# A new way to understand computational complexity

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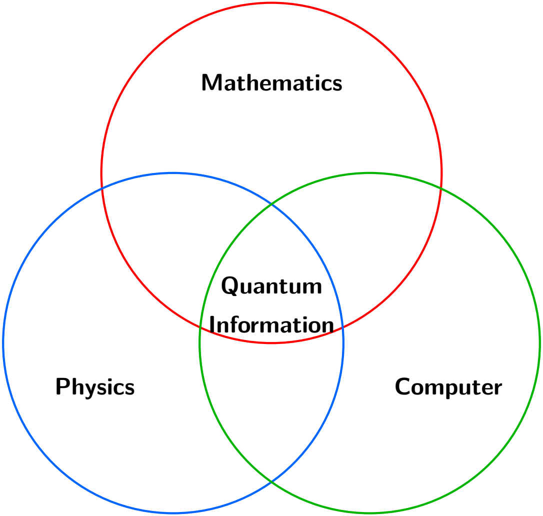

Mathematics
Quantum
Information
Physics
Computer

# Computational Complexity: Classical VS Quantum

Classical computing is approaching the limits imposed by Moore's law [Wal16]. Quantum computing has emerged as a promising alternative, inspired by the concept of quantum simulation introduced by Manin and Feynman [Man80, Fey82].

A landmark breakthrough was Shor's discovery of a quantum algorithm for integer factorization, which offers exponential speedup over known classical algorithms [Sho94, Sho97].

In the Noisy Intermediate-Scale Quantum (NISQ) era [Pre18, BCLK$^+$22, CCHL22], several experimental demonstrations have claimed quantum advantage [A$^+$19, M$^+$23, Z$^+$20, Z$^+$21, L$^+$22, Q$^+$23, M$^+$22, K$^+$23].

On the other hand, numerous research groups have developed classical simulation techniques to challenge and benchmark these experiments using high-performance classical computation [ZSW20, PWZP21, HLF$^+$20, GK21, CC20, OHJ21, W$^+$21, MTB$^+$19, TFSS23, BC23, ATKZ23, SWCL24, BC24].

Understanding the limitations of classical computation relative to quantum computation is a central question in quantum information science.
Two well-known classes of quantum circuits that admit efficient classical simulation are **Clifford** circuits, which can be simulated using the stabilizer formalism [Got99], and **Matchgate** circuits, which correspond to noninteracting fermionic systems and are efficiently simulable via Pfaffian matrix methods [Val02, TD02].
A further powerful approach involves the classical simulation of tensor network states with low **entanglement**. When the entanglement entropy scales according to an area law, as is typical for gapped ground states in one and two dimensions, tensor network methods such as Matrix Product States (MPS) and Projected Entangled Pair States (PEPS) enable efficient simulation by reducing the computational cost to the system's boundary degrees of freedom [ECP10, VMC08, Or 14].

# Universality and Magic

The union of Clifford gates and Matchgates constitutes a universal gate set for quantum circuits [JM08].
Significant efforts have been devoted to extending classical simulation techniques for Clifford circuits to accommodate a limited number of non-Clifford, or so-called Magic, gates [BK05, BSS16, HWVE14, SC19, BGK19], and to generalizing Matchgate-based simulation frameworks beyond non-interacting fermionic systems [TD02, JM08].
A central challenge in the field is to develop a unified classical simulation scheme capable of simulating circuits that combine both Clifford and Matchgate operations, ideally achieving polynomial time and space complexity for both Clifford circuits and Matchgate circuits.

# Quon Classical Simulation (QCS)

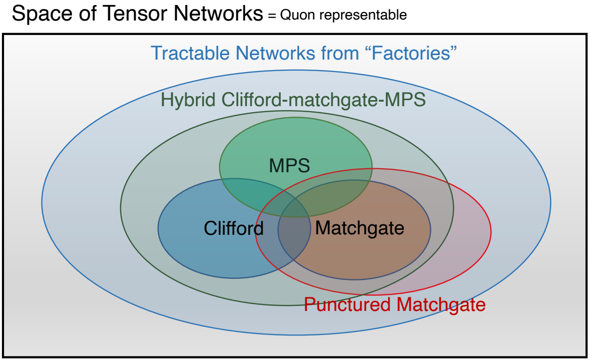
In recent two arXiv preprints posted on May 12, 2025,

- [KZL$^+$25] Byungmin Kang, Chen Zhao, Zhengwei Liu, Xun Gao, Soonwon Choi
  2D Quon Language: Unifying Framework for Cliffords, Matchgates, and Beyond
  https://arxiv.org/abs/2505.06336
- [FLLW25] Zixuan Feng, Zhengwei Liu, Fan Lu, Ningfeng Wang
  Quon Classical Simulation: Unifying Clifford, Matchgates and Entanglement
  https://arxiv.org/abs/2505.07804

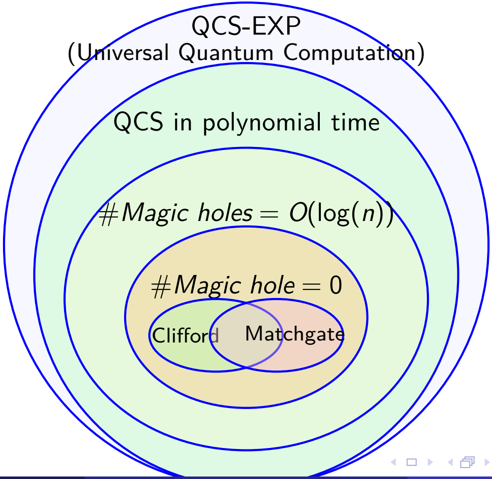
we develop a unified classical simulation framework of quantum circuits and tensor networks based on

- Quon Language [LWJ17], joint with Arthur Jaffe and Alex Woznikowski;
- Alterfold Topological Quantum Field Theory [LMWW23a, LMWW23b, Liu24], joint work with Shuangming, Yilong Wang and Jinsong Wu.

In [KZL+25], we propose a unified framework to present quantum circuits and tensor networks, with efficient classical simulation of **Punctured Matchgates** under planar compositions.



In [FLLW25], we propose a unified algorithm to classically simulate quantum circuits and tensor networks, achieving polynomial time and space complexity for both Clifford circuits and Matchgate circuits. It resolves this central challenge in the field.
The algorithm is designed by new **Topological Complexity** and the essential source of exponential computational difficulty is captured by the number of **Magic Holes**.



# Highlights of QCS

Quon language is 3D picture language designed for quantum information. It is a 2+1D string-decorated topological quantum field theory with space time boundary in alterfold theory.

Quon Classical Simulation (QCS) provides a natural unified framework to represent hybird-Clifford-Matchgate tensor networks, which include universal gate sets of quantum circuits.

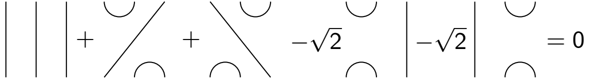
Our algorithm unifies the efficient classical simulations of both Cliffords and Matchgates:

- Clifford reductions are captured by generalized Surgery Moves in the 3-manifold;
- Matchgate reductions are captured by Reidemeister Moves (Yang-Baxter relations) of string-decortations on the 2D space boundary.

Given a 3-disc $D^3$ with a space boundary $S^2$ decorated by a set $C$ of embedded non-intersecting closed strings $C_i, i = 1, ..., m$, we define its partition function as:

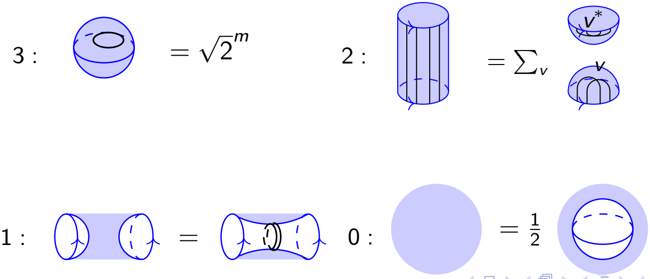
$$Z(D^3 \supset S^2 \supset C) = \sqrt{2}^m.$$

This partition function remains unchanged under the following local relation:




(1)

As shown in the alterfold topological quantum field theory, the partition function on the 3-disc $D^3$ can be uniquely extend to decorated 3-manifolds with string decorations on their space boundaries, which is invariant under the following four moves.



# Tensornetworks in Alterfold

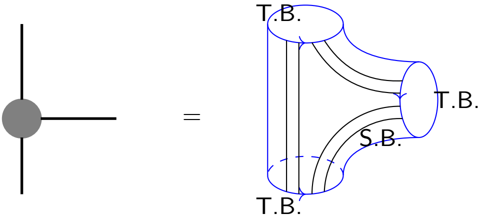
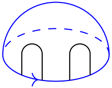
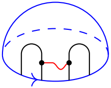


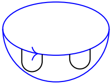
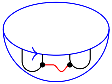

Figure: Quon representation of the Copy tensor: The boundary surface of the 3-manifold separates into time boundary (TB) and space boundary (SB). The space boundary is decorated by strings. The time boundary is the union of three input/output discs. Every disc has four points on its boundary, corresponding to a qubit space. In general, a disc with 2m boundary points corresponds to a $2^{m-1}$ dimensional vector space.

A 3-manifold arising as a neighborhood of a tensor network naturally forms a handlebody, which can be interpreted as a fractional tensor network endowed with refined topological and algebraic decorations. This refined structure provides new insights into various foundational concepts in quantum information. More general 3-manifolds extend beyond conventional tensor networks, offering a broader mathematical framework for modeling quantum phenomena.

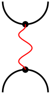
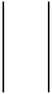
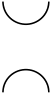
# Quon Language: braids and charges

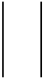
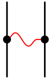
We simulate the qubit basis vectors $|0\rangle, |1\rangle, \langle 0|, \langle 1|$ as:

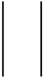
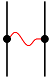
$|0\rangle = \frac{1}{\sqrt{2}}$  $|1\rangle = \frac{1}{\sqrt{2}}$ 

$\langle 0| = \frac{1}{\sqrt{2}}$  $\langle 1| = \frac{1}{\sqrt{2}}$  (2)

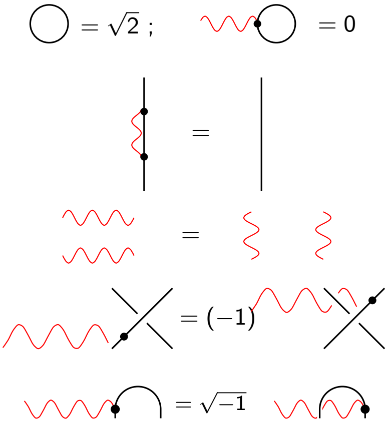
where the wire (as red tilde) is defined as follows:

 $:= \sqrt{2}$  $-$  (3)

 $:= \frac{e^{-i\frac{\pi}{8}}}{\sqrt{2}}\Big($  $+\sqrt{-1}$  $\Big)$ (4)

 $:= \frac{e^{i\frac{\pi}{8}}}{\sqrt{2}}\Big($  $-\sqrt{-1}$  $\Big)$ (5)

Charges behave like Majorana fermions, and move freely up to a phase. They have low computational complexity.




(6)

(7)

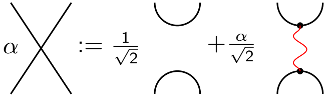
(8)

(9)

(10)

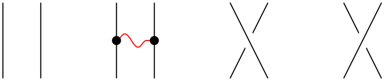
# Quon Language: Crossings

A crossing is a generalized braid defined as follows:



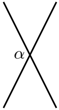

(11)

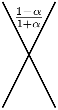
A crossing is called **Clifford**, if $\alpha = 1, -1, i, -i$, represented as follows respectively:



# Quon Language: Yang-Baxter Relation

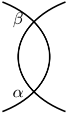
The following **Reidemeister Moves** or called **Yang-Baxter Relations** are crucial to reduce the computational complexity for diagrams with crossings.



$$= \frac{1+\alpha}{\sqrt{2}}$$


$$, \quad \alpha \neq -1 \qquad \text{(R0)}$$

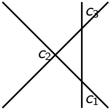
$$= \frac{1+\alpha}{\sqrt{2}} \Big| \qquad \text{(R1)}$$

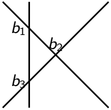


$$= \frac{1+\alpha}{\sqrt{2}}$$


(R2)

and except for a zero measure set of $c_1, c_2, c_3 \in \mathbb{C}$, there exist $b_1, b_2, b_3, k \in \mathbb{C}$, such that

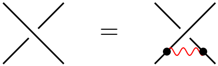


$$= k \times$$


(R3)

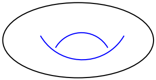
# Quan Language: Two distinguished relations

The following relation allows up to untying the link to several layers of unknots with pairs of charges.



(12)

The following **string-genus relation** allows up to change the shape of the 3-manifold.

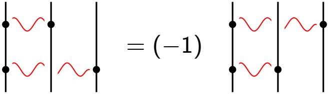


$$= \frac{1}{\sqrt{2}} \tag{13}$$

These two relations are crucial to reduce computational complexity, but still not enough.

As discussed in the 2-string model in [JLW18], the charges are Majorana fermions, which can be seen as the generators of the CAR algebra, a $C^*$-algebra with $2n$ generators $\{c_1, c_2, ..., c_{2n}\}$ satisfying

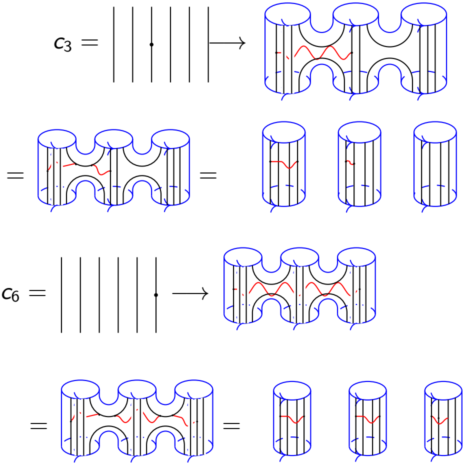
$$\begin{aligned} c_i^2 &= 1 \\ c_i^* &= c_i \\ c_i c_j &= -c_j c_i \quad (i \neq j) \end{aligned} \tag{14}$$



(15)

# Jordan-Wigner Transformation

The CAR algebra can be represented as decorated 3-manifolds in Quon language, which illustrates the Jordan-Wigner transformation[JW28].

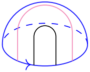
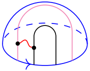
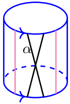


# Matchgates in Quon

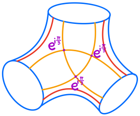
The group generated by unitary crossings in the CAR algebra are all Matchgates.

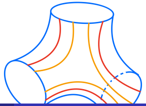
**Theorem 1 ([KZL+25, FLLW25])**

*A tensor network is a planar Matchgate tensor network if and only if it can be represented as a Crossing-Decorated 3-Manifold (CDM) whose bulk 3-manifold is a 3-Disk, i.e. a genus-zero CDM.*

$|0\rangle_X = \frac{1}{\sqrt{2}}$   $|1\rangle_X = \frac{1}{\sqrt{2}}$   $e^{i\alpha Z} =$ 

$= \begin{pmatrix} 1 & 0 \\ 0 & \alpha \end{pmatrix}_X$

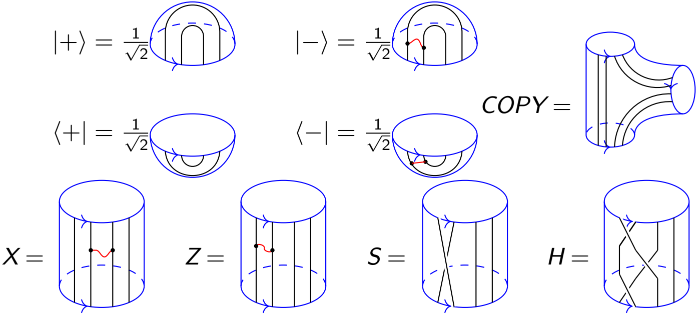
$|\widehat{w}\rangle := |011\rangle_X + |101\rangle_X + |110\rangle_X =$ 

$|Max\rangle = |000\rangle_X |011\rangle_X + |101\rangle_X + |110\rangle_X =$ 

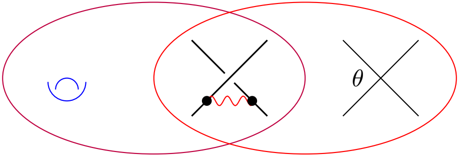
Kitaev map: The Pauli matrices are represented by pair of charges
The stabilizer group of the Pauli group is the Clifford group.

**Theorem 2 ([KZL+25, FLLW25])**

*A tensor network is Clifford iff it is represented as a Crossing Decorated 3-Manifold (CDM) in which all crossings are Clifford crossings.*



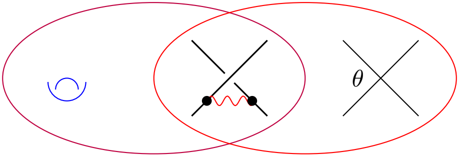
# Classical Simulation problem



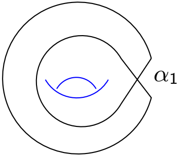

- **Clifford circuits: charged-braids+holes**
  - Efficiently classically simulable via symplectic formalism.
  - phase gate (e.g. $T$ gate) is the resource (*magic*).
- **Matchgate circuits:charged-braids+crossings**
  - Efficiently classically simulable via FKT algorithm of Pfaffian (on planar graphs).
  - Non-planarity is the resource exponential difficulty (magic).
- **Open question:**
  - Can we design a classical simulation framework where both **Clifford circuits and matchgate circuits are free**?
  - If so, what is the magic?

# Topological Complexity: Unified Framework

We propose **Topological Complexity** to characterize the computational complexity of a tensor network by QCS. The resource of exponential difficulty is neither the number of crossings, nor the number of holes, but the topological feature how crossing moving round holes.



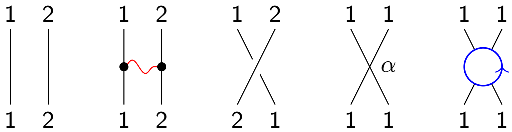

A minimal example of such resource is the **topological spin**:



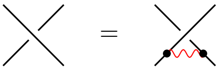

To simplify the terminology, we will consider the 2D projection of the 3D quon language in the rest, while the concepts are indeed intrinsic in the 3 dimensions.

# Layers

We define a connected component as a collection of strings that are interconnected in the following sense ($\alpha \notin \{\pm 1, \pm i\}$):




where strings in one connected component are labeled as the same number. In the case with boundary, all boundary points of an output disc are defined to be connected.
To resolve possible links between distinct connected components, we employ the relation



(16)

to reduce positive and negative braidings into standard forms, potentially introducing additional charges.
After this step, each connected component of strings defines what we refer to as a **Layer**.
We say that a hole is **involved** in a given layer if there does not exist any path from the hole to infinity that avoids intersecting the layer.

**Definition 3**

A hole $g$ is called odd if there exists a layer of strings $L$ and a ray $r$ from $g$ to infinity without passing through crossing points such that $r$ intersects with $L$ transversally at an odd number of points. Otherwise, it is called even.

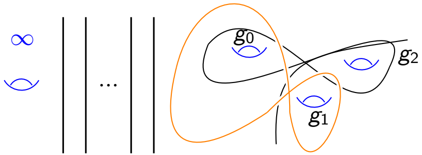



Figure: The orange layer makes the holdes $g_0$ and $g_1$ to be odd.

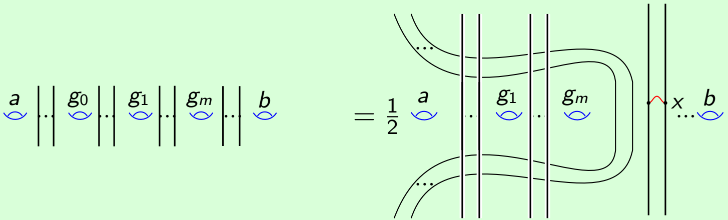
Each layer induces a constraint equation for inside odd holes $g_0 + g_1 = 0$, while considering the lable $g_0$ and $g_0$ as a variable in the field $\mathbb{F}_2$.

# Odd hole handleslides

The following relation is a generalization of the string genus relation, allowing us to eliminate odd holes efficiently.

## Lemma 4 (Odd hole Handle slides)

*Given an odd hole $g_0$ and pick a layer constraint $g_0 + g_1 + ... + g_m = x$, where $x = 0$ or $1$. Then $g_0$ can be removed by the following identity:*




*The above identity provides an efficient diagrammatic transformation, whose time complexity is $O(n)$, where $n$ is an upper bound on the number of strings between any pair of involved holes.*

# Even holes

Once all odd holes are eliminated, only even holes remain. Among them, certain genus cuts can still be simulated classically, provided they are of a specific type:

**Definition 5**

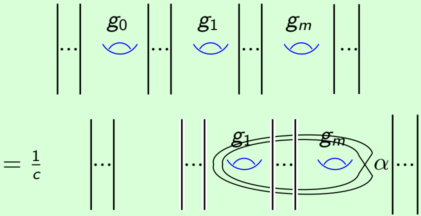
A genus cut from a hole $g$ to another hole is called **Clifford** if all intersecting strings along this cut intersect with all the other strings (not necessarily along this cut) only via Clifford crossings. A hole is called **Clifford** if such a cut exists.

**Definition 6**

A hole is called **magic** if it is even and not Clifford.

**Theorem 7**

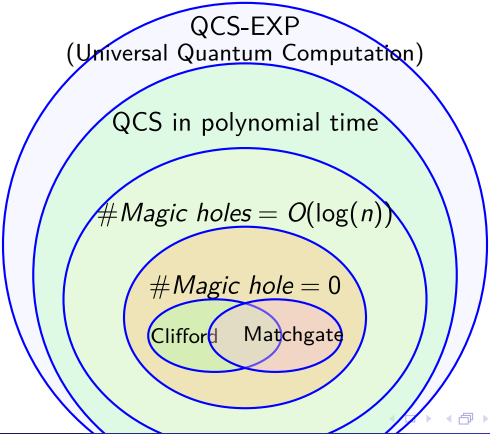
*Let $g_0$ be an even Clifford hole. Then there exists a set of holes $g_1, g_2, \ldots, g_m$, such that the hole can be eliminated by making up a double-parallel circle with a Clifford crossing as follows:*




*for a Clifford crosssing $\alpha$.*

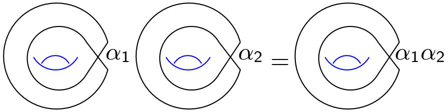
**Theorem 8 ([FLLW25])**

*We establish a unified algorithm to classically simulate quantum circuits and tensor networks by QCS, achieving polynomial time and space complexity when the number of magic holes is $O(\log n)$. In particular, both Clifford circuits and Matchgate circuits are free, namely they have no magic holes.*



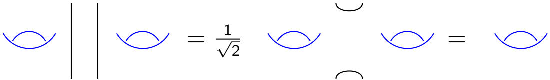
When all holes are magic, we can further eliminate some through the following relations:

1. Merging topological spins:

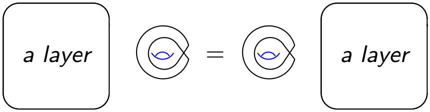



(17)

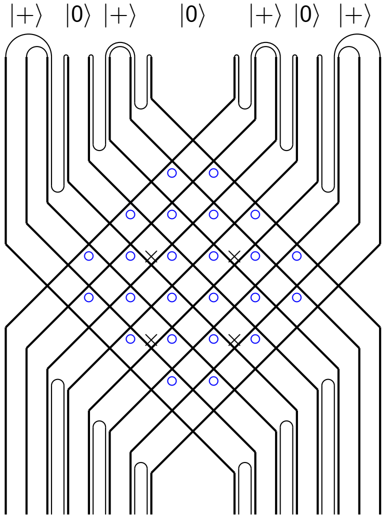
2. Two string cut:

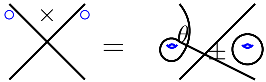



3. Layer free move:



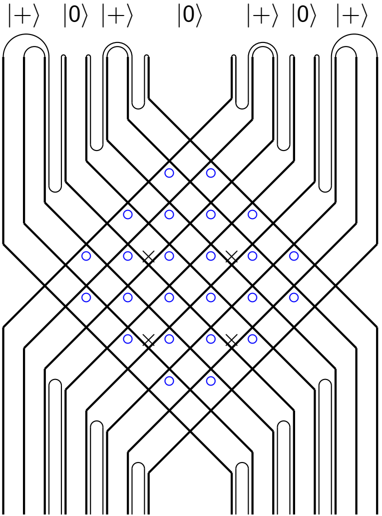
# Examples




where 


- This is a family of quantum circuits with $O(n)$ width and depth, $O(n^2)$ Clifford gates and Matchgates as shown in the following figure.
- The input state is an alternating product state of $|+\rangle$ and $|0\rangle$.
- The measurement of the outcome state can be classically simulated if one can compute its conditional expectation for measurements on every single qubit.

# Examples




- When the measurement of qubits are all in $Z$ basis, the conditional expectation can be computed efficiently in QCS, due to the local move of Yang-Baxter relations.
- When the measurement of qubits are in $X$ basis and $Z$ basis alternatively, the conditional expectation can be computed efficiently in QCS, due to the global layer free move of magical hole, even if it has $O(n^2)$ magic holes! This trick requires global property, and local moves do not work in this case.
- When the measurement of qubits are all in $X$ basis, the conditional expectation can not be computed efficiently in QCS.

# Summary

Quon Classical Simulation:

- presentations of tensor networks (3D Cliffords and 2D Matchgates)
- topological complexity
- algorithms (polynomial vs exponential)
  (1) crossing reduction (Yang-Baxter relations on 2D space boundary)
  (2) genus reduction (surgery moves of 3-manifolds)
  (3) magic hole reduction (topological entanglement)

We are essentially applying the Quon language as a 2+1 topological quantum field theory with space-time boundary.
The efficiency of simulating Cliffords is captured by generalized surgery moves of 3-manifolds, and the efficiency of simulating Matchgates is captured by Yang-Baxter relations on 2D space boundary. The two operations are parallel, which is the main reason that we can combine both algorithms in an efficient and natural way.
The essential exponential difficulty is neither the number of crossings nor holes, but the topological entanglement of the network, captured by the magic holes and their interactions in layers.

# Outlook

- classical vs quantum (e.g. Shor's algorithm)
- Quon Classical Simulation
- topological complexity (topological orders, QECC, lang range entanglement etc)
- topological tensor network
- global algorithm (circuit simplification, QECC, machine learning etc)

**Thank you for your attention!**

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