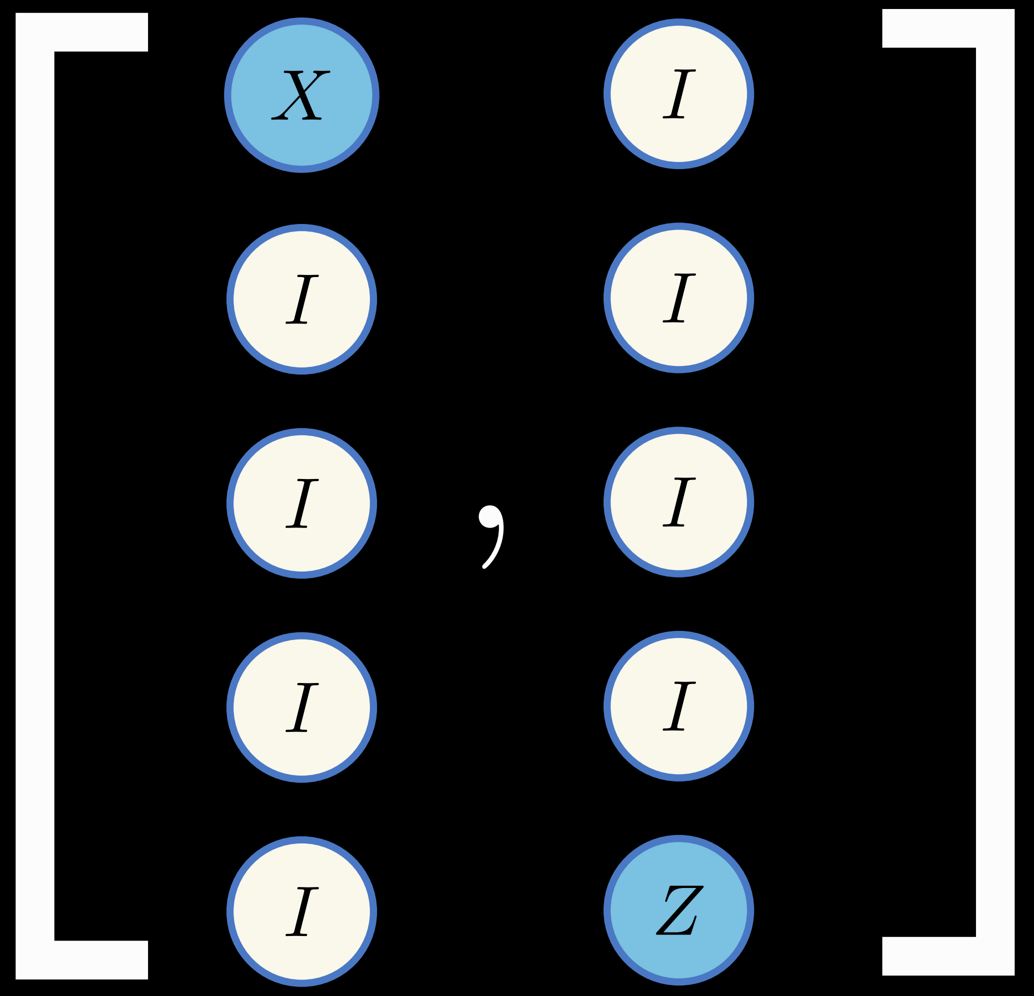
How do we quantify scrambling?

Commutator norm

$$C(t) = \left|\left|[U^\dagger X_1 U, Z_n]\right|\right|_{\text{HS}}^2$$

$U =$ 

$t = 0$

$[X_1, Z_n]$ → 

$= 0$ → $C(0) = 0$

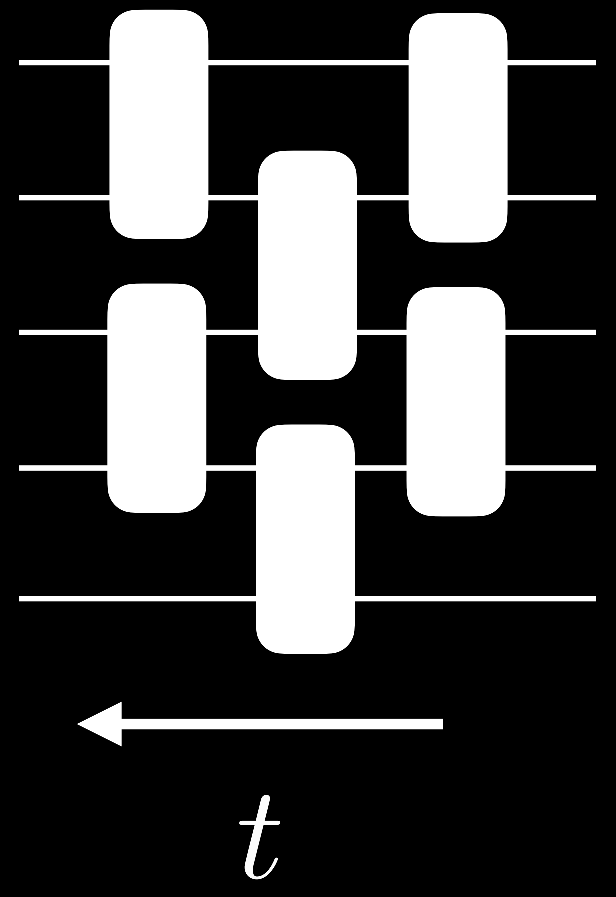
(No overlap)

How do we quantify scrambling?

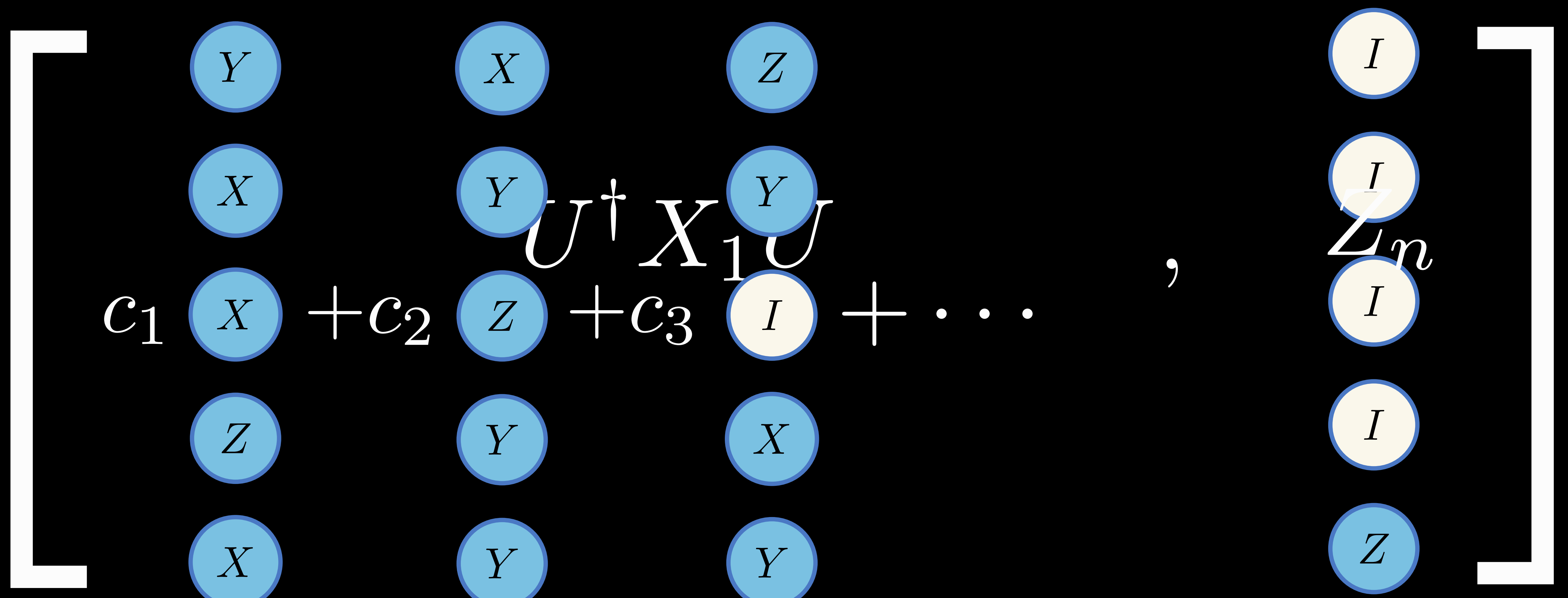
Commutator norm

$$C(t) = \left|\left|[U^\dagger X_1 U, Z_n]\right|\right|^2_{\mathrm{HS}}$$

$U =$



Large t

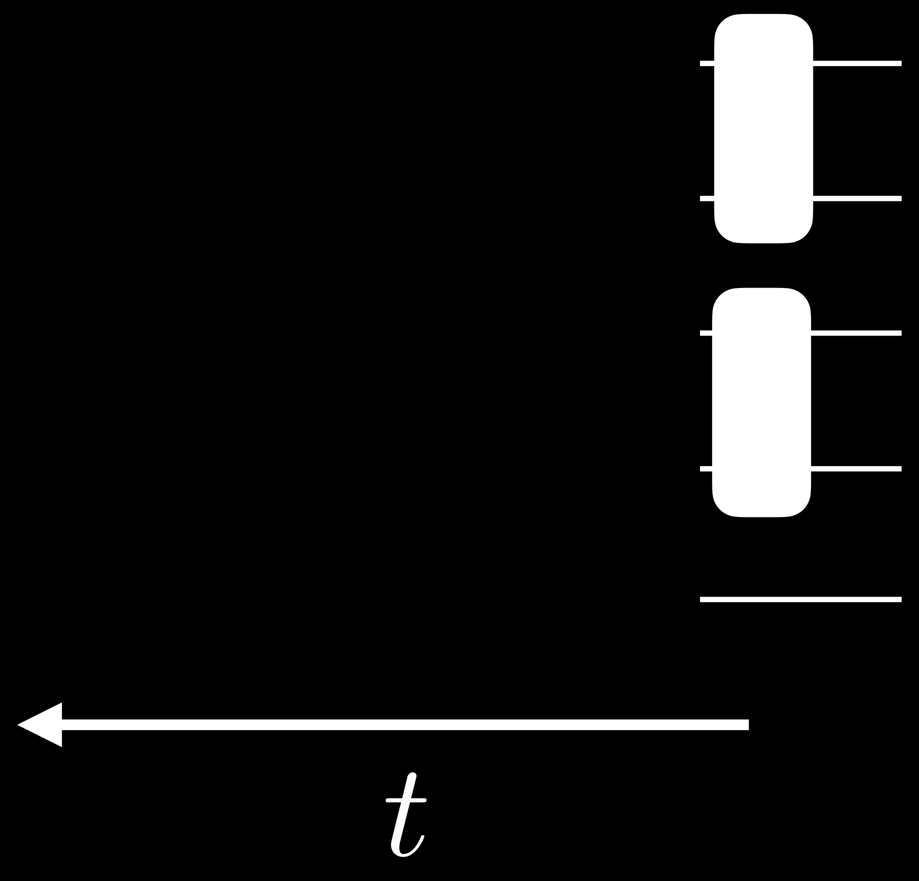


(Much overlap)

$\Rightarrow$ $C(t)$ **is large**

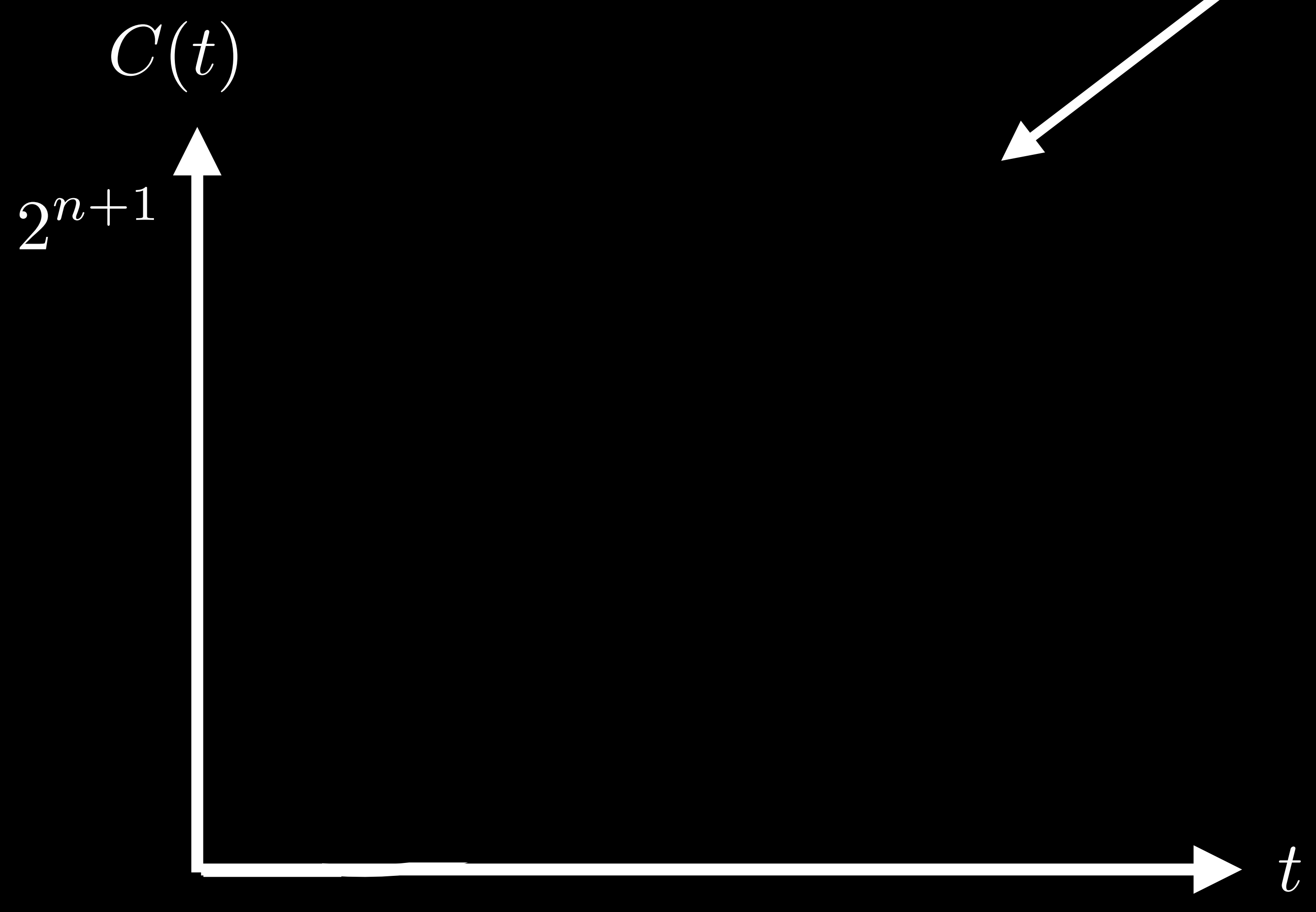
How do we quantify scrambling?

$U =$

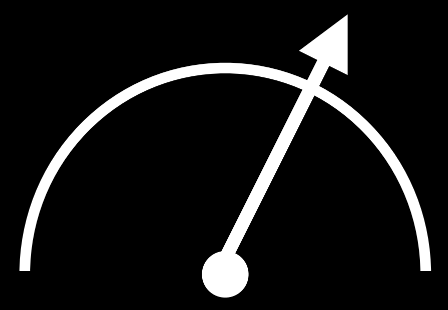


Large overlap

Scrambling signature



How do we measure scrambling in experiment?



Out-of-time-ordered correlator

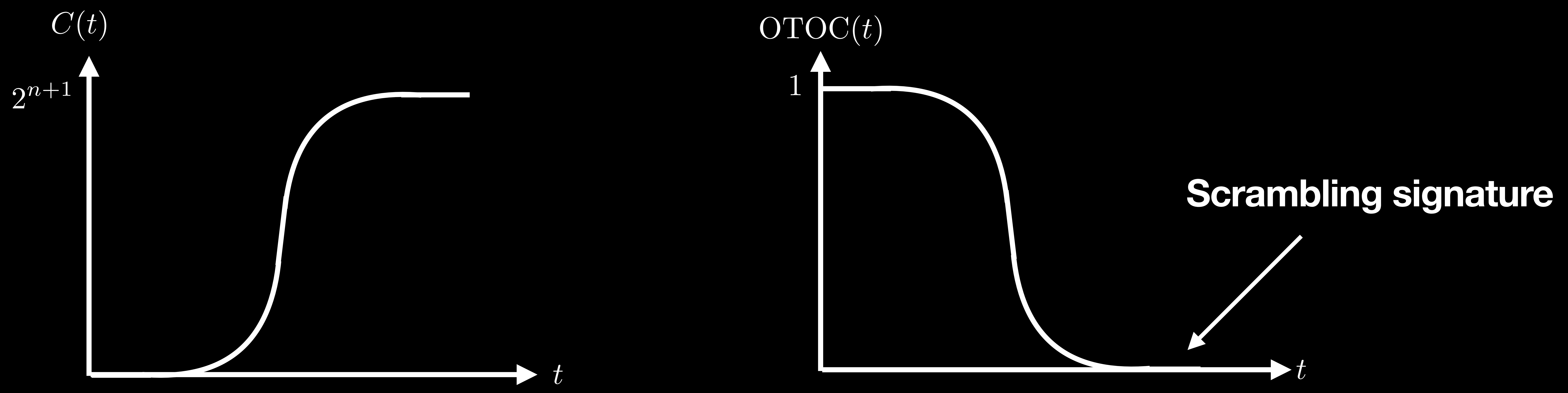
$$\boxed{\mathrm{OTOC}(t) = \langle U^\dagger X_1 U Z_n U^\dagger X_1 U Z_n \rangle}$$

Related to commutator norm

$$C(t) \propto 1 - \mathrm{OTOC}(t)$$

$$\langle \cdot \rangle = \mathrm{Tr}\left\{ \frac{I}{2^n} \cdot \right\}$$

How do we measure scrambling in experiment?



Related to commutator norm

$$C(t) \propto 1 - \mathrm{OTOC}(t)$$

How do we measure scrambling in experiment?

Measurement protocol

$$\mathrm{OTOC}(t) = \langle U^\dagger X_1 U Z_n U^\dagger X_1 U Z_n \rangle$$

Shadow tomography

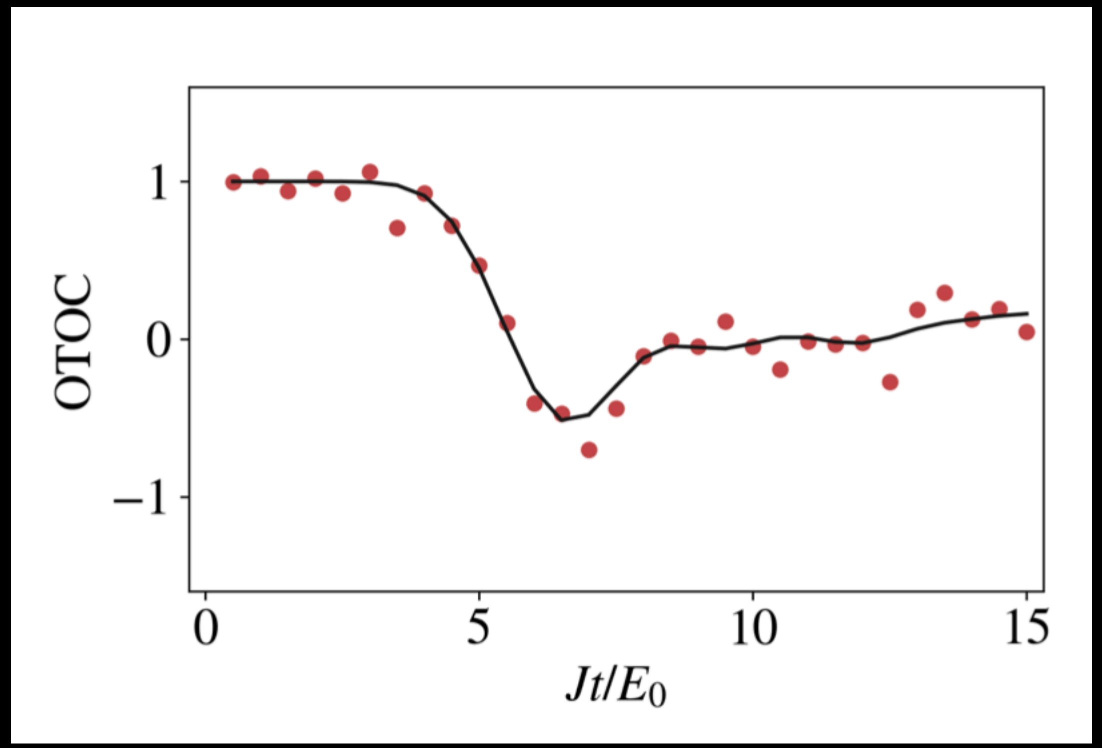
Technique to measure observables

$$\mathrm{Tr}\left\{O\rho^{\otimes 2}\right\}$$

$$\mathrm{OTOC} = \mathrm{Tr}\left\{O\rho^{\otimes 2}\right\}$$

contains X & Z

contains U & $U^\dagger$



R.J. Garcia et al., PR Research 3, 033155 (2021).

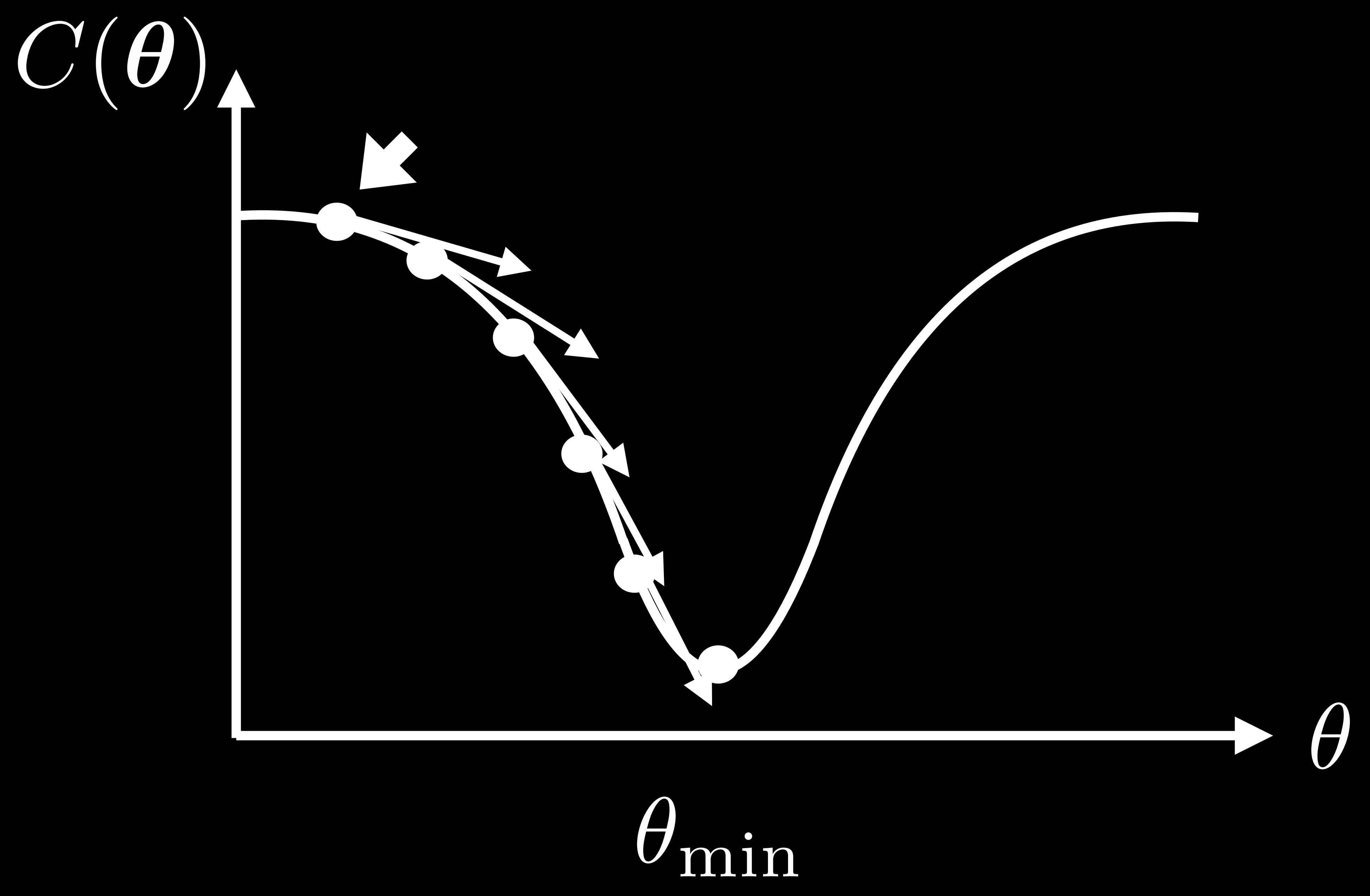
Why study scrambling?

Scrambling

- **Quantum Machine Learning**
  - *Z. Holmes et al., PRL 126, 190501, (2021).*
  - *R.J. Garcia et al., JHEP 2022, 27 (2022).*
- **Black Holes**
  - *P. Hayden and J. Preskill, JHEP 2007, 120 (2007).*
- **Shadow tomography**
  - *H. Hu et al., PR Research 4, 013054 (2022).*
- **Quantum state designs**
  - *W.W. Ho and S. Choi, PRL 128, 060601 (2022).*
- **Quantum error correction**
  - *S. Choi et al., PRL 125, 030505 (2020).*

Quantum Machine Learning

Goal: Minimize a function

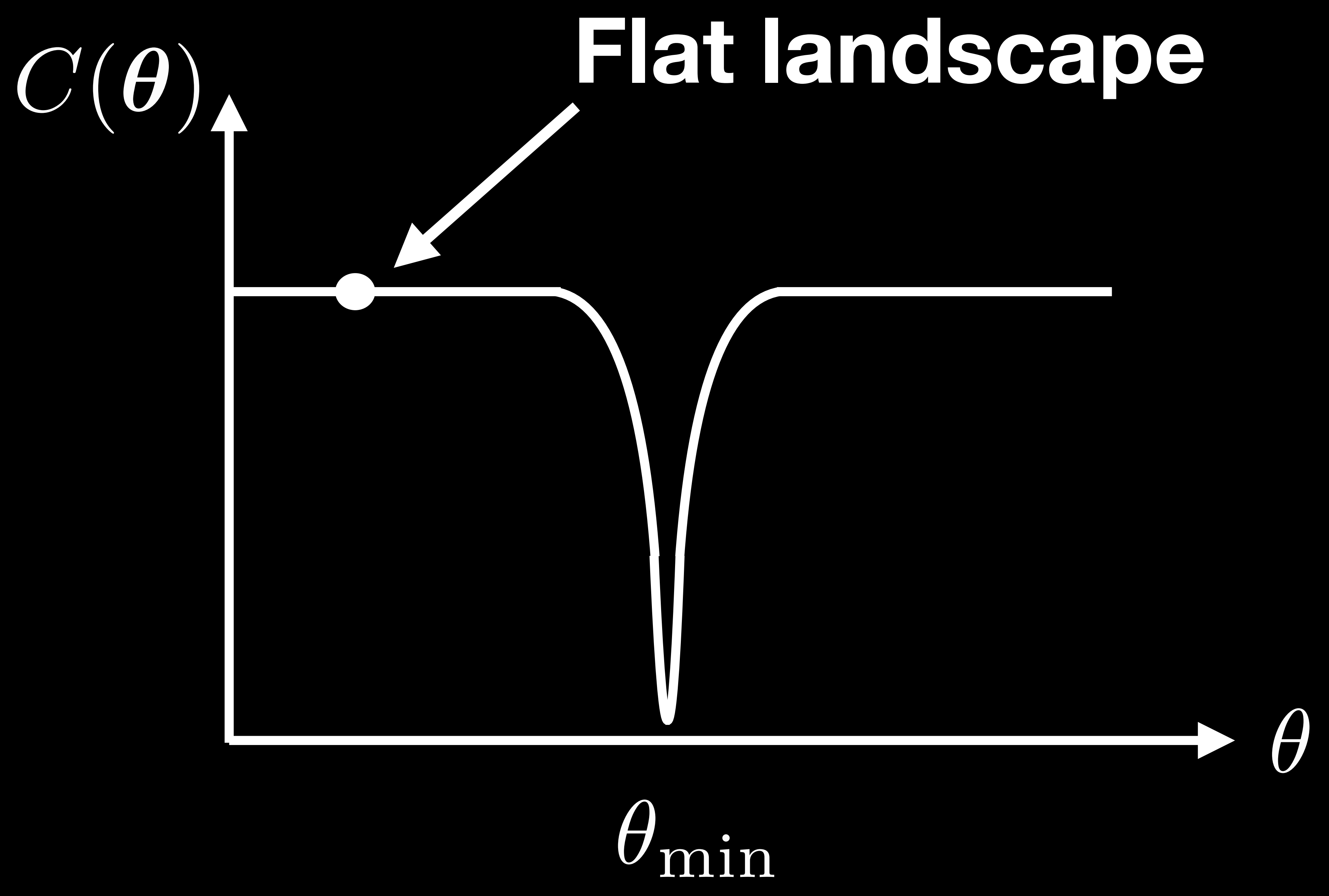


Method: Gradient descent

1. Randomly pick θ
2. Step down gradient $-\partial_\theta C(\theta)$

Quantum Machine Learning

Goal: Minimize a function ➡ **Method: Gradient descent**

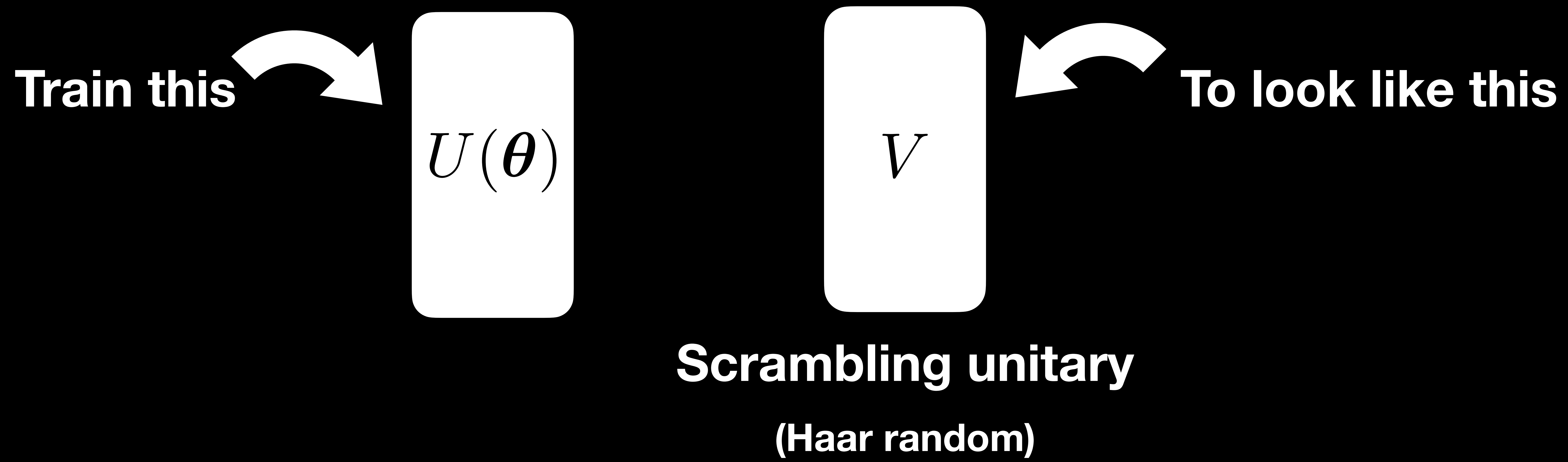


1. **Randomly pick** θ
2. **Step down gradient** $-\partial_\theta C(\theta)$

Barren Plateau: $\partial_\theta C(\theta) = 0$ **with high probability** ➡ **Can't find min** (efficiently)

Quantum Machine Learning (Application)

Learning Task

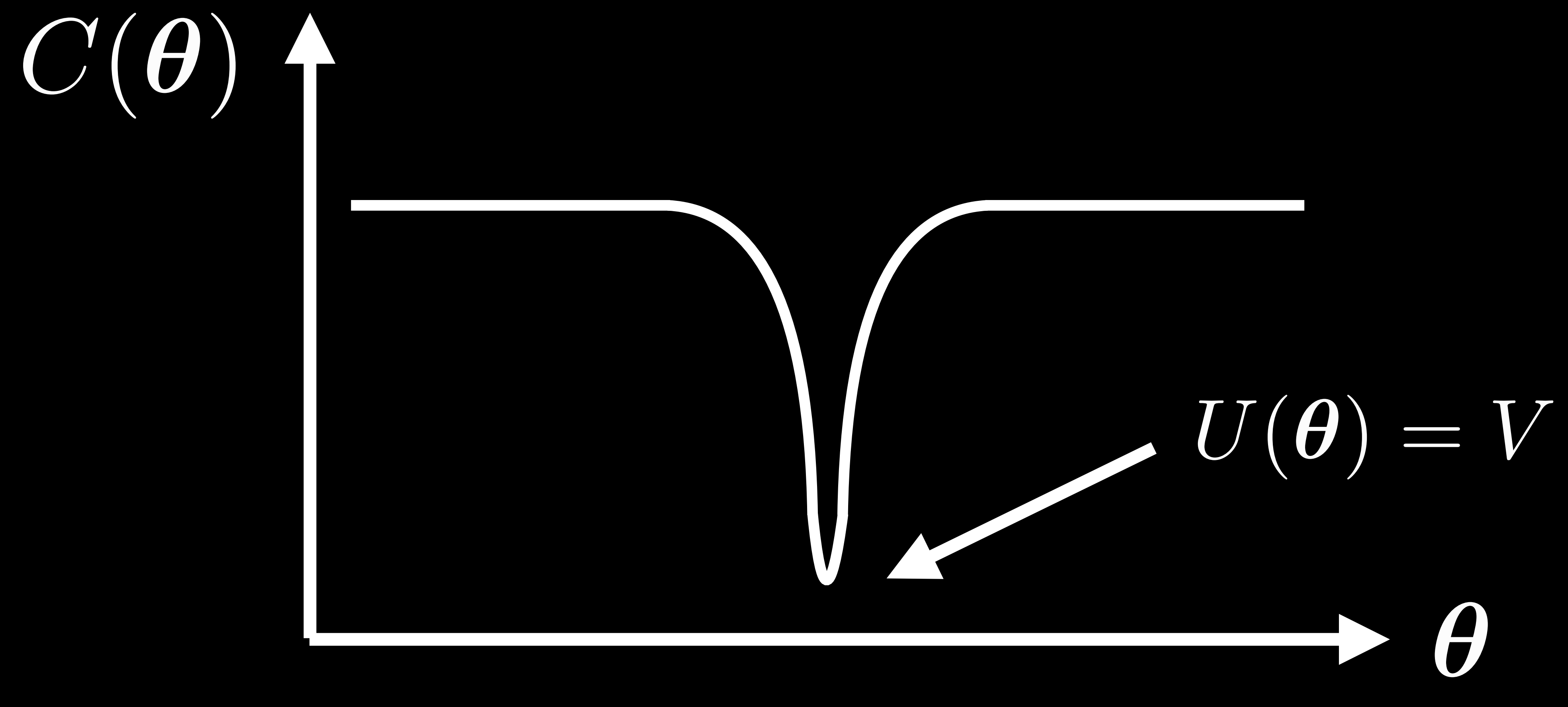


Method: Gradient descent

Quantum Machine Learning (Application)

$VMV^\dagger$ → O ; $U|0\rangle\langle 0|U^\dagger$ → ρ

Cost function: $C(\boldsymbol{\theta}) = \mathrm{Tr}\left\{O\rho\right\}$



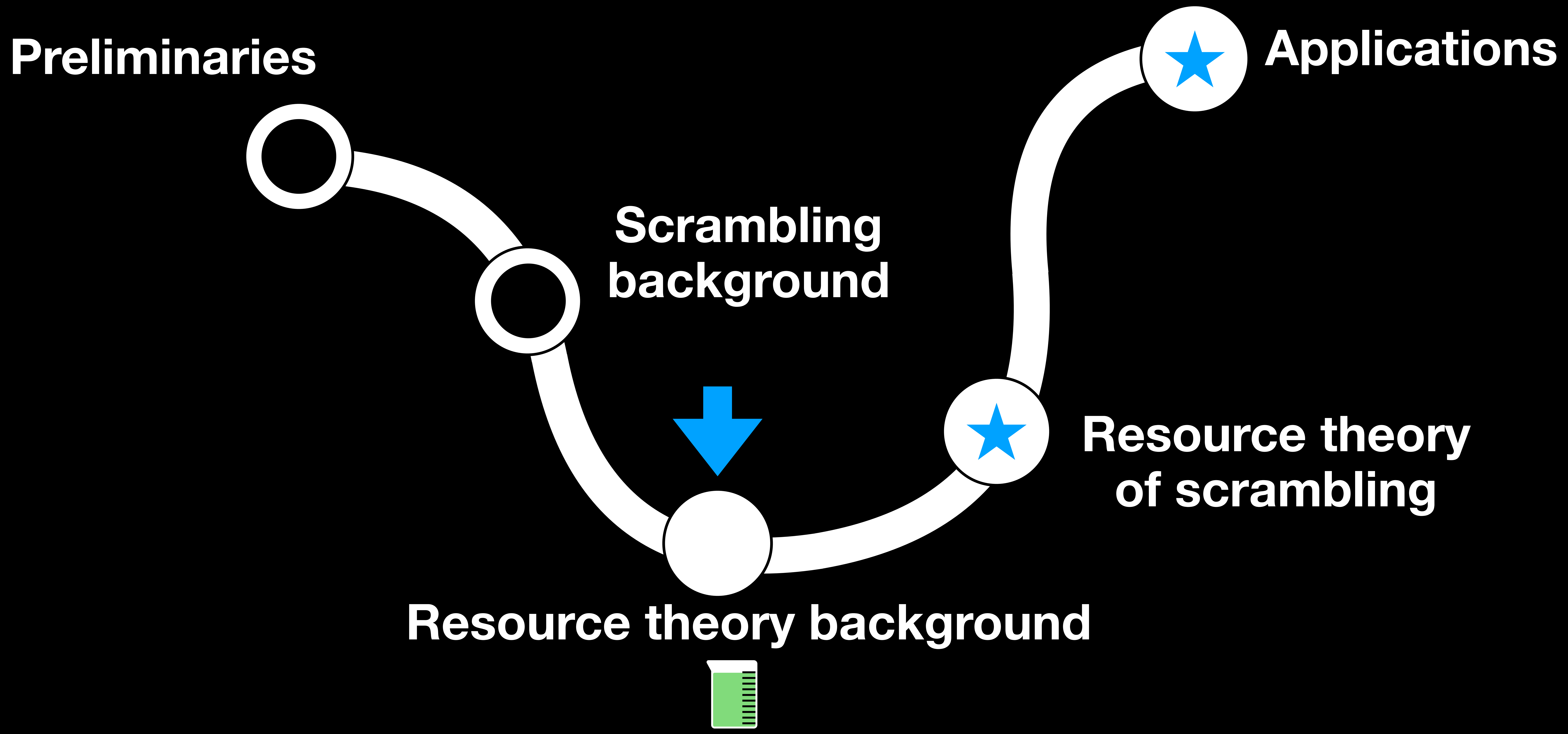
Barren Plateau

Can't learn scrambling unitaries.

(via gradient descent)

Z. Holmes et al., PRL 126, 190501, (2021).
R.J. Garcia et al., arXiv:2205.06679 (2022).

Where we're going

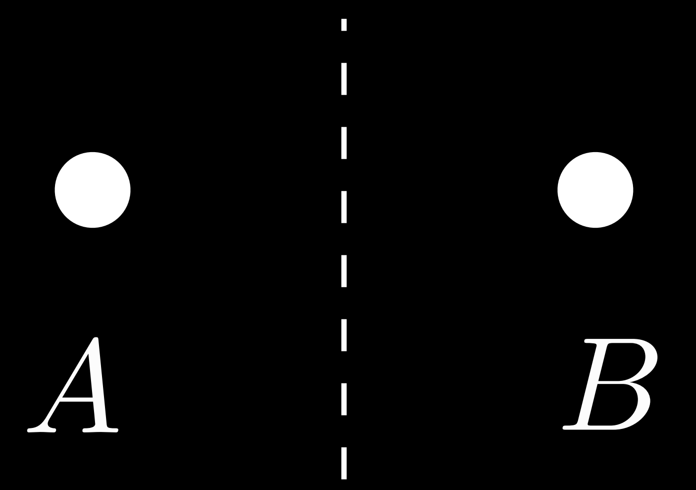


How do we define a scrambler?

We answer this question using resource theory

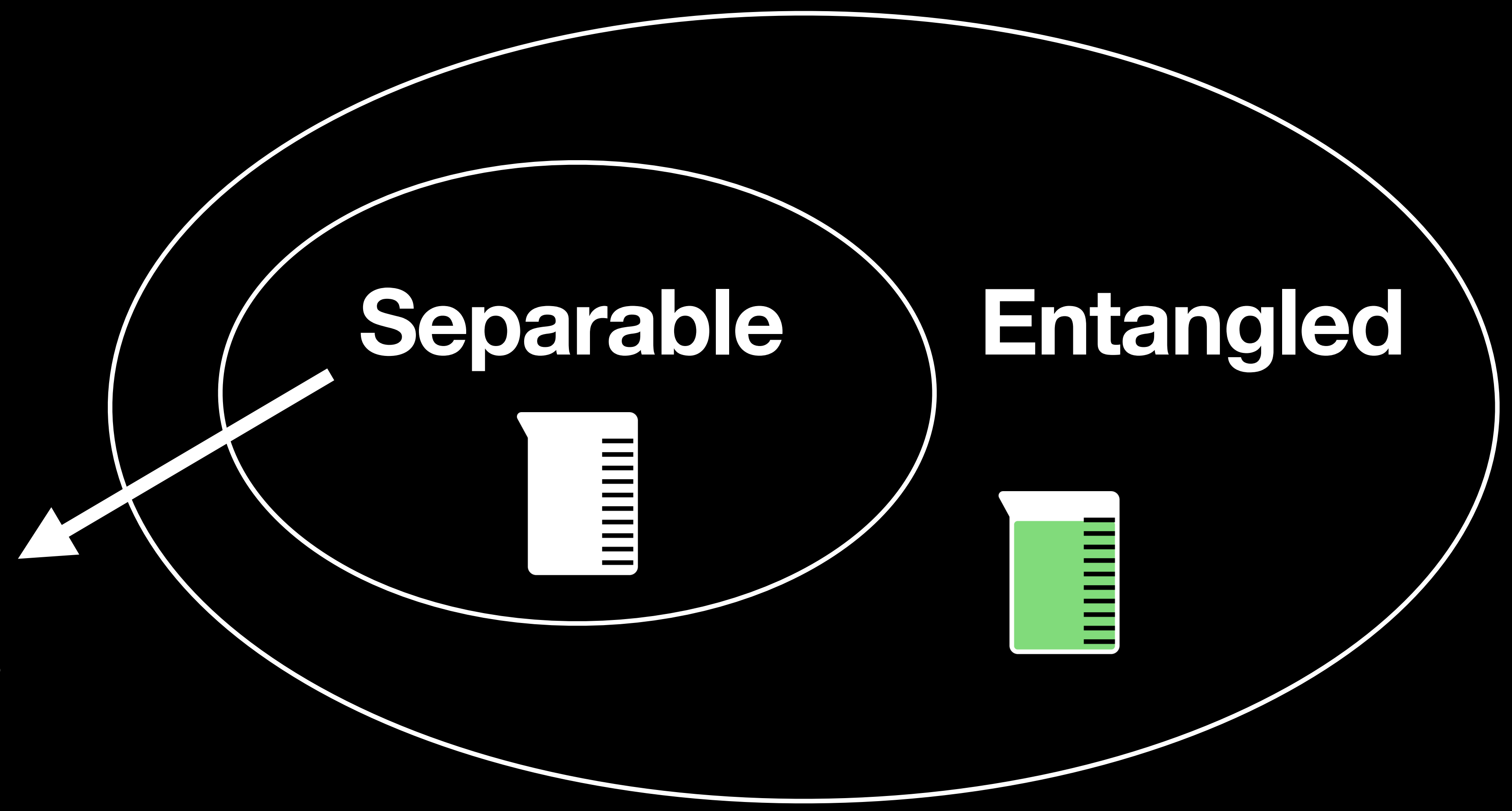
Resource theory of entanglement

Bipartite system



$$|\text{Bell}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

How do we define an entangled state?



$$|\psi\rangle_A \otimes |\psi\rangle_B$$

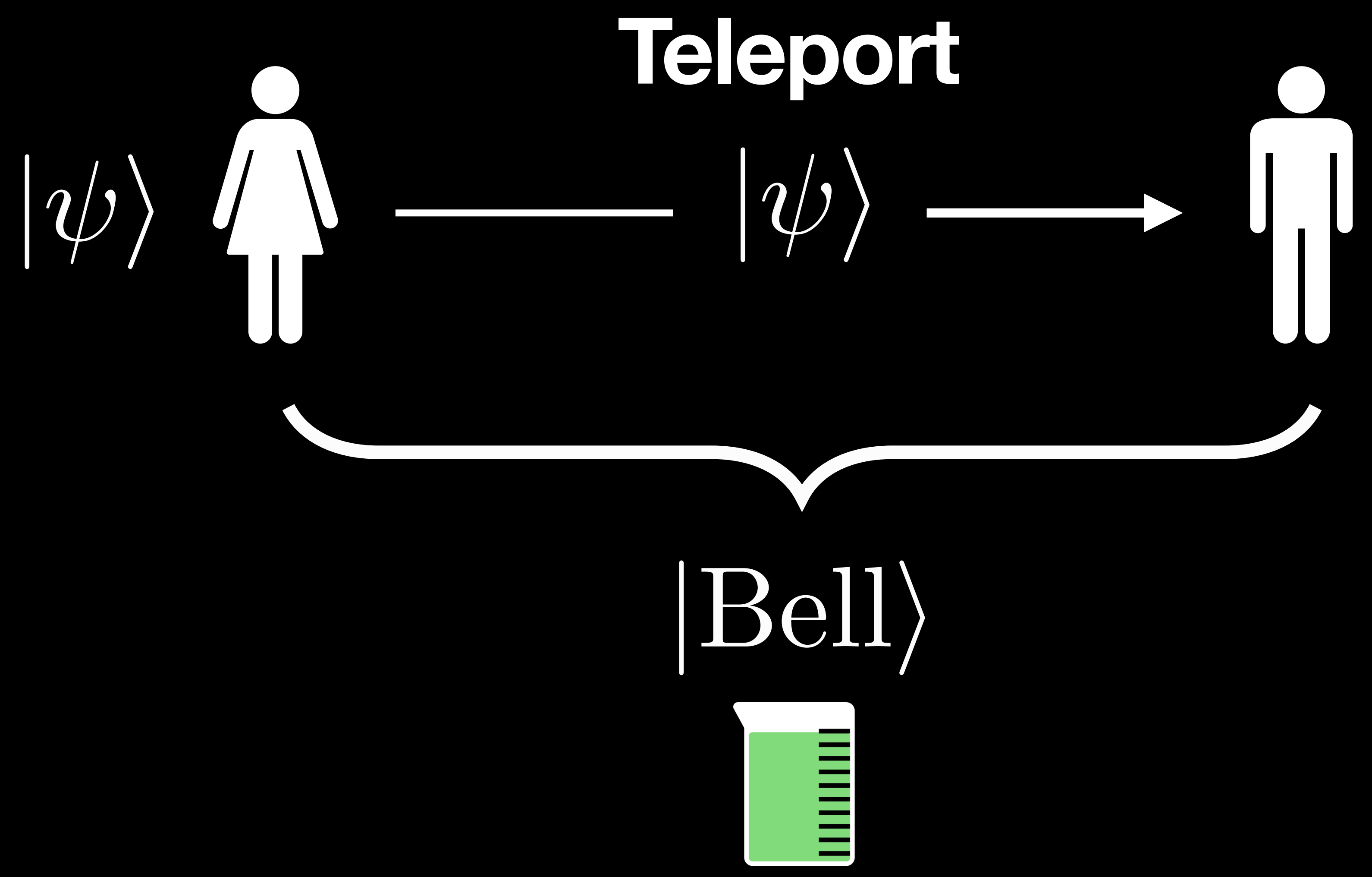
How to measure entanglement?

$$S(\rho_A) = -\text{Tr}\left\{\rho_A \log(\rho_A)\right\}$$

Resource theory of entanglement

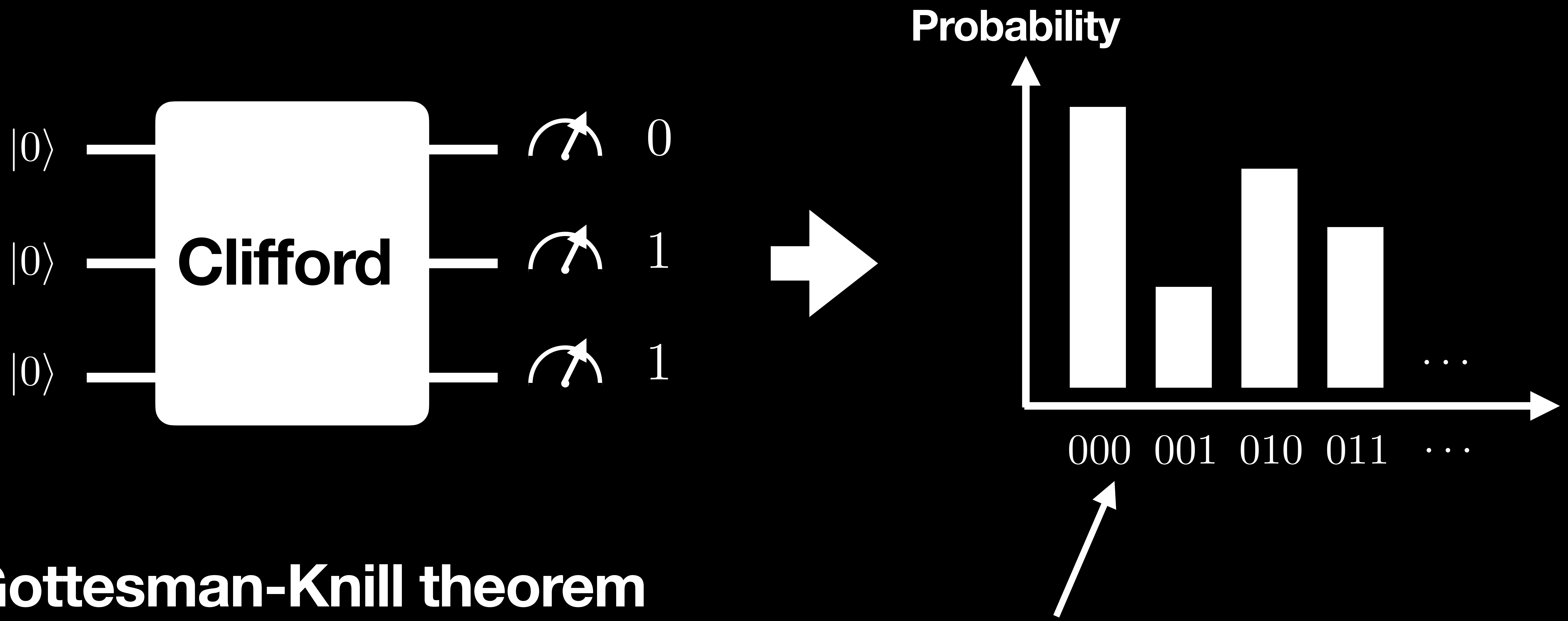
Why is entanglement a resource?

Has many applications!



Entanglement is essential for quantum teleportation

What is Magic?



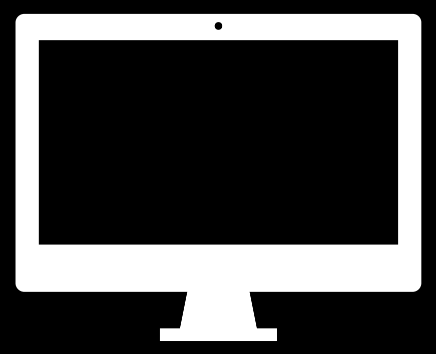
Gottesman-Knill theorem

Can be efficiently simulated classically.

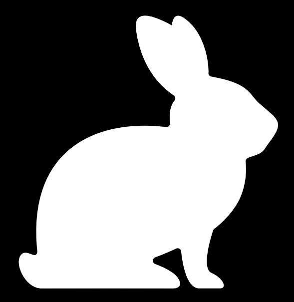
What is Magic?

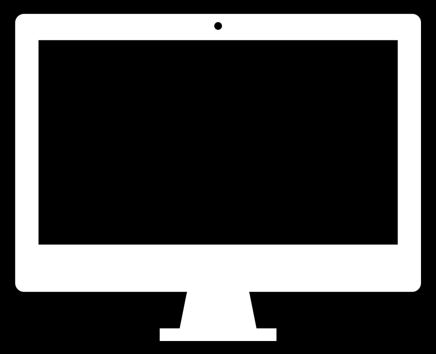
Want quantum advantage → **Use non-Clifford gates**

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$

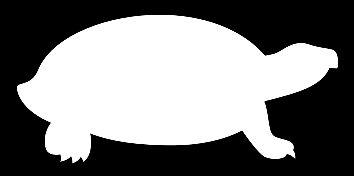


Quantum

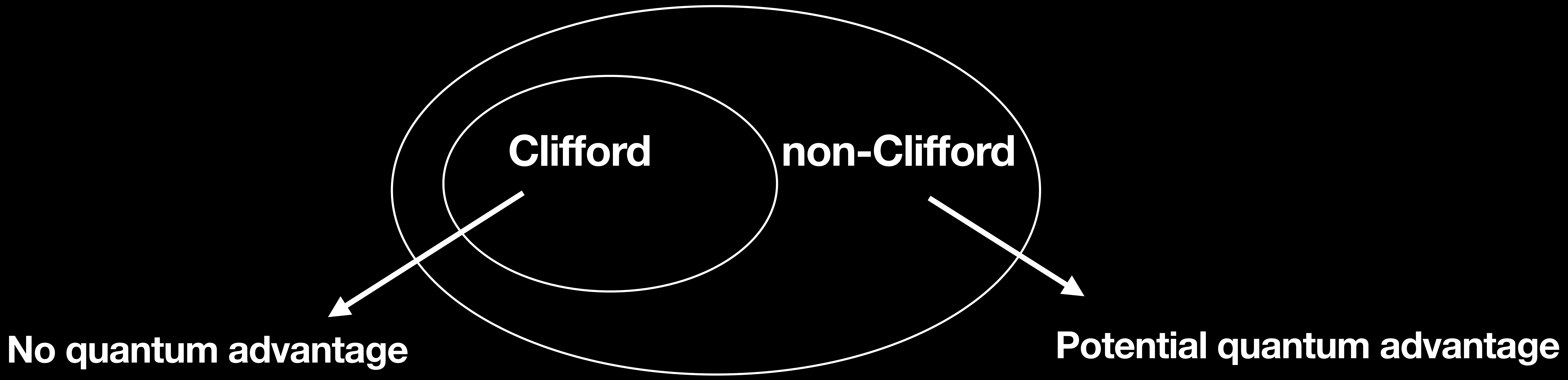




Classical



Resource theory of magic



Resource: magic

Magic monotone

Function which measures magic

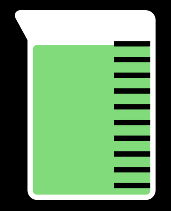
Resource theory framework

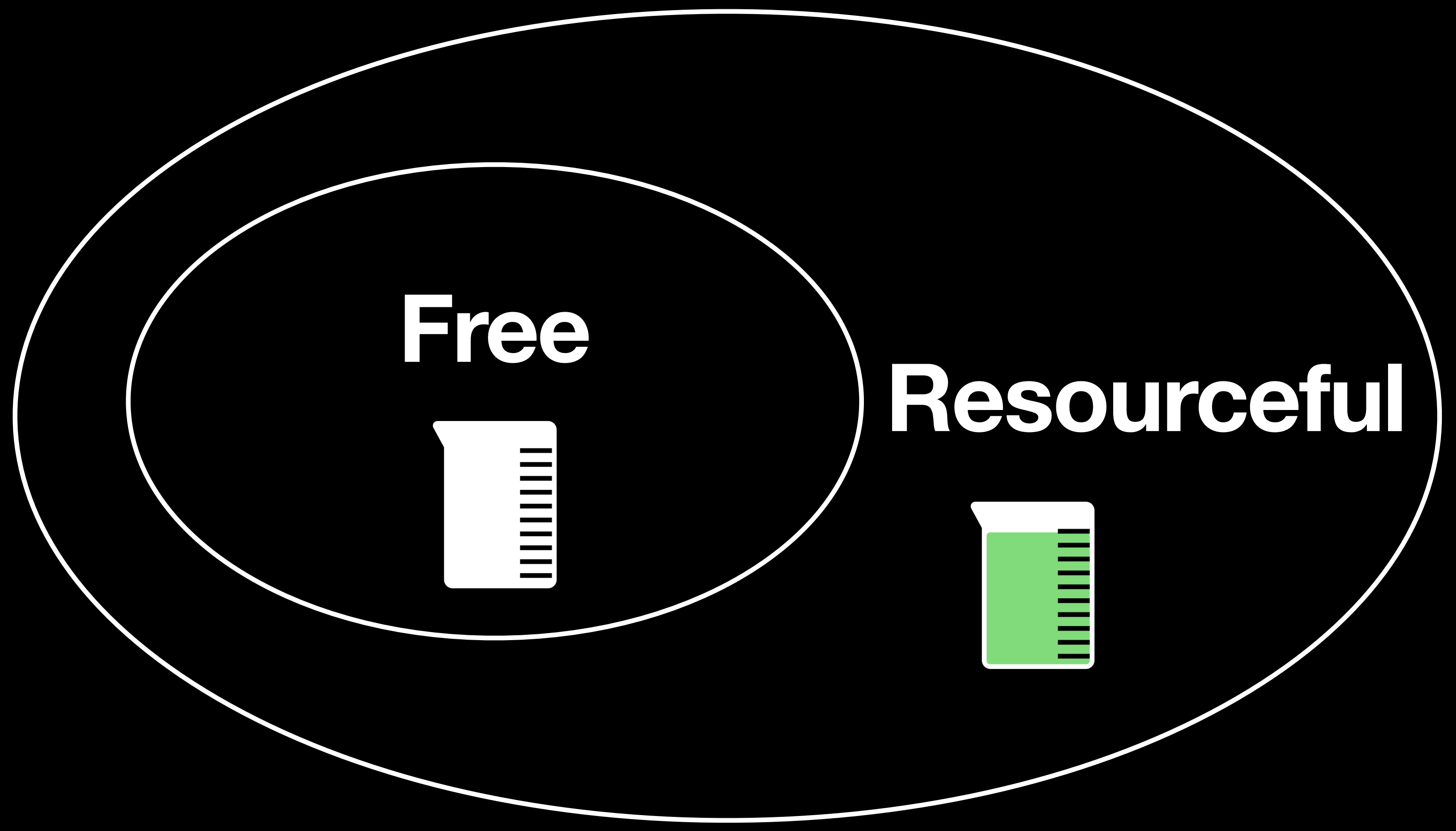
Resource

A useful quantum process/phenomenon

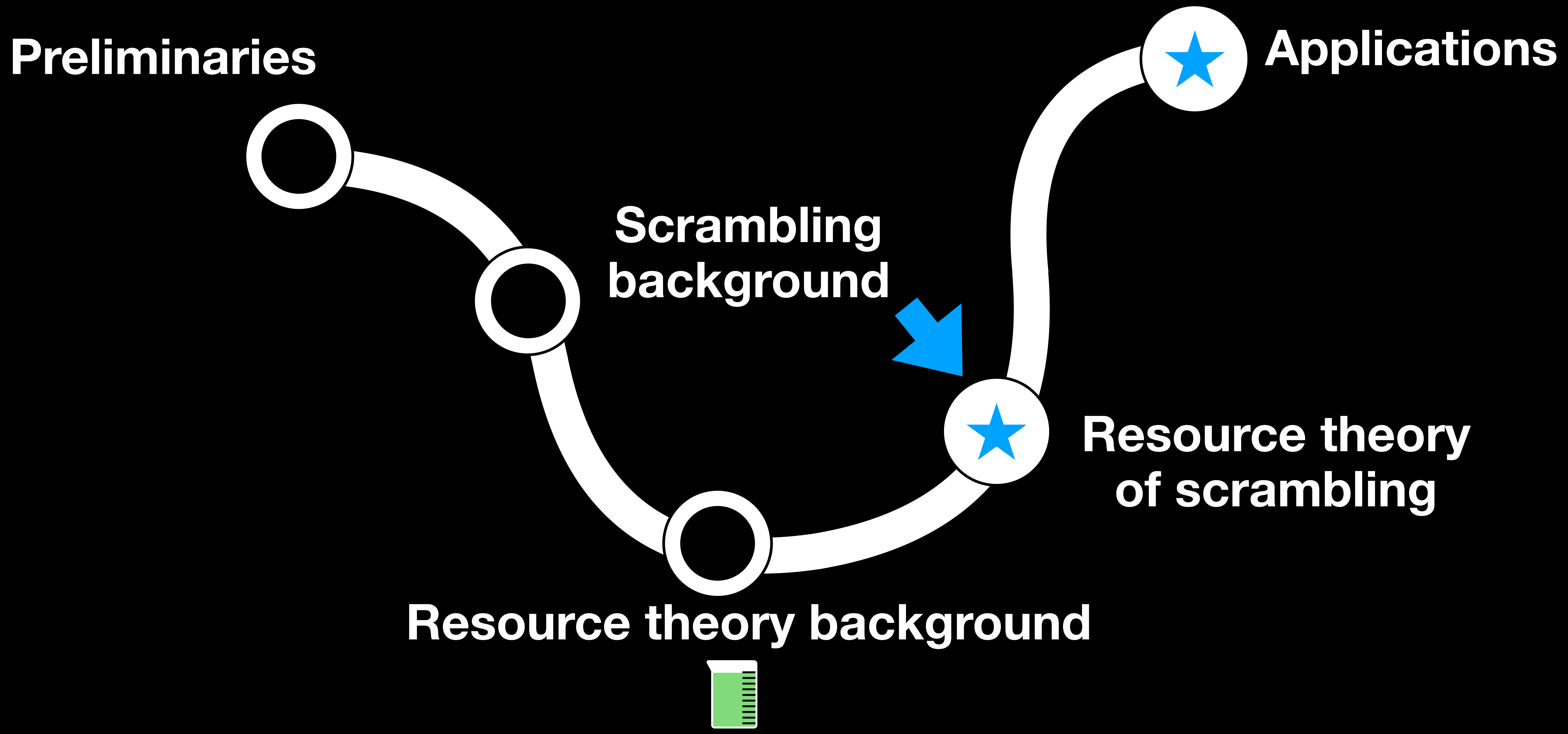
Components of a resource theory

1. Free unitaries
2. Resourceful unitaries
3. Resource monotone

 "Measuring cup"



Where we're going



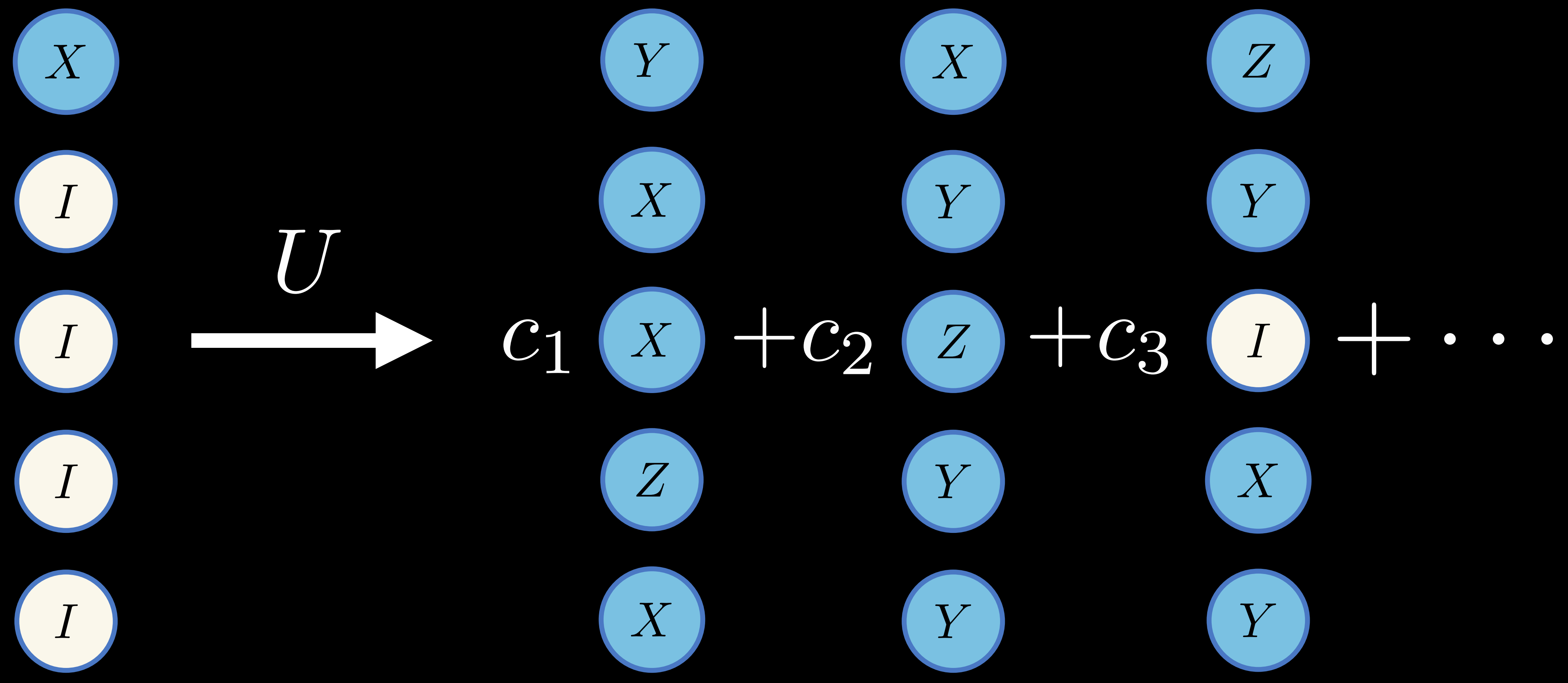
Why define a resource theory of scrambling?

- Gives us a definition of a scrambler
- Let’s us measure scrambling
- Helps us find applications for scrambling

What are the “resources” in scrambling?

Recall:

Scrambling unitaries do the following

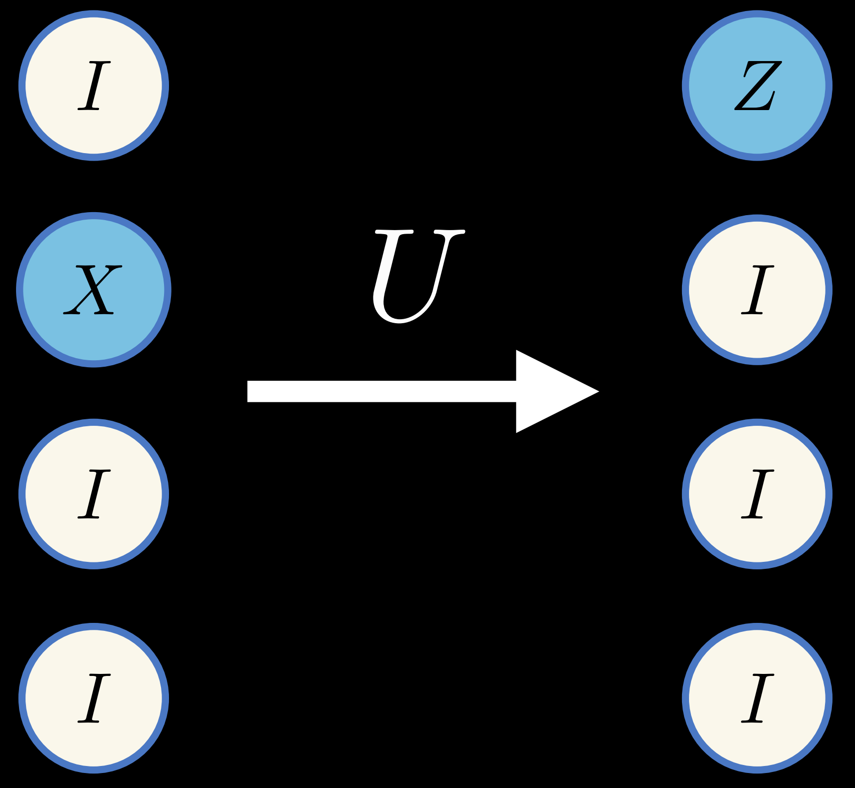


1. Increase # of strings ⇒ **Magic scrambling**
2. Increase # of blue dots ⇒ **Entanglement scrambling**

Entanglement scrambling

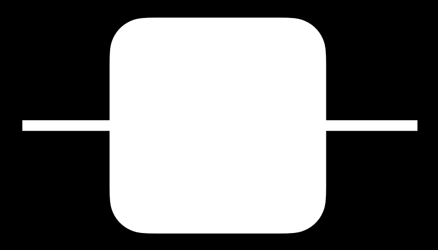
Resource: operator spreading

1. Free unitaries

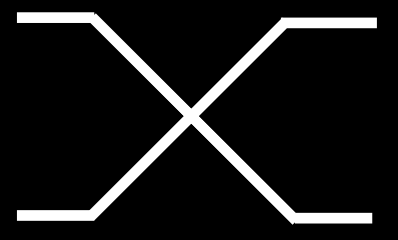


Generated by

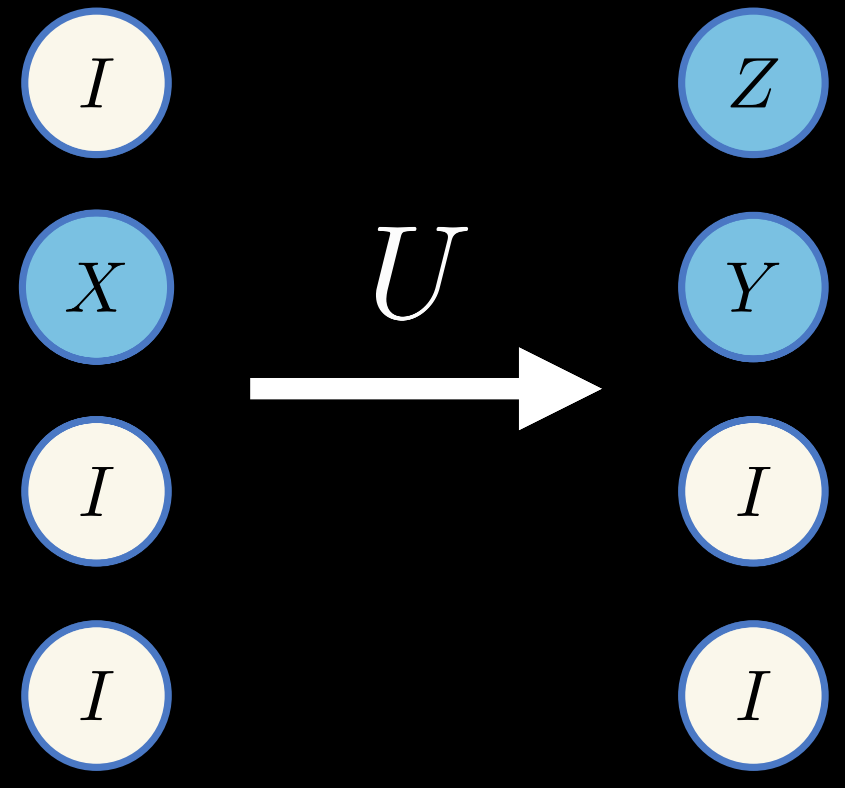
Local gates



Swap gates



2. Resourceful unitaries



↑ # of blue dots

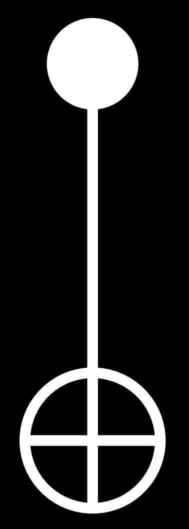
Every other unitary

3. Monotone

Pauli Growth

Ex:

CNOT

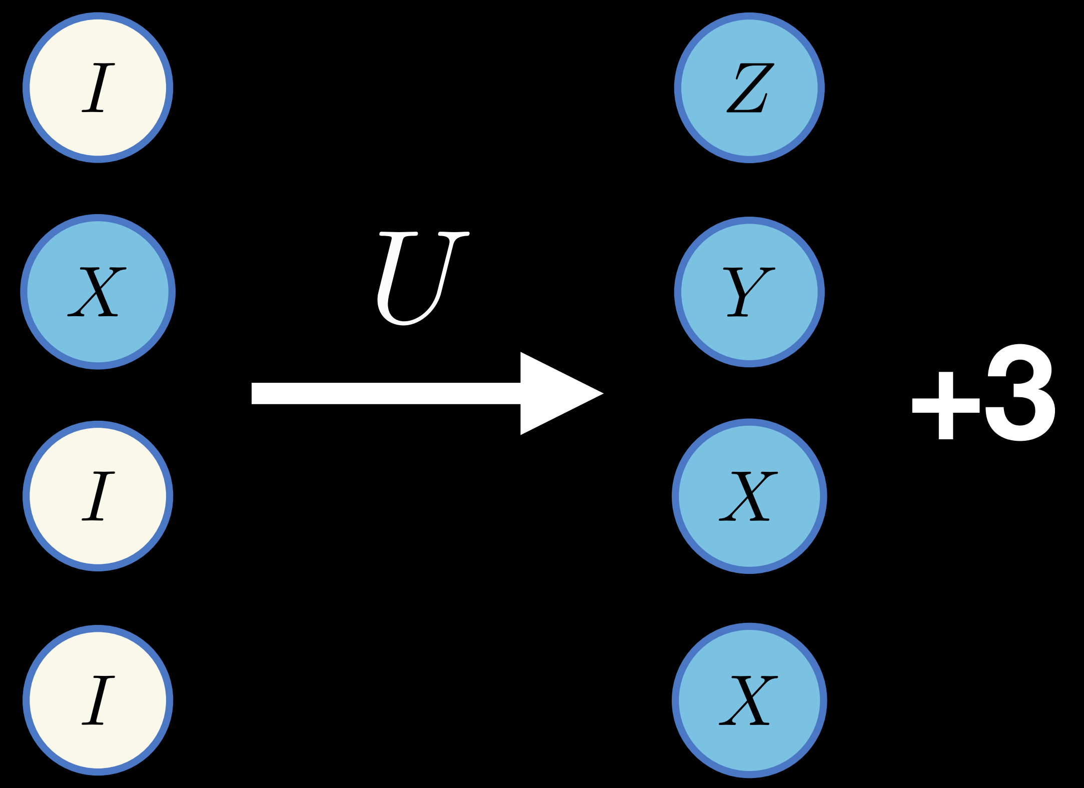


Entanglement scrambling

Pauli growth

$$G(U) \equiv \max_{\substack{O:||O||_2=1,\\ W(O)=1,\mathrm{Tr}\{O\}=0}} \left[W(U^\dagger O U) - W(O)\right]$$

Counts increase in blue dots



Entanglement scrambling

Pauli growth

$$G(U) \equiv \max_{\substack{O:||O||_2=1,\\ W(O)=1,\mathrm{Tr}\{O\}=0}} \left[W(U^\dagger O U) - W(O)\right]$$

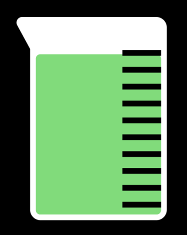
$G = 0$ **Free unitaries**

$G > 0$ **Resourceful unitaries**

Entanglement scrambling

How to experimentally measure Pauli growth?

$$G(U) \geq \frac{3n}{4}\Big(1 - \mathbb{E}\mathrm{OTOC}(U)\Big) - 1$$



Measured in experiment

Similar bounds established in

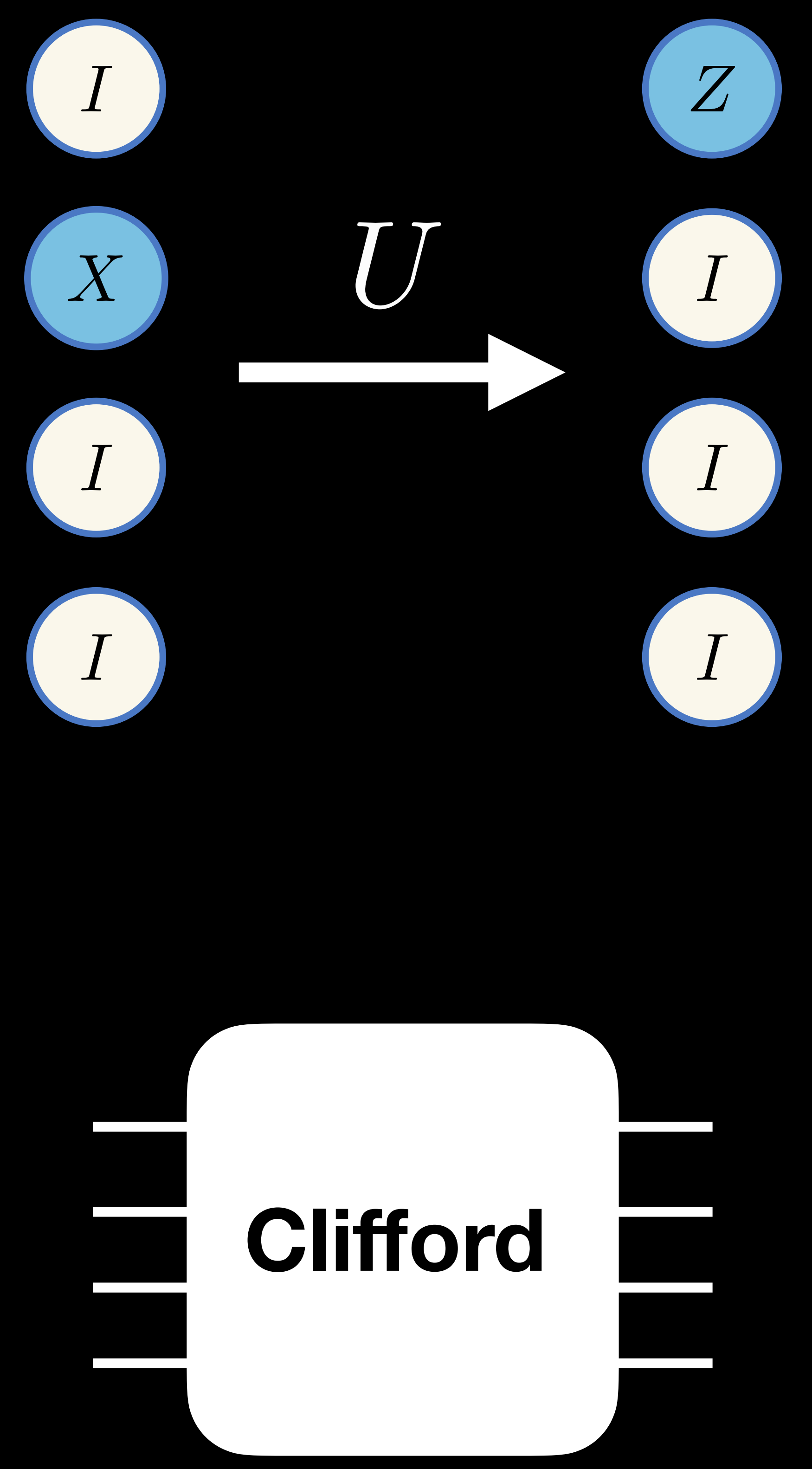
T. Zhou and X. Chen, PRE 99, 052212 (2019).

K. Bu et al., arXiv:2204.12051, (2022).

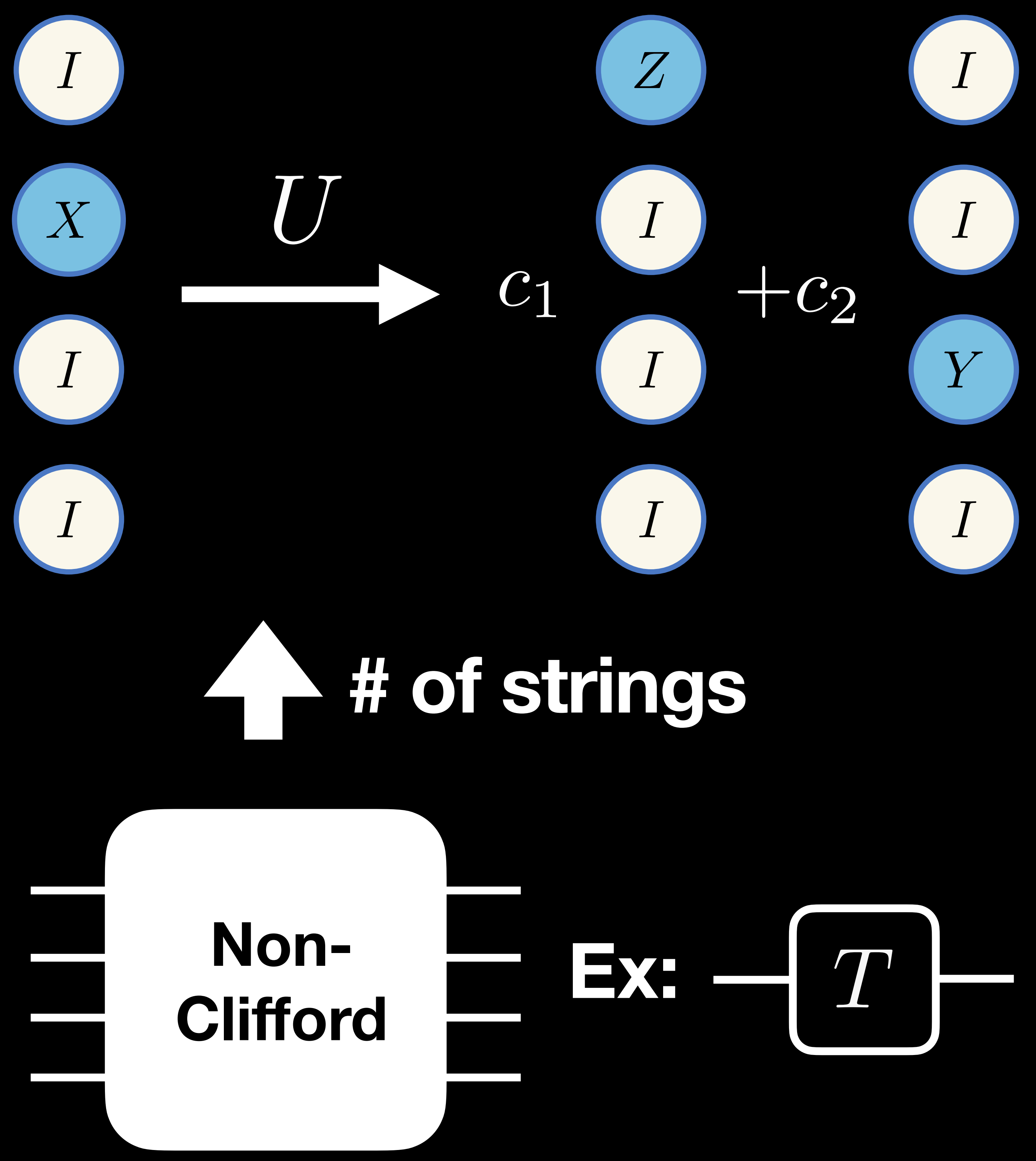
Magic scrambling

Resource: magic

1. Free unitaries



2. Resourceful unitaries



3. Monotone

Entanglement scrambling

OTOC magic

$$O_M(U) \equiv \max_{P,P' \in \mathcal{P}_n} [1 - |\mathrm{OTOC}(U)|]$$

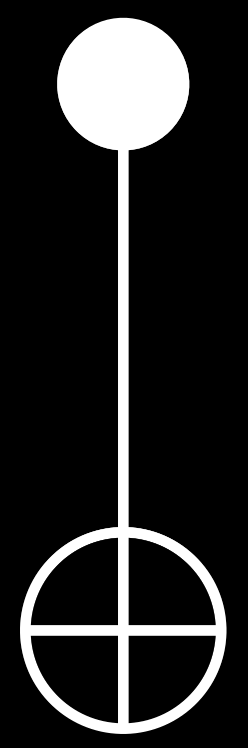
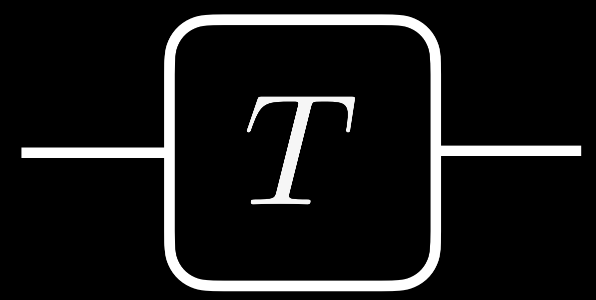
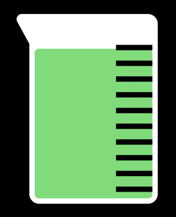
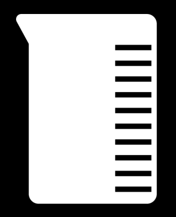
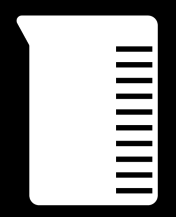
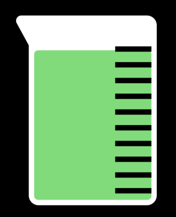
Measured in experiment

$O_M = 0$ **Clifford unitaries**

$O_M > 0$ **Non-Clifford unitaries**

Why define a resource theory with two mechanisms?

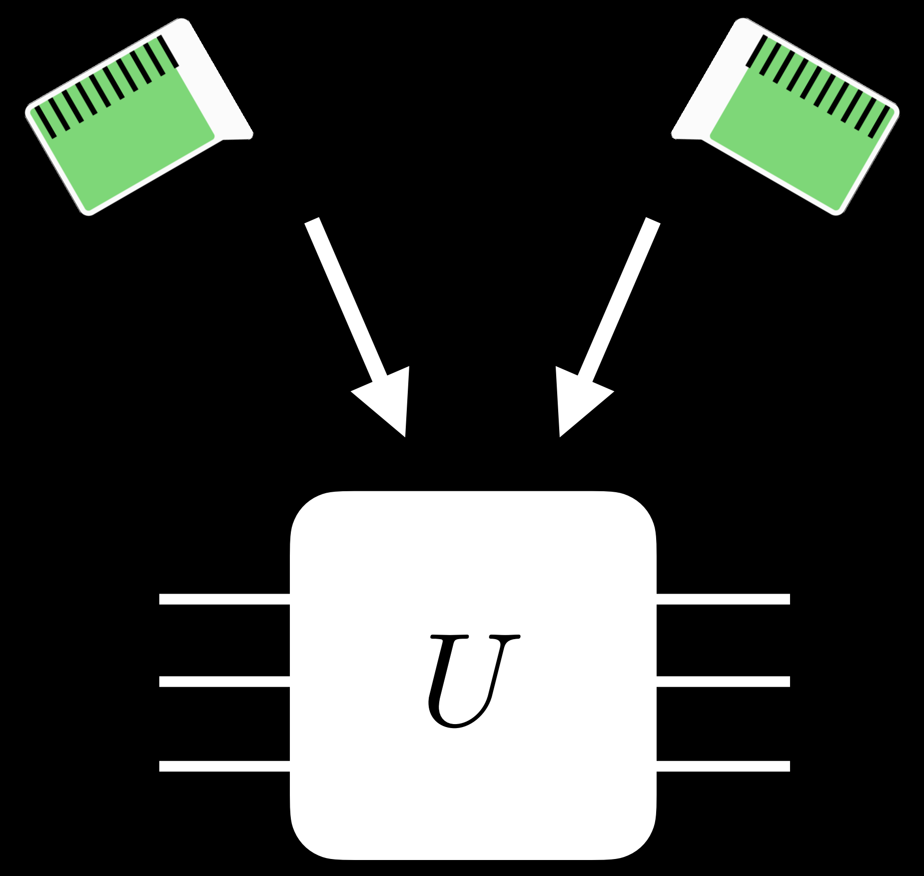
They are independent.

	CNOT 	T-gate 
Entanglement scrambling		
Magic scrambling		

What is a scrambler?

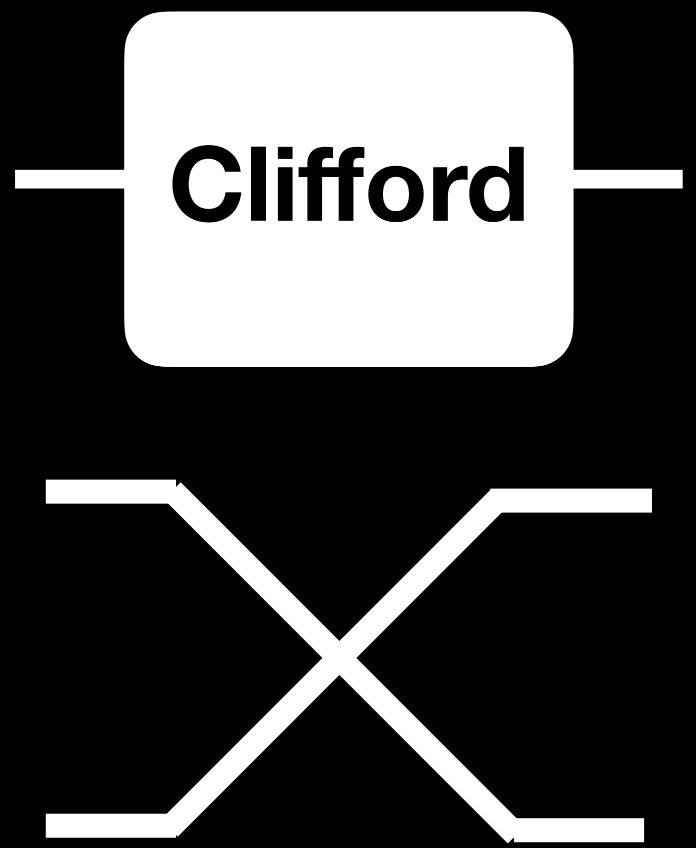
A unitary with at least one mechanism.

Entanglement scrambling or **Magic scrambling**



What is a scrambler?

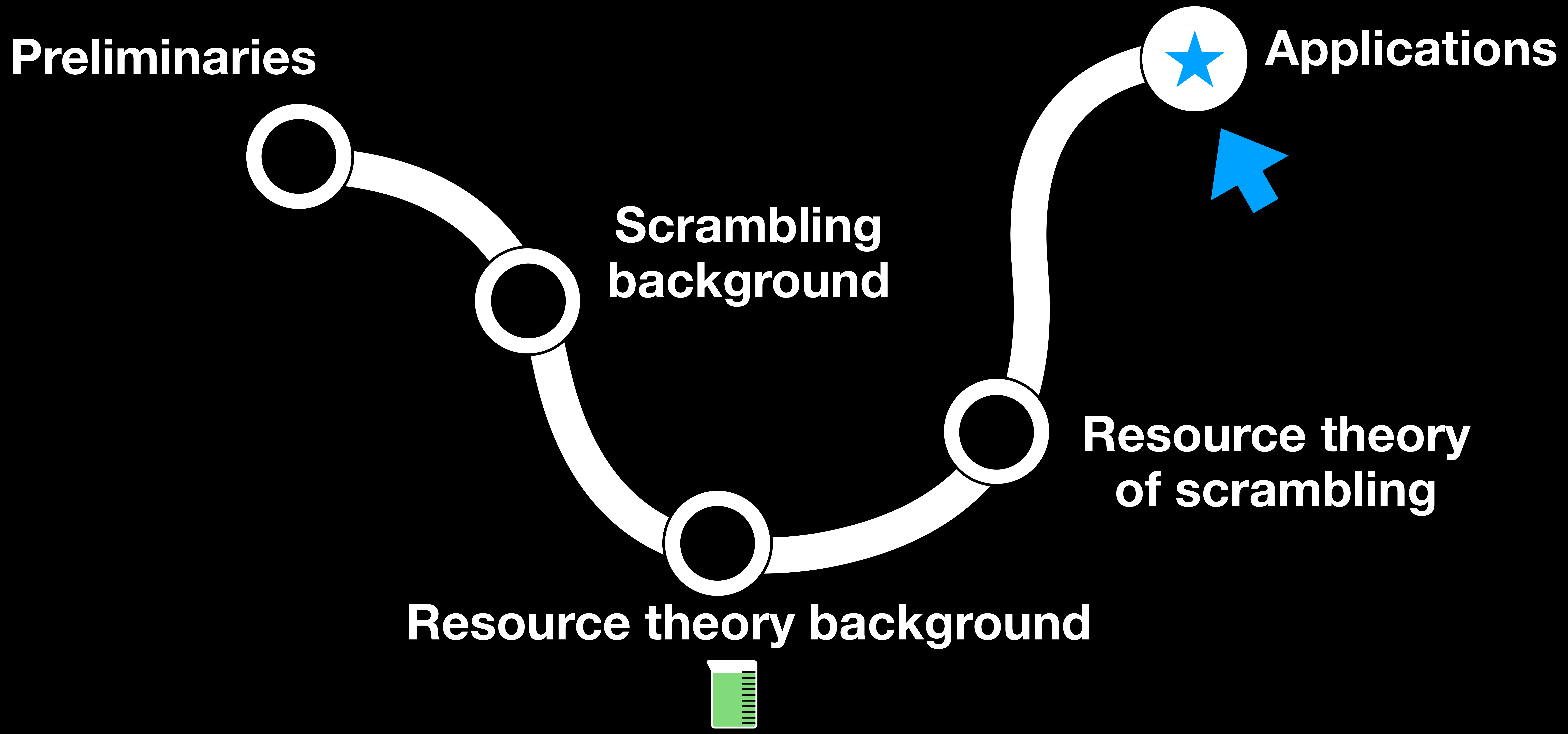
Non-scramblers

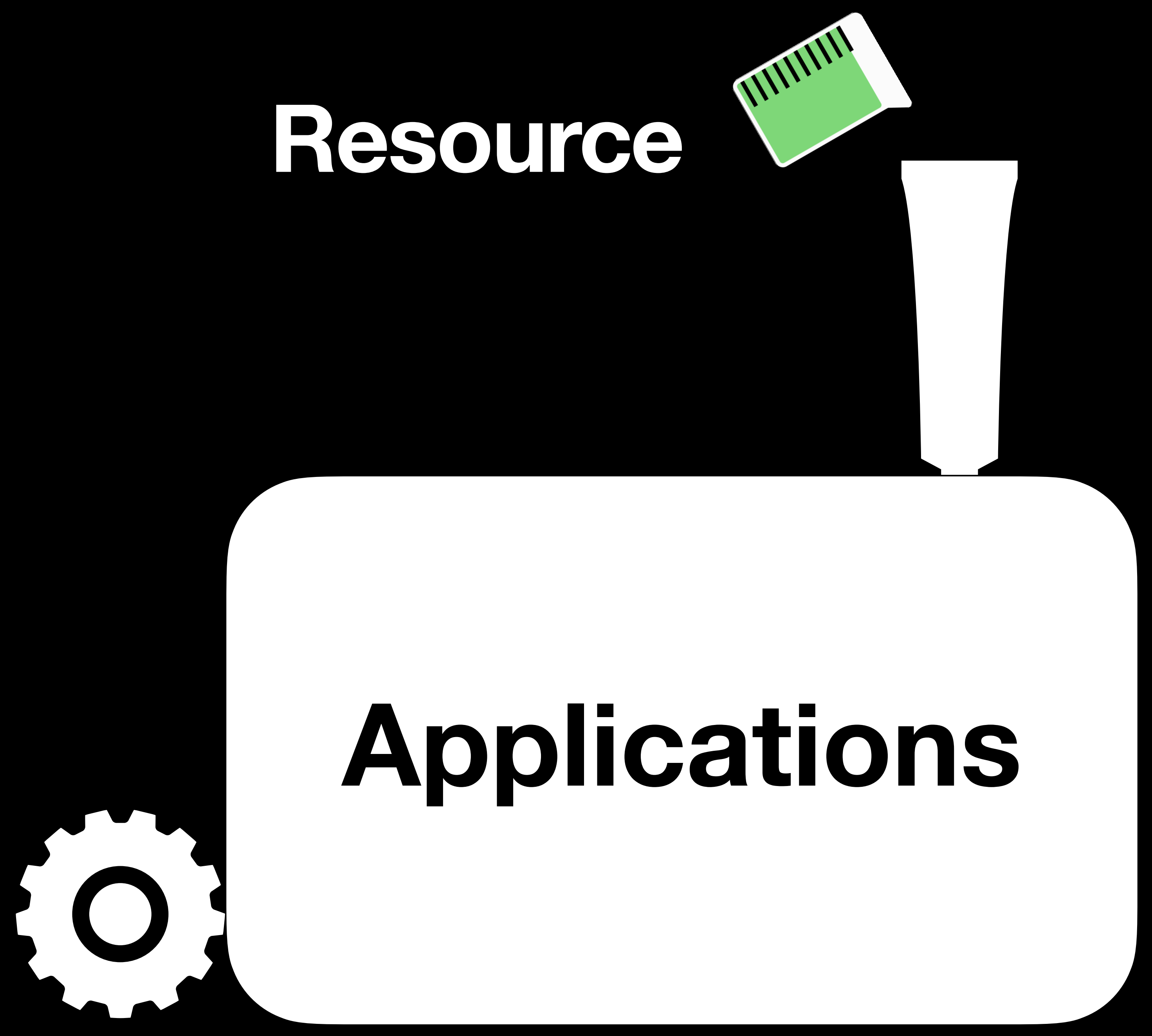


Scramblers

Can’t be
generated by

Where we're going



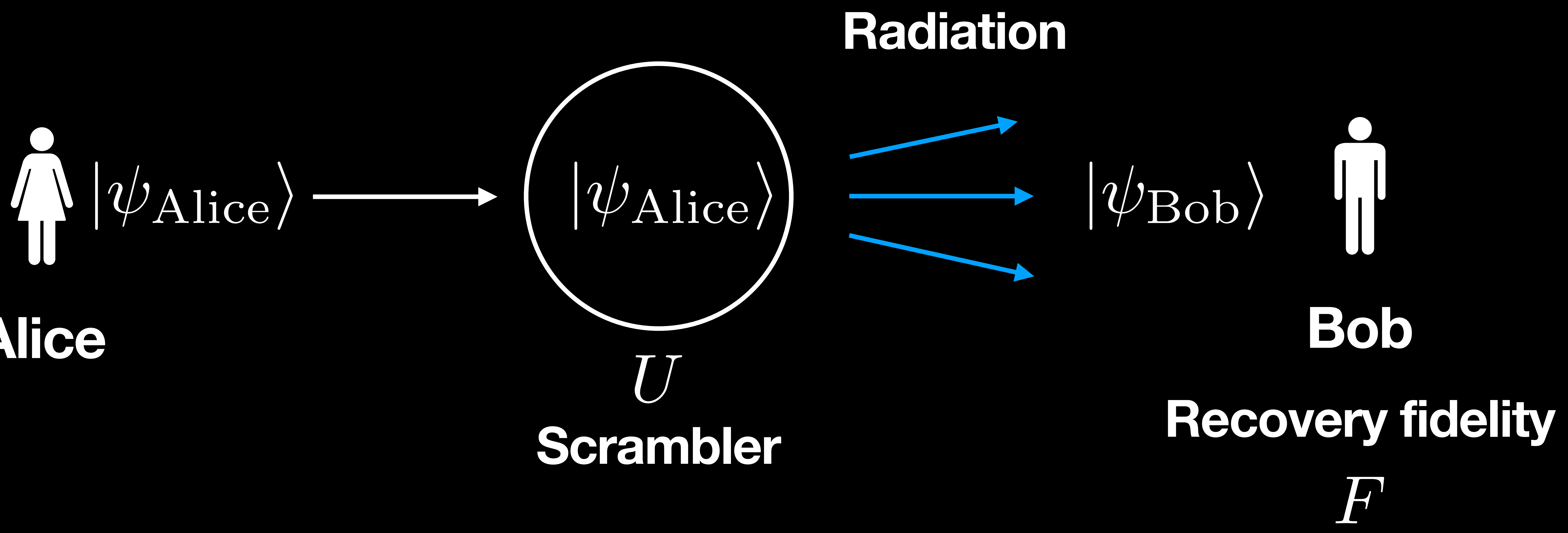


1. Recover information from a black hole

2. Explain Google's experiment

Application to Hayden-Preskill Protocol

Task: Recover state thrown into black hole



P. Hayden and J. Preskill, JHEP 2007, 120 (2007).

B. Yoshida and A. Kitaev, arXiv:1710.03363 (2017).

Application to Hayden-Preskill Protocol

Our contribution:

Entanglement scrambling

G

Measures operator spreading in black hole

Magic scrambling

O_M

Measures magic in black hole

Bound fidelity F

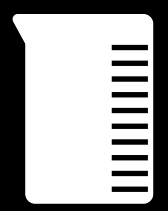
Scrambling resources can help Bob recover Alice’s state.

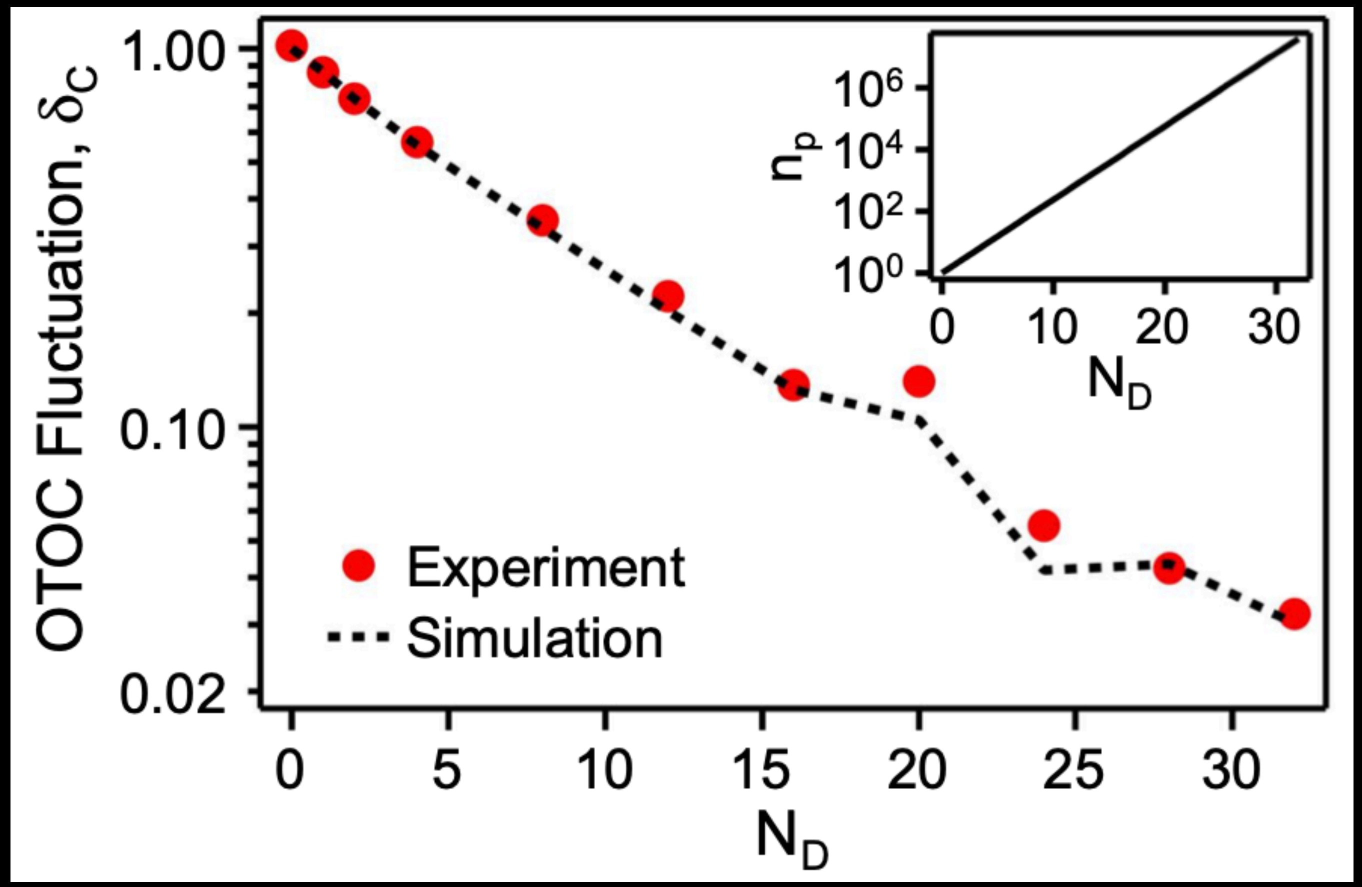
Google's experiment

Google measured signatures of magic in experiment.

Our contribution:

Use resource theory

 O_M

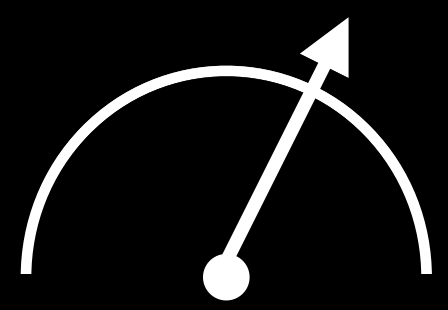


To explain magic signature

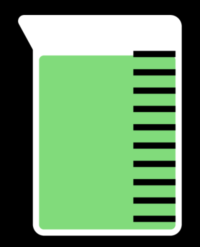
X. Mi et al., Science 374, 1479 (2021).

Google's experiment

Google found

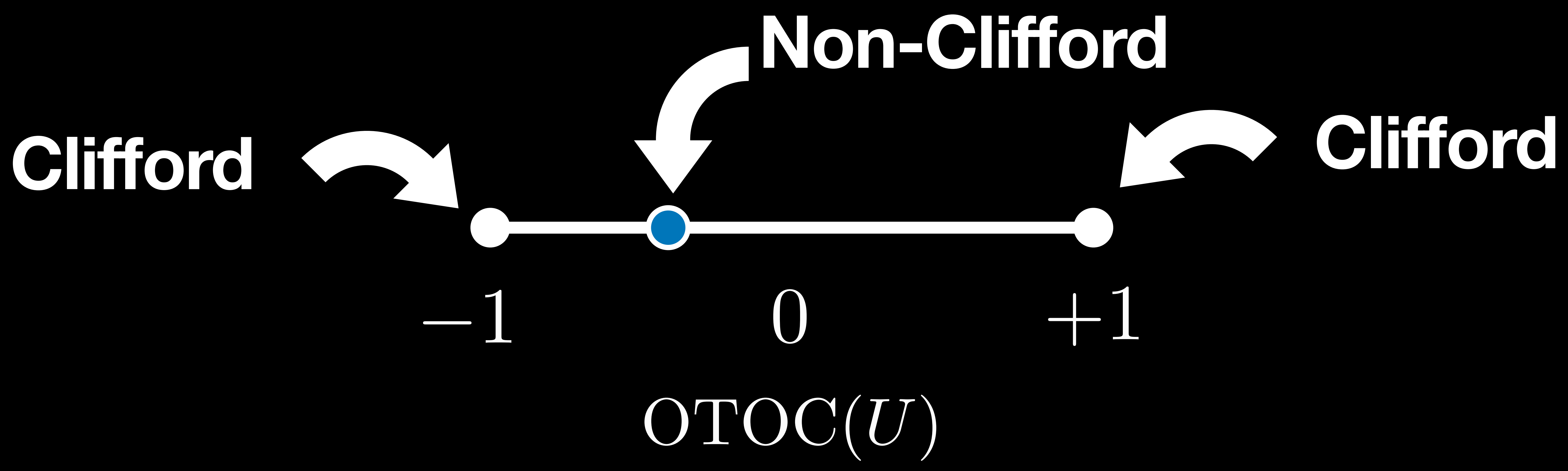


If $\mathrm{OTOC}(U) = 0$
with high probability



U
has a large amount of magic.

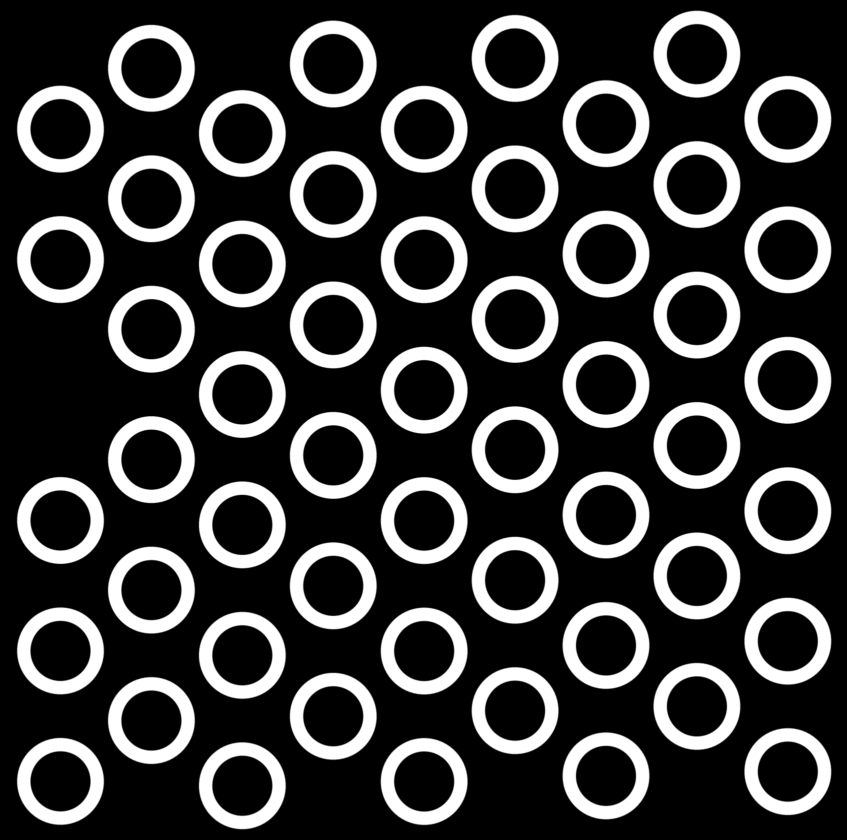
Google's experiment

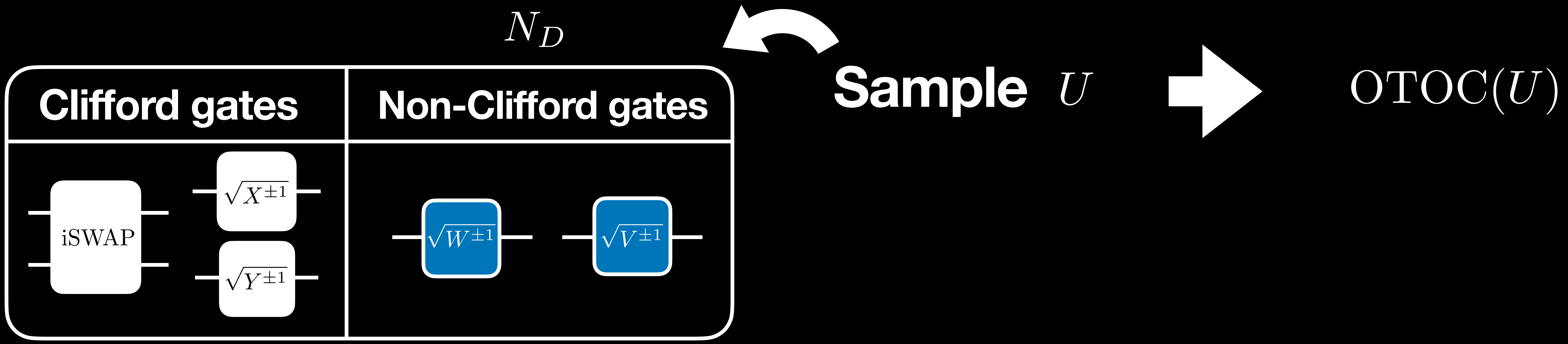


Google's experiment

Basics of the experiment

Sycamore quantum processor





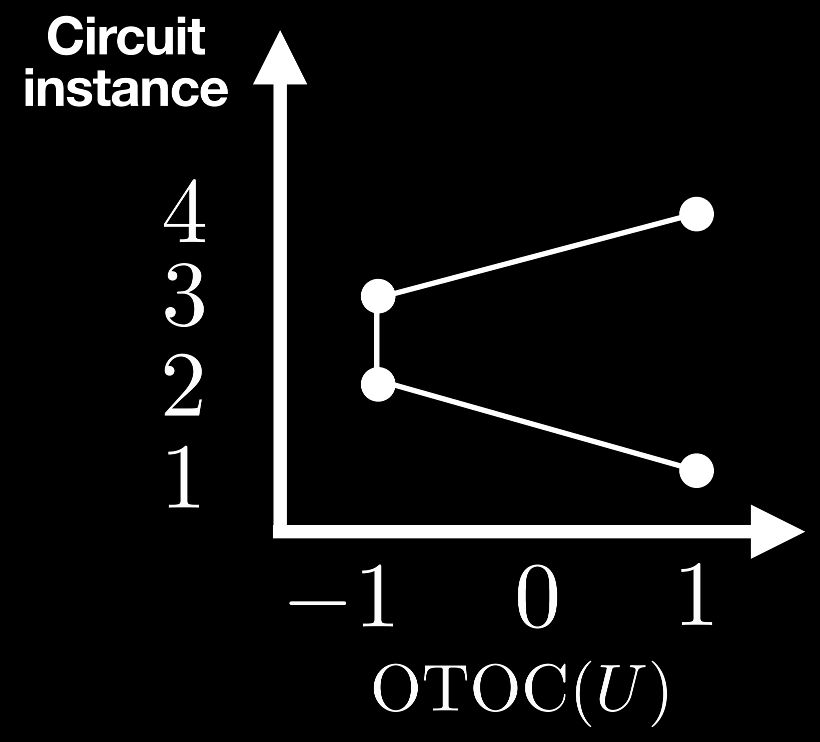
Google's experiment

Example (Not real data)

Fix $N_D = 0$

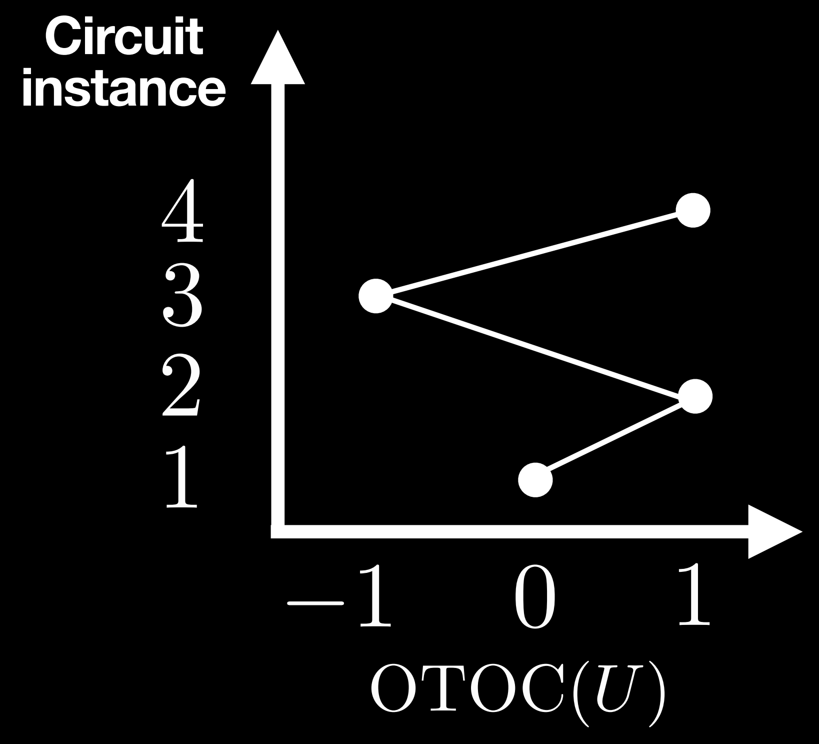
(Clifford circuit)

Random Circuit	$\mathrm{OTOC}(U)$
U_1	1
U_2	-1
U_3	-1
U_4	1
⋮	⋮



Fix $N_D = 1$

Random Circuit	$\mathrm{OTOC}(U)$
U_1	0
U_2	1
U_3	-1
U_4	1
⋮	⋮

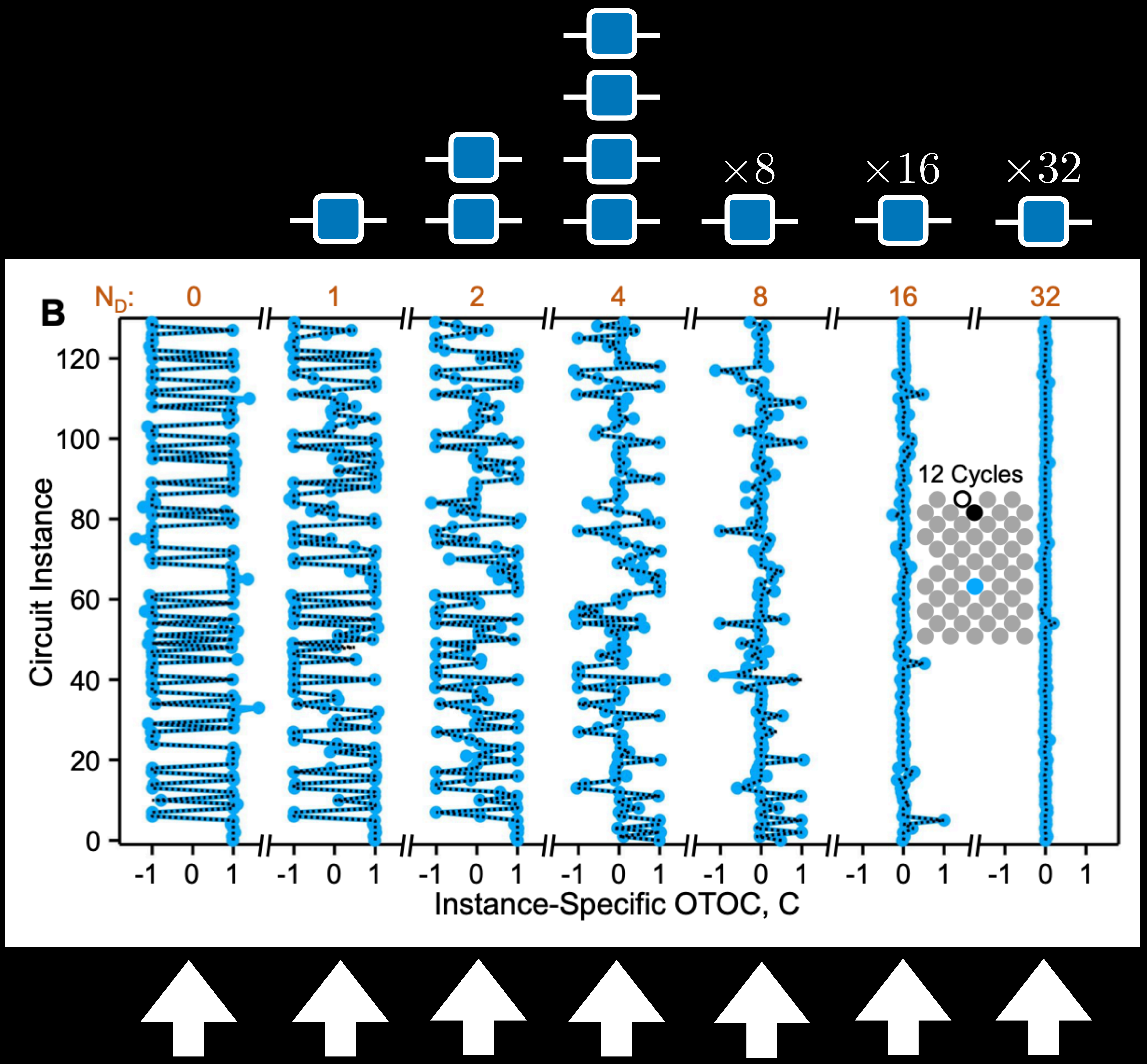


Fix $N_D = 2$

Repeat

⋮

Google's data



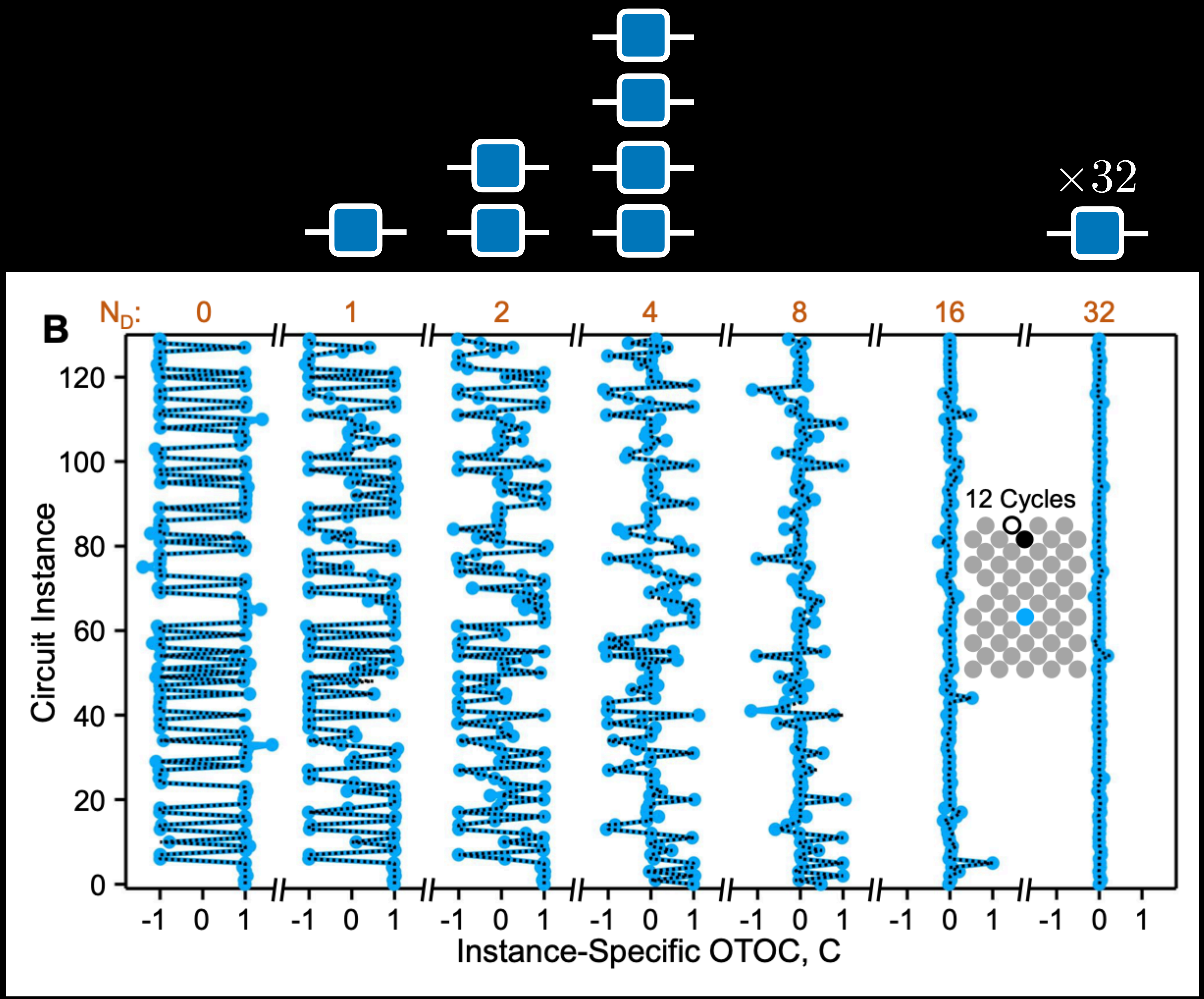
Fluctuations decay as magic gates added

$$\mathrm{OTOC}(U) = 0$$

with high probability

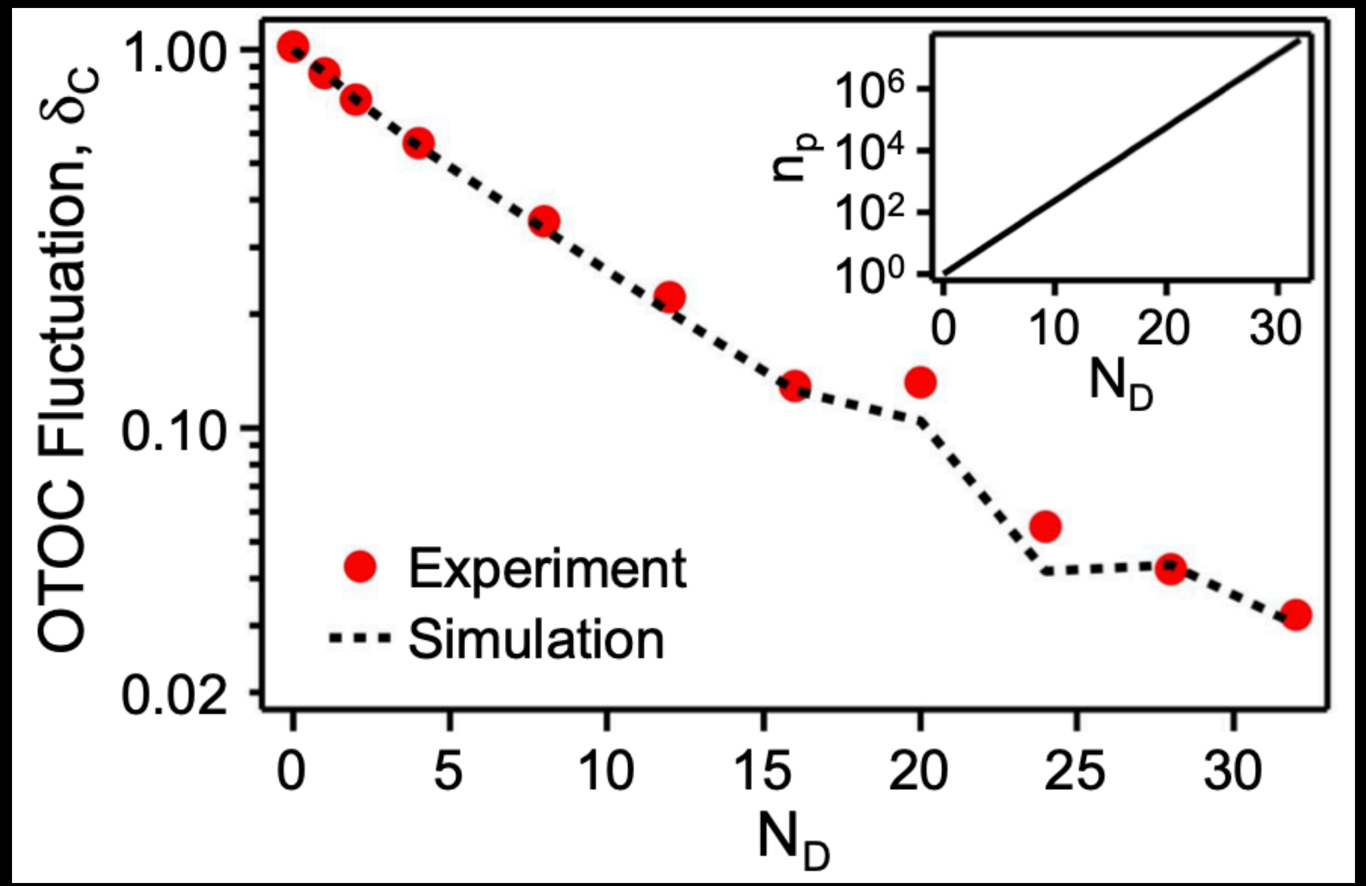
X. Mi et al., Science 374, 1479 (2021).

Google's data



OTOC fluctuations

$\delta_C \equiv$ **Standard deviation of OTOC**

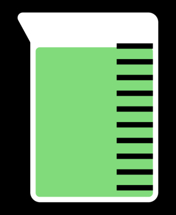


X. Mi et al., Science 374, 1479 (2021).

Our results

Use OTOC magic to explain data

$$\mathbb{E}_{U} O_M(U) \geq 1 - \delta_C$$



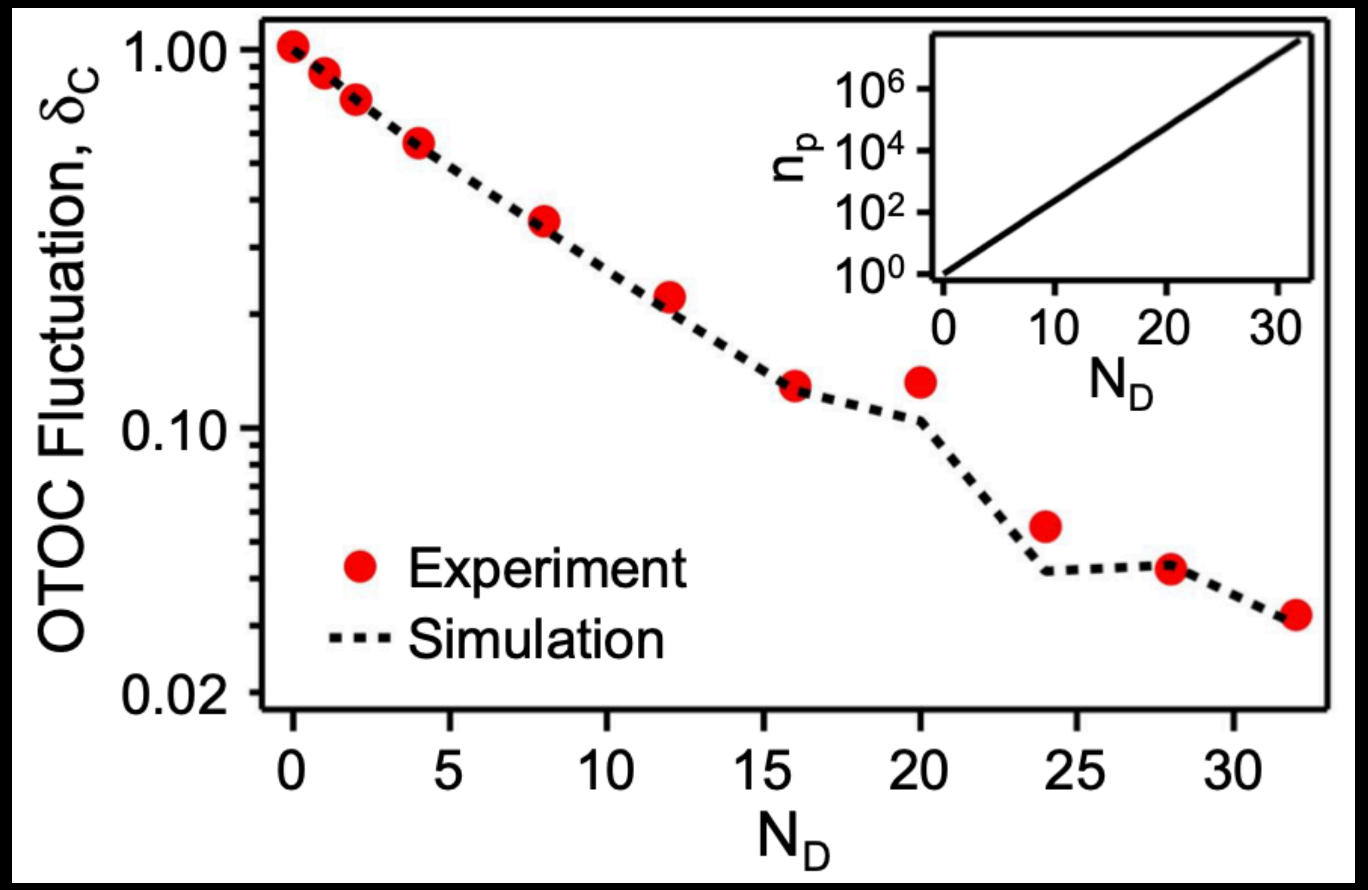
Magic monotone

Faithful

⬆ Large

⬇ Small

Not a magic monotone
but can be measured



X. Mi et al., Science 374, 1479 (2021).

Closing Remarks

How is scrambling a resource in other QI tasks?

Generalize to quantum channels

How is scrambling related to other resources theories?

Thank you!