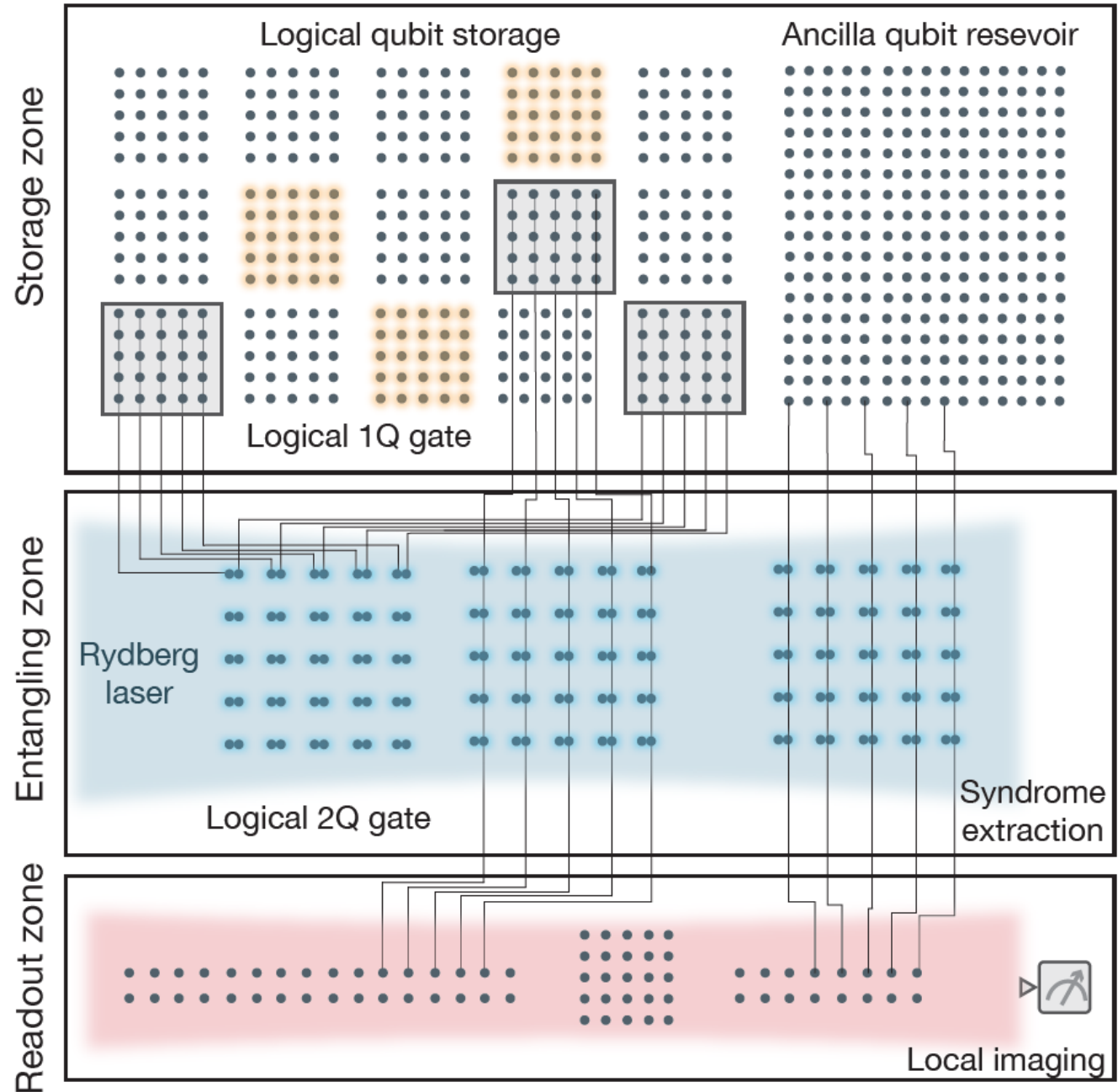


# Logical quantum processor based on reconfigurable atom arrays

Dolev Bluvstein  
Harvard atom array team  
Lukin, Greiner, and Vuletic collaboration  
Mathematical picture language seminar  
April 16 2024



# Frontiers of experimental quantum information

~2010s, exploring small-scale quantum computations with physical qubits  
(physical qubit: atoms, ions, defects, superconductors ...)

“Age of quantum discovery”

Many-body systems and new quantum phenomena

Error correction frontier

First 1-2 *logical* qubits / gates

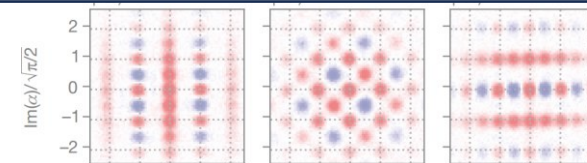
**Fighting decoherence is **the central challenge** in large-scale quantum computation**

**Quantum error correction is the only known realistic route to suppress gate errors to the required levels for useful algorithms ( $10^{-3} \rightarrow 10^{-10}$ )**

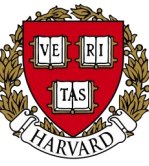
...before QEC, people thought quantum computing would be fundamentally impossible



Xanadu Nature 2022  
(see also USTC Science 2020)



Yale 2023  
(Bosonic)

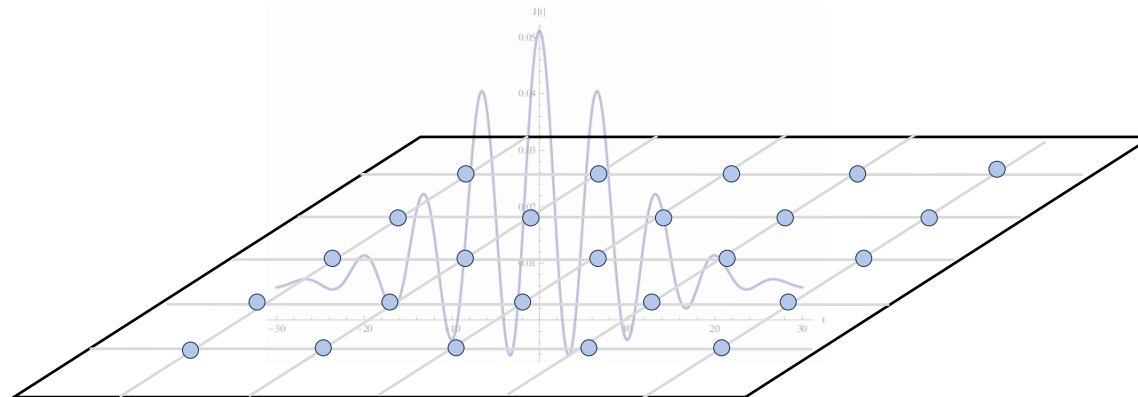


# Quantum error correction

- Classical error-correction: make copies!  $0 \rightarrow 000$
- Quantum error-correction: *conceptual challenges*
  - no-cloning theorem ☹️, can't duplicate quantum information
  - How to check for error without collapsing state?

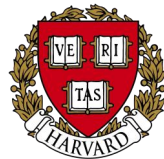
## So how to do quantum error correction?

- Use *entanglement* to store information *nonlocally* to encode a *logical qubit*
- *By being delocalized, logical qubit degree of freedom hard to accidentally manipulate*
- *Measuring products of qubits (stabilizers) detects errors while preserving encoded q. info*



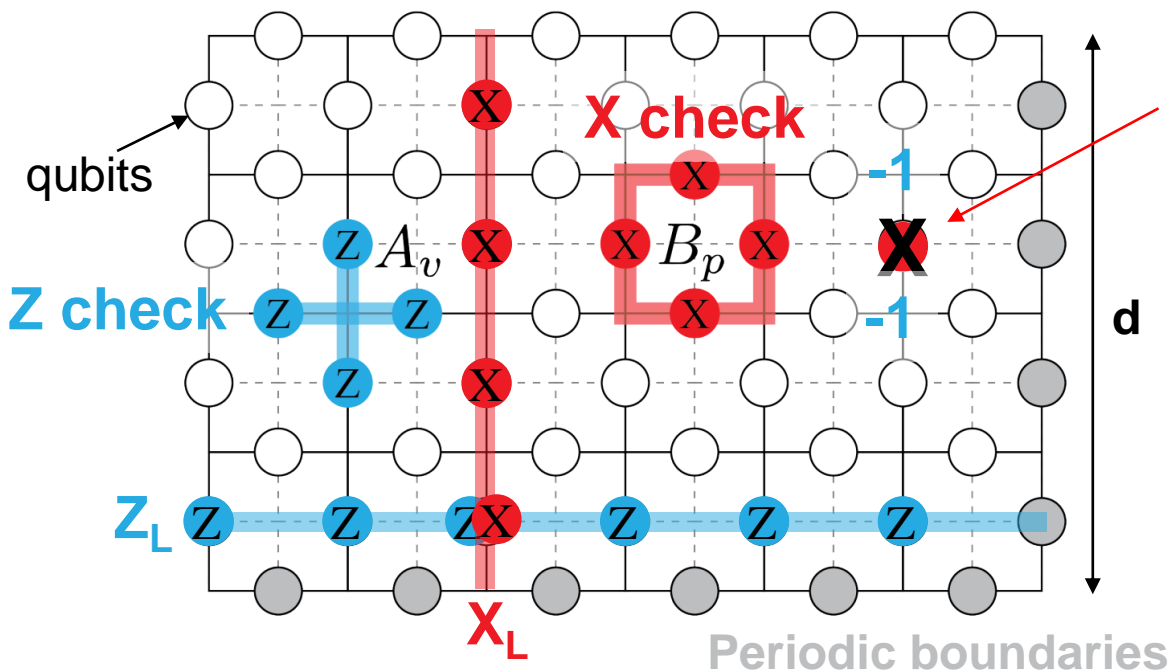
Logical qubit

Shor 95, Steane 96, Kitaev 97, Preskill, Laflamme, Calderbank, Gottesman ...



# QEC example: the toric code

Error-free state is with all X and Z  
“stabilizer” products (checks) = +1



(technically two logical qubits for the torus)

Physical qubit errors will cause checks to show an error happened at a specific location – infer (decode), and undo.

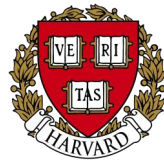
As lattice size (**code distance  $d$** ) increases: more opportunities for errors, but more checks – *threshold behavior*

$$\text{Logical error probability} \sim \left(\frac{p}{p_{th}}\right)^{\frac{d+1}{2}}$$

→  $p_{th} \approx 1\%$

→ Offers **realistic route** to extremely small errors

- It was the theoretical breakthrough of quantum error correction that really allowed the field of quantum computing to take off
- **And it is understood that eventually, we will need to switch to performing algorithms with *logical qubits*, instead of physical qubits**

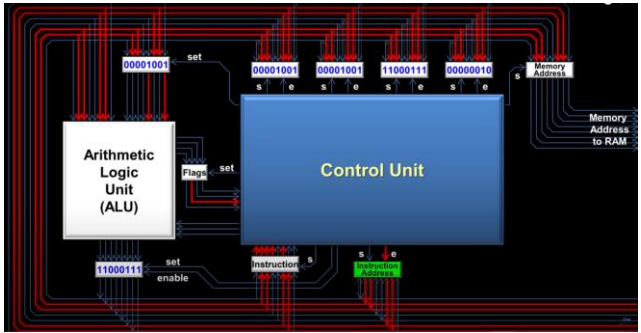


# Quantum error correction is *challenging*

(Optimistic) estimates<sup>1</sup> for large-scale problems: million physical qubits and logical error rate  $10^{-10}$

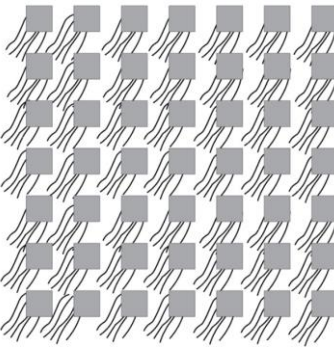
Efficient classical control is a major challenge  
*Classical computers: ~1000 wires for ~billion bits*  
*Quantum computers: several “wires” per qubit*

Classical architecture



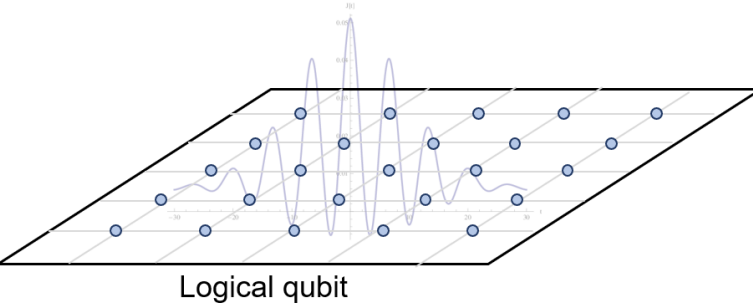
“How a CPU works” by InOneLesson

Quantum architecture



Challenge 1: “Wire problem” poses significant challenge to large-scale control.

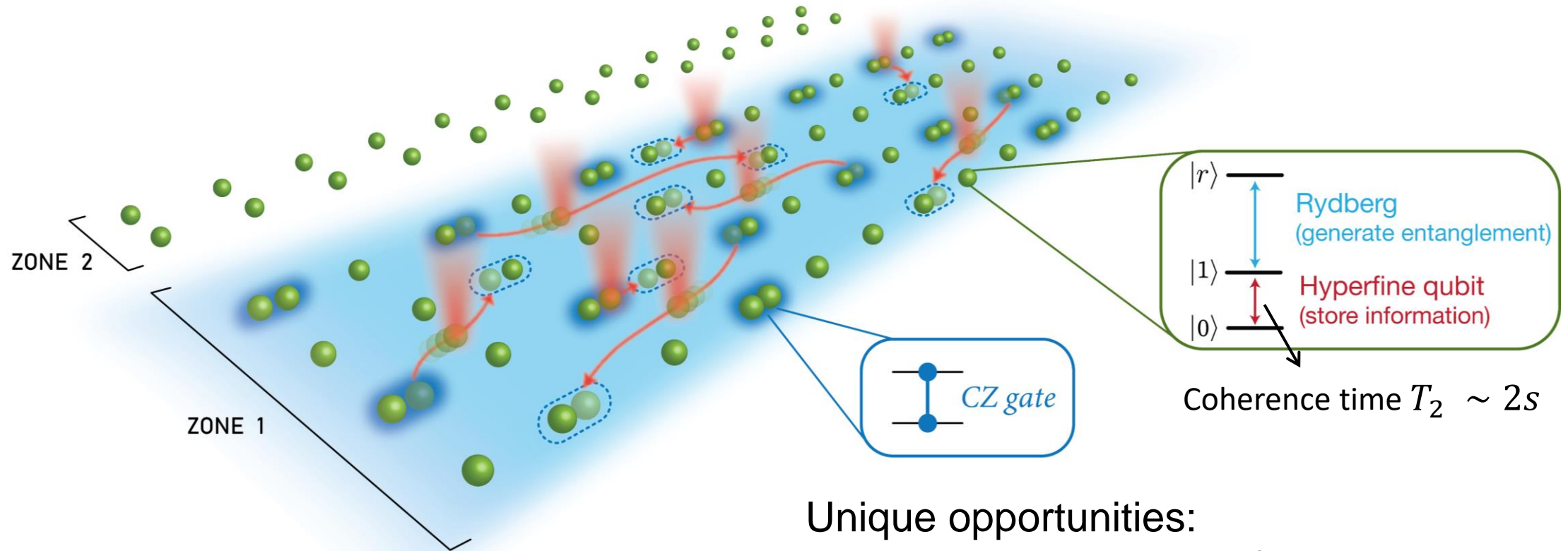
Challenge 2: Once logical qubit is delocalized, it becomes *hard to operate on*.



Large overheads + complexity of *operating* on logical qubits have focused studies to ~1-2 logical qubits / gates

1: Gidney Ekeru Quantum 2021

# Our approach: reconfigurable atom arrays



DB *et al.* *Nature* **604**, 451-456 (2022)

Unique opportunities:

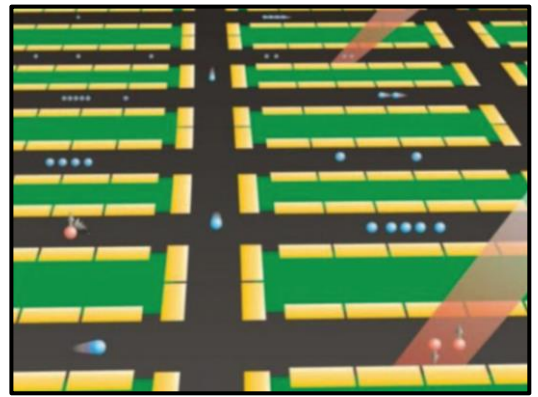
- Nonlocal connectivity (turns out, extremely useful for error correction)
- Parallel, efficient classical control

Pioneering work and recent exciting developments: Weiss, Saffman, Browaeys, Grangier, Regal, Endres, Kaufman, Bernien, Thompson, Ni, Bakr, Bloch, Covey ... See also optical lattice parallelism (Deutsch)

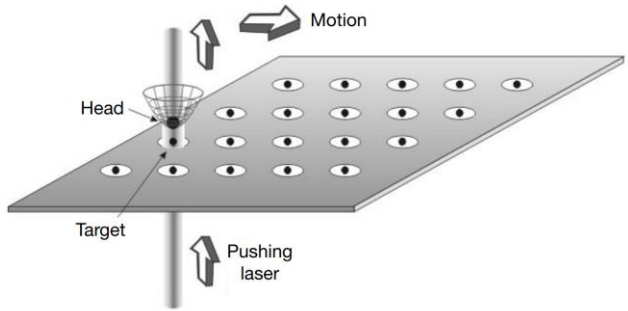


# We are building off tremendous progress from the community...

Ions: 20+ year vision of shuttling-based architecture (+ recent expts) – nonlocal connectivity



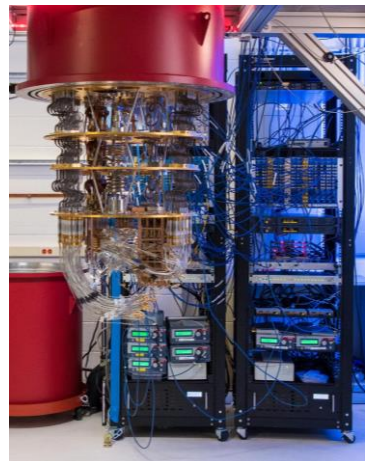
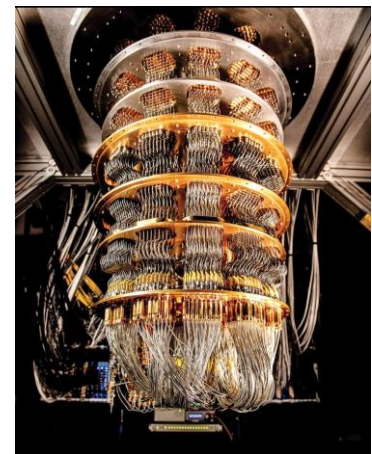
Monroe, Kim



Cirac, Zoller Nature 2000

Wineland 1998, recent Quantinuum expts...

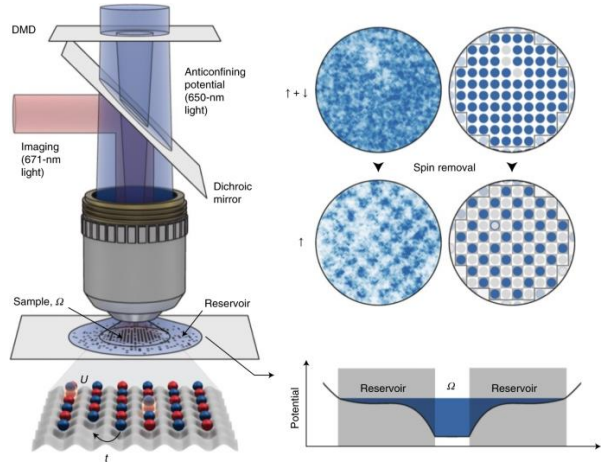
Superconducting qubits: early experience with controlling moderately large systems (50-70 qubits) – taught importance of the “wire problem” of control



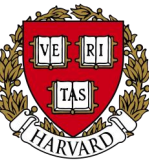
Yale, Google, IBM, Wallraff, Oliver ...

## Decades of cold atom research

- Pioneering work in neutral atom tweezer community
- Ultracold atoms / optical lattice quantum simulators (the true pioneers of parallel, efficient control)



Greiner, Gross, Bakr, Bloch, Deutsch ...



1. Programming a quantum circuit with neutral atoms
2. Logical quantum processing



# Neutral atoms

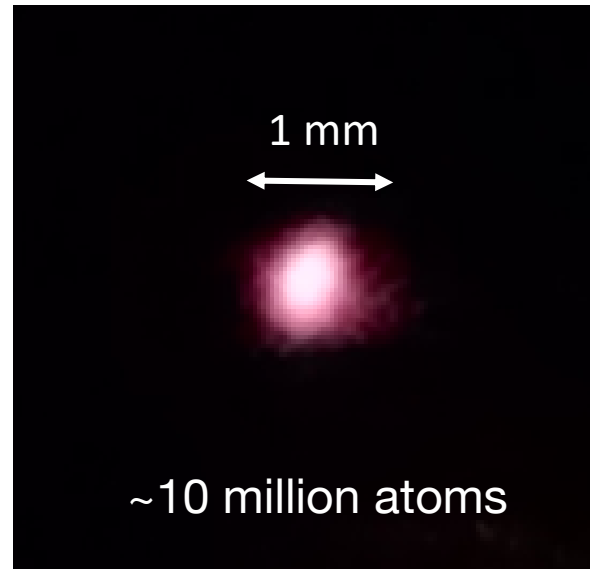
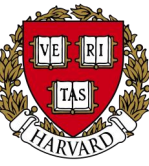
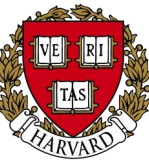


Image of atoms in a magneto optical trap (MOT)

Cold, identical neutral atom qubits are essentially unlimited  
*The key challenge is efficient classical control*

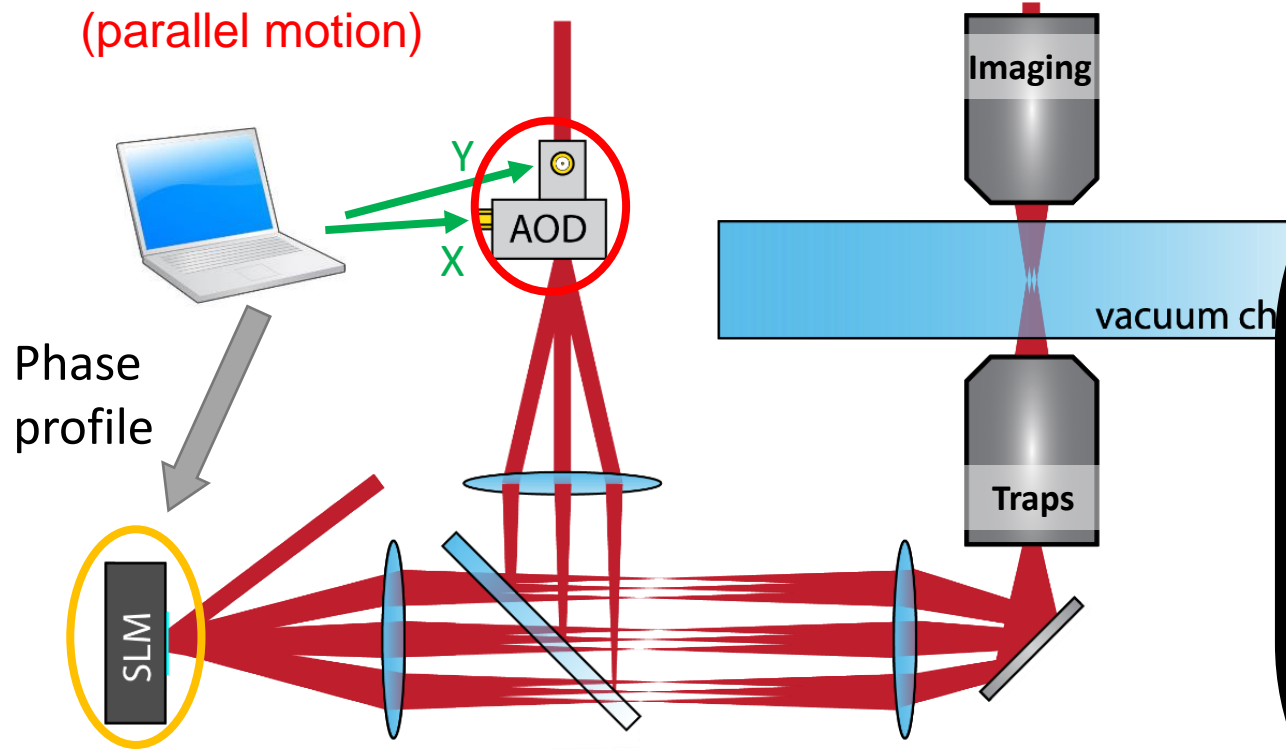
# Thousand neutral atoms in 2D tweezer arrays

*Efficient classical control of many qubit positions*

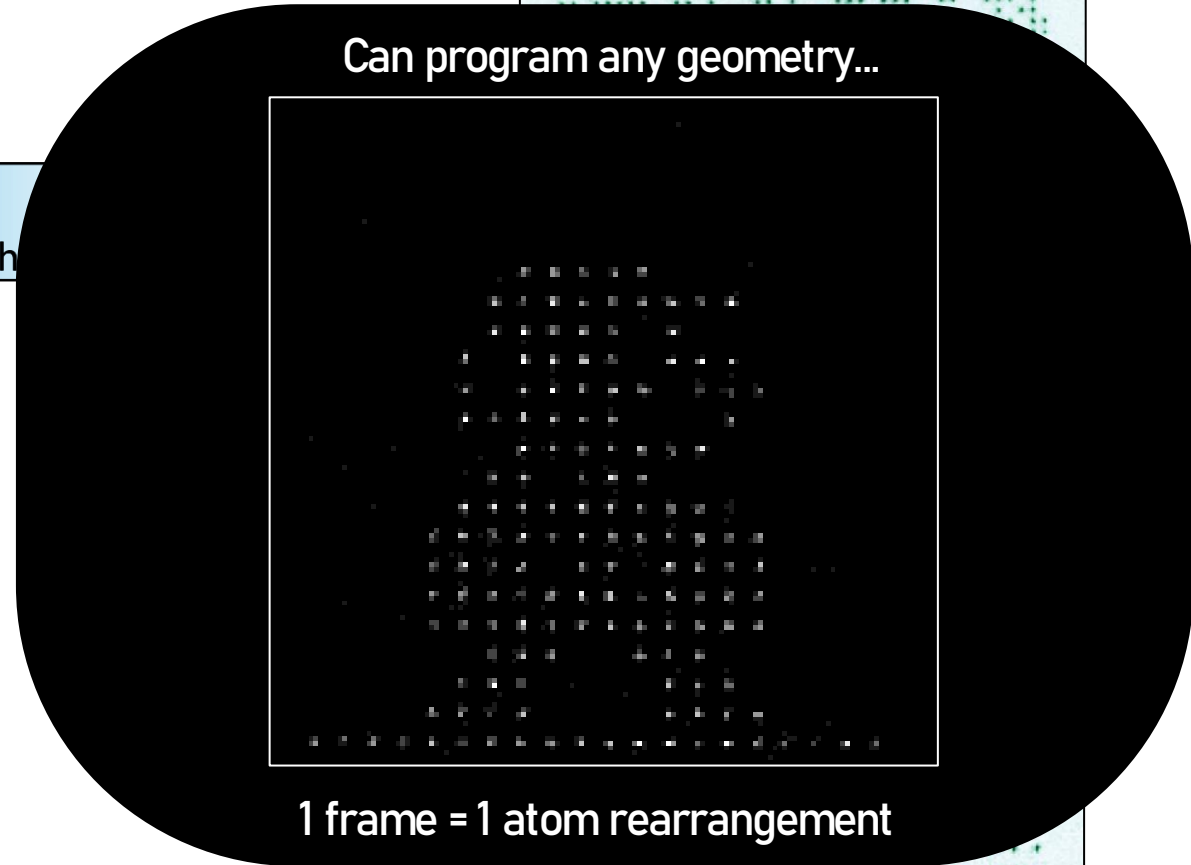


>1000 atoms

**Crossed Acousto-Optic Deflectors**  
(parallel motion)



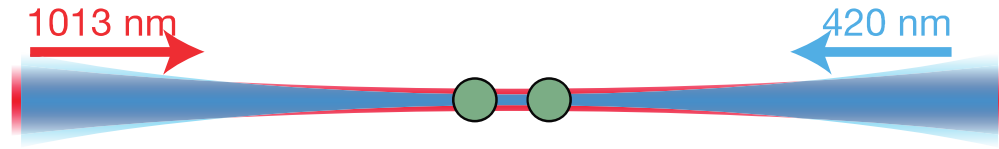
**Spatial Light Modulator**  
(efficient programmable generation of many tweezers)



Ebadi et al Nature 2021

Related work: Scholl et al Nature 2021 (Browaeys),  
Endres, Kaufman, Bernien, Saffman, Thompson, Ni, Bakr, Bloch ...

# Rydberg atoms and entanglement with blockade



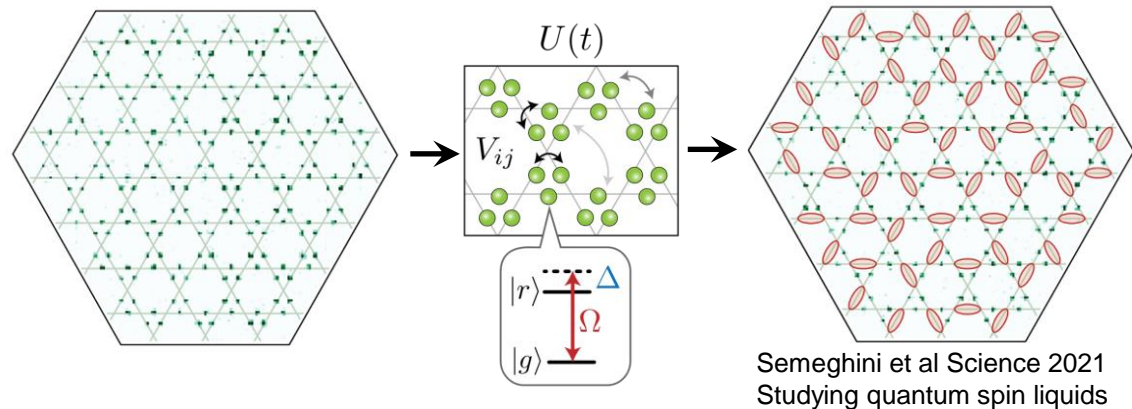
Ground state  $|g\rangle$   $\rightarrow$  ● Rydberg state  $|r\rangle$   $\rightarrow$  ●  
( $n=50-70$ )

Two atoms:  $|rr\rangle$  shifts by Van der Waals interaction ( $\propto 1/R^6$ )

$\rightarrow$  adjacent atoms cannot be simultaneously excited, **Rydberg blockade**

$$|gg\rangle \longrightarrow \text{Entangled state } (|rg\rangle + |gr\rangle)/\sqrt{2}$$

## Analog (Hamiltonian evolution)



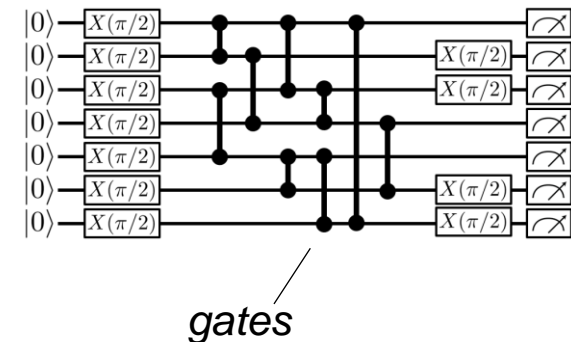
- Position atom, turn on global excitation laser (run Hamiltonian), study quantum dynamics

Phase transitions (Ebadi et al Nature 2021)

Also: Nonequilibrium dynamics (Bluvstein et al Science 2021)  
Combinatorial optimization (Ebadi et al Science 2022)

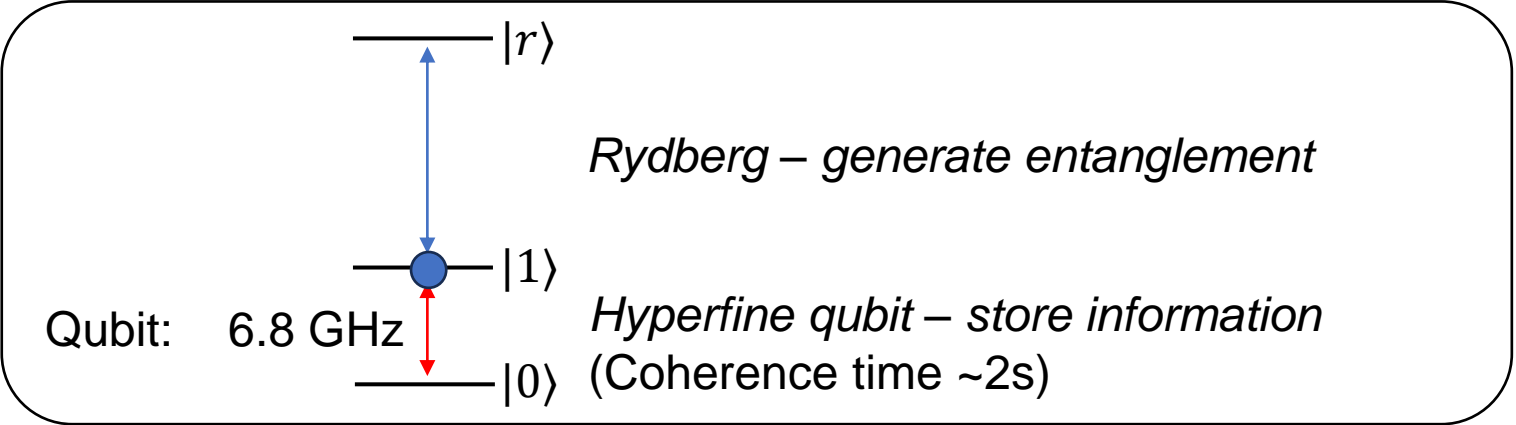
## Digital

Quantum circuit:



- Controllable quantum operations (gates) to realize *universal quantum computation*
- Although more degrees of freedom, *still want efficient classical control*

# Neutral atom quantum circuits: two-qubit gates



Rydberg state  $|r\rangle$   $\rightarrow$  (n=50-70)

Ground state  $|1\rangle$   $\rightarrow$

**Levine-Pichler CZ gate:**

$|00\rangle \rightarrow |00\rangle$   
 $|01\rangle \rightarrow |01\rangle$   
 $|10\rangle \rightarrow |10\rangle$   
 $|11\rangle \rightarrow -|11\rangle$

$\Omega$  entangled coupling to  $\frac{|1r\rangle + |r1\rangle}{\sqrt{2}}$  generates a different *accumulated phase*  
 $\sqrt{2}\Omega$

Due to Rydberg blockade, if both in  $|1\rangle$ ,

- Global pulses
- Blockade: if two atoms are next to each other, they will do a CZ gate!



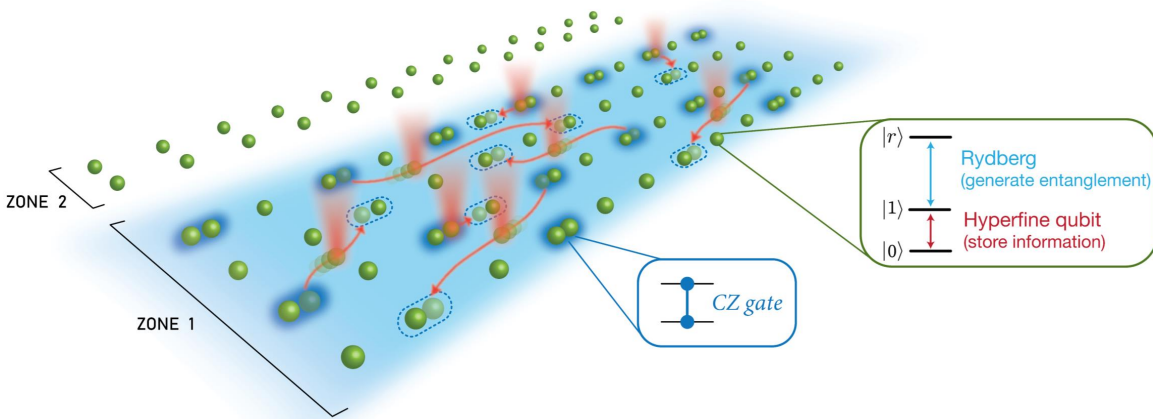
Global Rydberg laser drive



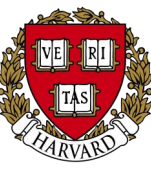
Levine, et. al. *PRL* 123, 170503 (2019)

Harry Levine

Rabi frequency  $\Omega$



# Programming a circuit with parallel controls: 12-atom cluster state



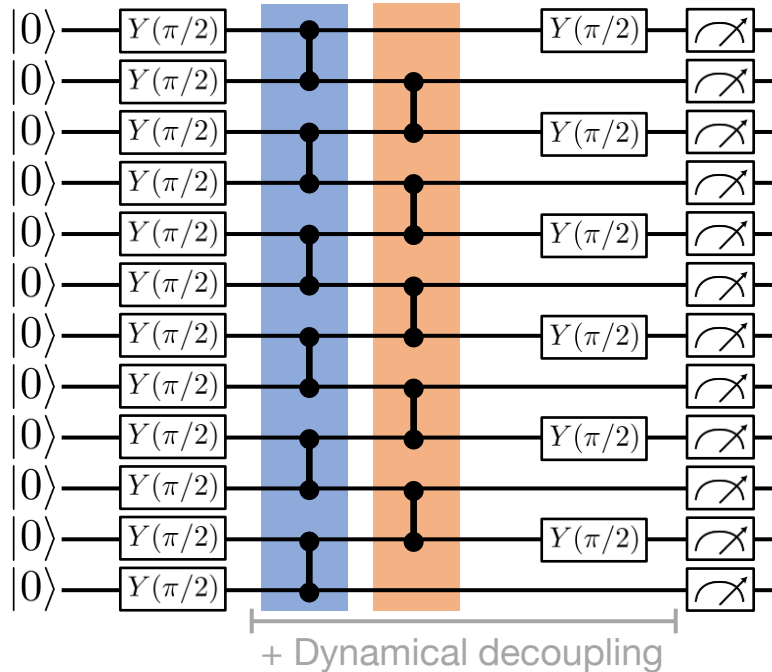
*Position defines gate (blockade) → efficient control over many qubit positions gives efficient control over complex quantum circuits*

SLM      AOD

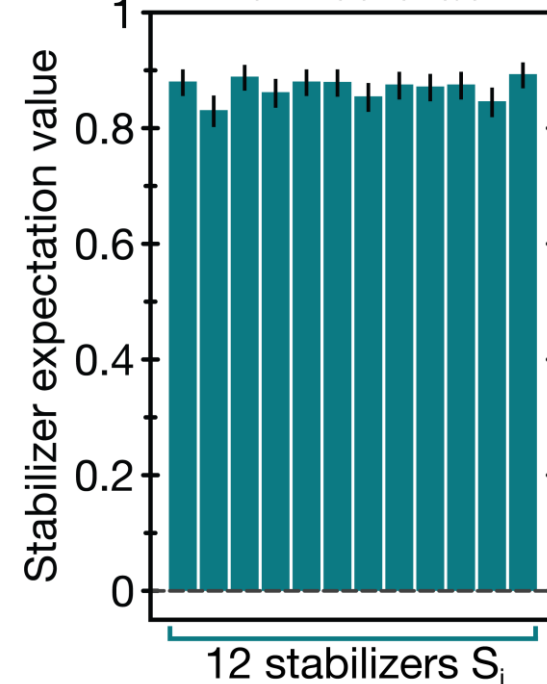
1<sup>st</sup> parallel layer:



2<sup>nd</sup> parallel layer:



$$S_i = Z_{i-1} X_i Z_{i+1}$$

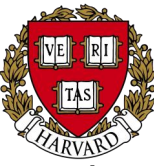


Shows successful creation of the state

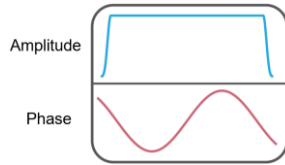
Fidelity limited by (old) 2Q gate fidelity ~97.5%

First coherent moving:  
Beugnion Nat Phys 2007  
See also:  
Schlosser Quant Inf Proc 2011

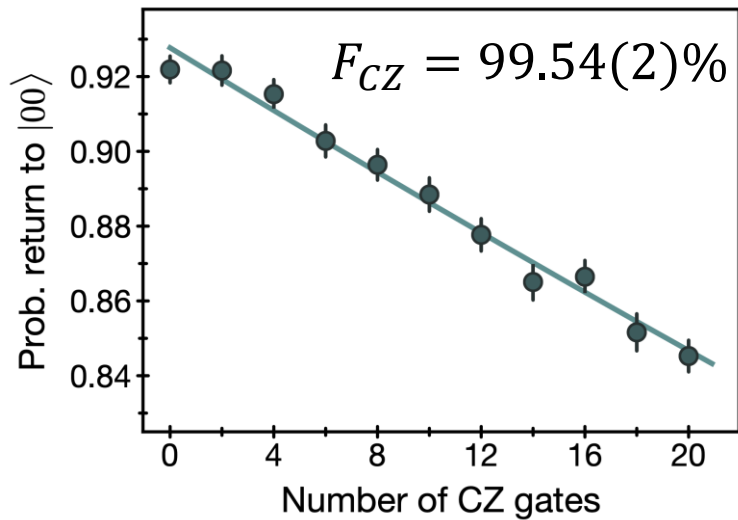
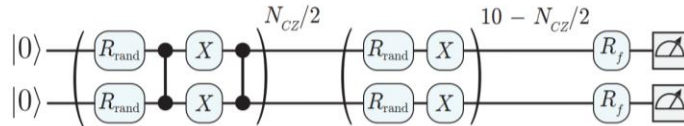
# Universal, high-fidelity digital circuits: technical upgrades (2023)



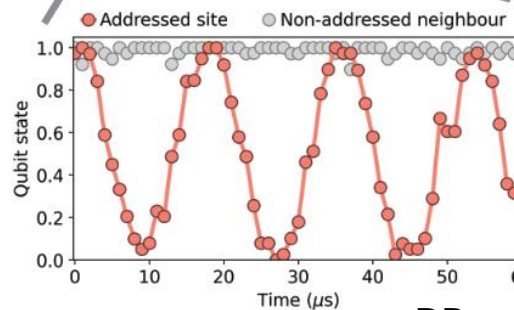
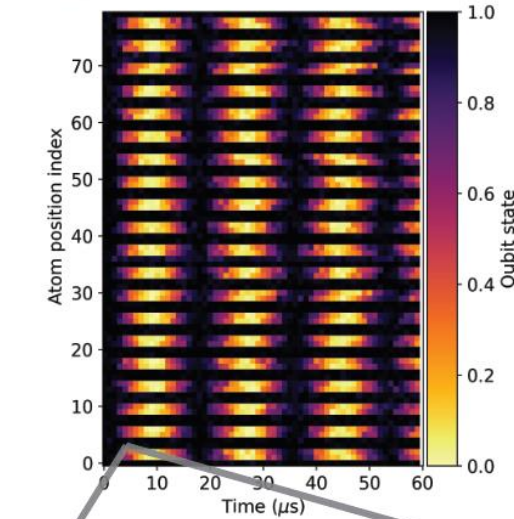
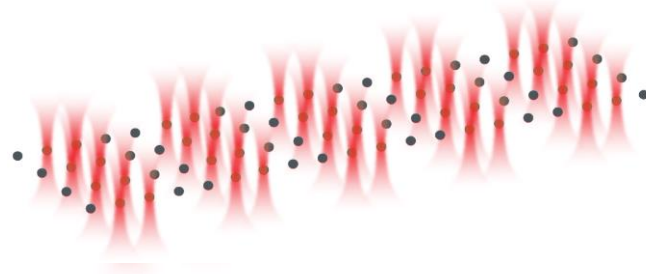
## 99.5% 2Q gates on 60 qubits in parallel



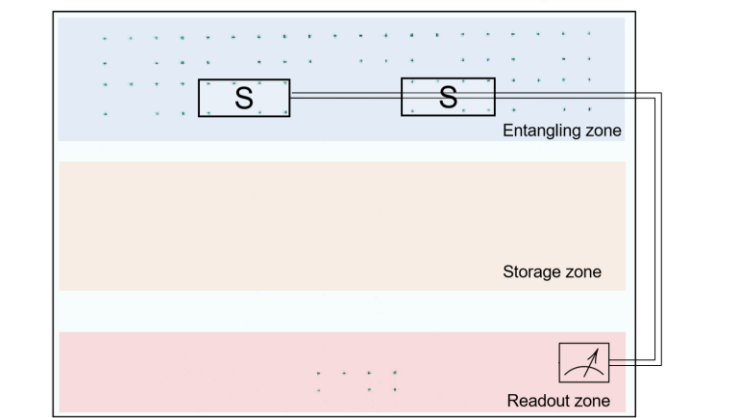
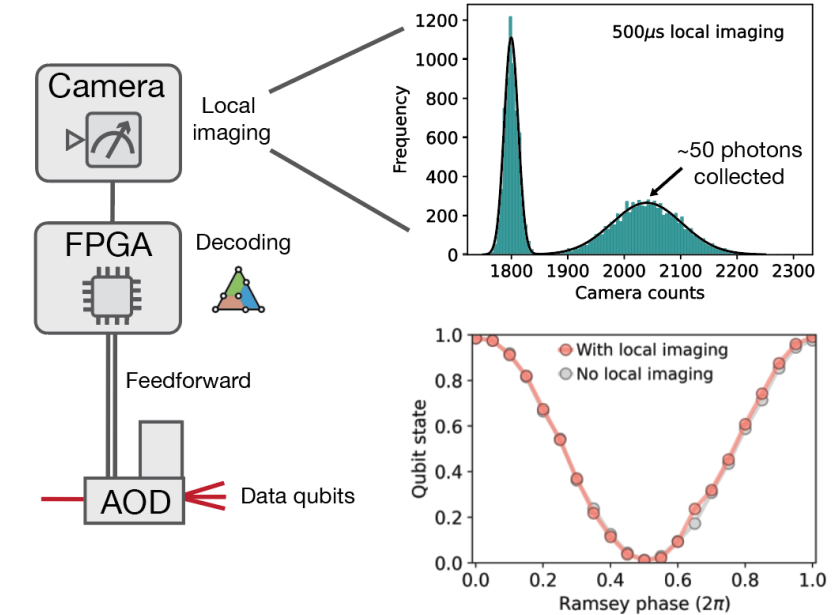
High power laser, improved gate technique (Jandura, Pupillo Quantum 2022)



## Fully programmable, parallel 1Q gates



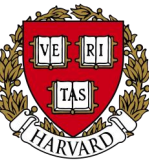
## Mid-circuit readout and feedforward



Harvard-MIT-QuEra collaboration  
 See also very nice works: Stamper-Kurn, Bernien, Saffman, Thompson, Kaufman, Atom Computing

S Evered\*, DB\*, M Kalinowski\* et al Nature 2023  
 See also Thompson, Endres papers (with erasure!)

DB et al Nature 2023



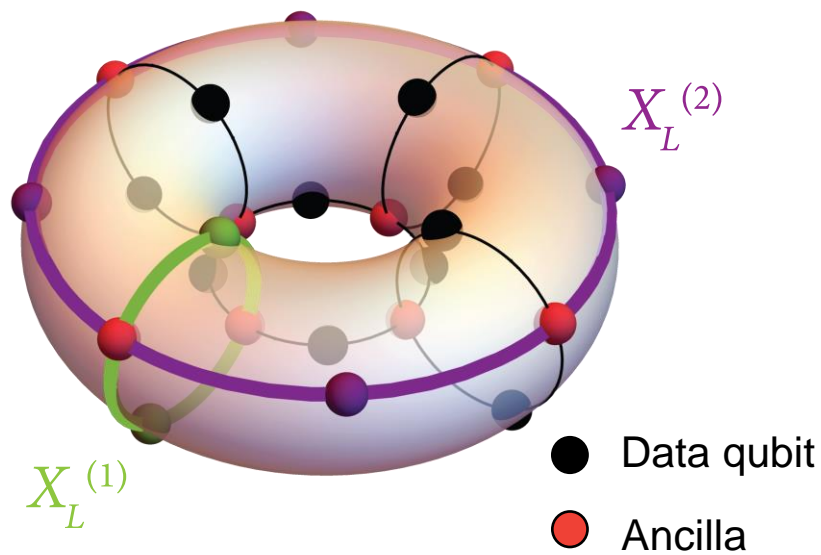
# Exploring quantum error correction with neutral atom devices



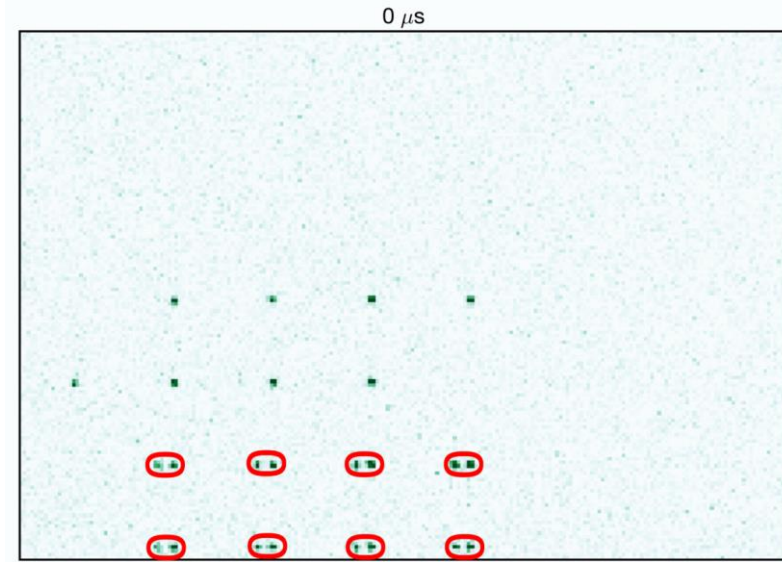
# 2022: Toric code (on a torus)

*Realizing error correction codes with nonlocal connectivity*

Circuit is simply programmed by specifying SLM profile and AOD waveform  
Parallel control over many qubits with  $O(1)$  classical controls



Z stabilizer measurement:





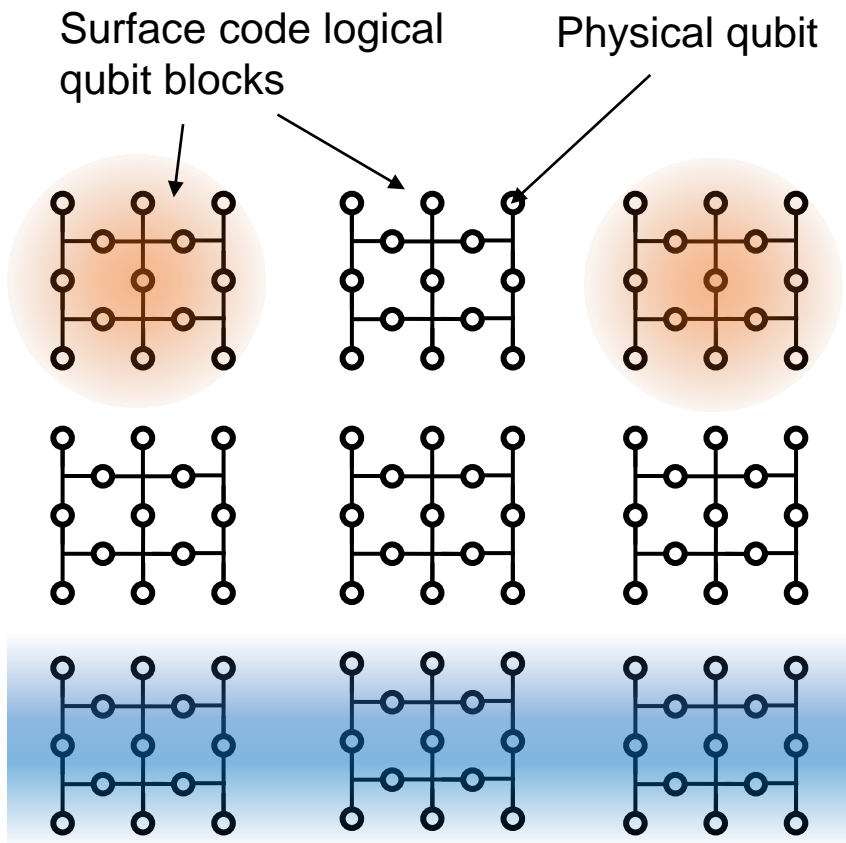


# Parallel logical qubit processing with $O(1)$ controls

Transversal single-qubit gate

Transversal entangling gate

Shor 1996, Dennis et al 2001



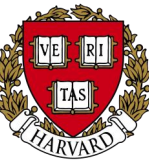
Single *logical* qubit control, instead of single *physical* qubit control  
*Naturally multiplexes with optics*

5. XOR Gates and  $\frac{\pi}{2}$  Rotations

Shor 1996 – *Fault-tolerant Quantum Computation*

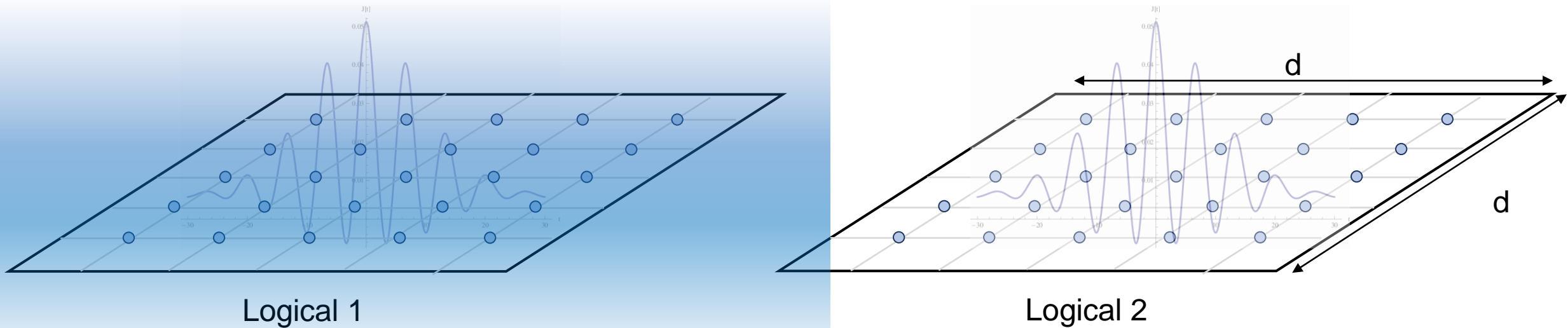
This operation is easily accomplished by elementary quantum gates and as it is a bitwise operation, it is fault-tolerant.

*Efficient, parallel computation with logical qubits*



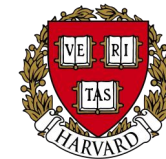
# Transversal CNOT based on parallel motion

By being delocalized, logical qubit degree of freedom hard to accidentally **or intentionally** manipulate



*Transversal CNOT: directly interact the delocalized degrees of freedom*

- *Inherently fault-tolerant* –  $d$  rounds of correction not required between each gate unlike lattice-based approaches
  - *Fault-tolerant: errors cannot spread within code block*
- *Long-range, direct connections between logical qubits* – can have significant savings for large-scale algorithms
- *Efficient control: all physical qubits receive the same instruction and act like one big atom*



# First generation logical processor based on zoned architecture

*Key focus: parallel control of many logical qubits with only a few wires*

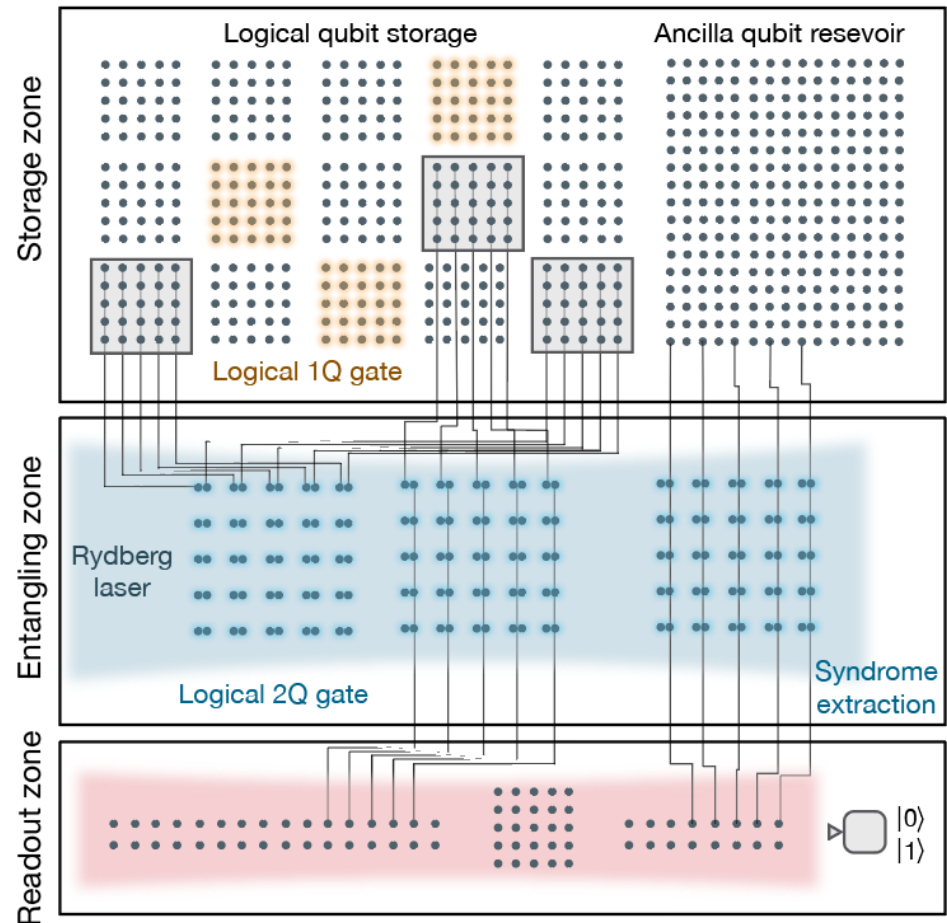
**Transversal gates:** inherently fault-tolerant

**Storage zone:** idle logical qubits are stored, safe from errors

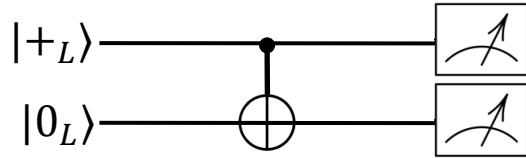
**Entangling zone:** transversal operations with few global beams

**Readout zone:** measure qubits without disturbing active qubits

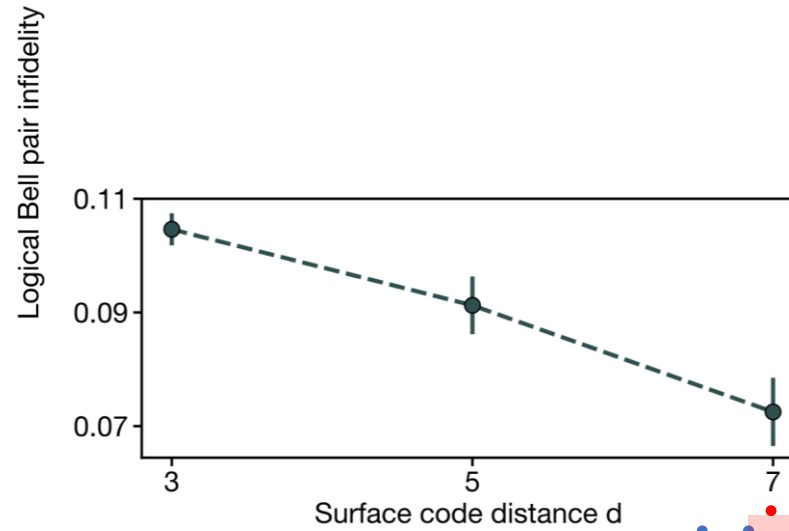
**Long-range connectivity:** opportunities for exotic codes



# Example: logical CNOT with surface code



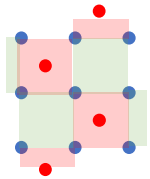
Note: effective threshold of entire circuit higher due to nearby time boundaries - future work will involve trying to *increase the number of applied transversal CNOTs* with repeated correction



1. Improved gate performance with increased code size

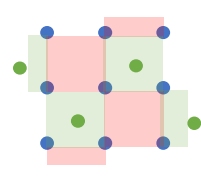
2. Decoding logical qubits *jointly* improves logical algorithm performance – M Cain ... DB, Lukin

measure Z stabilizers

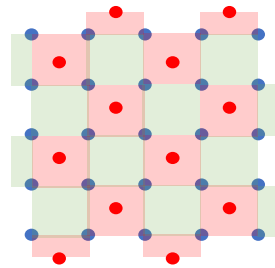


$d = 3$

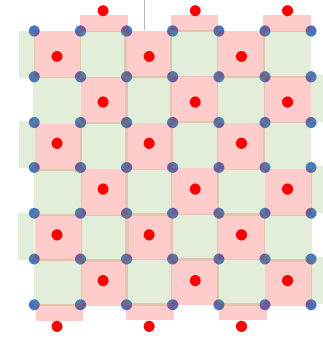
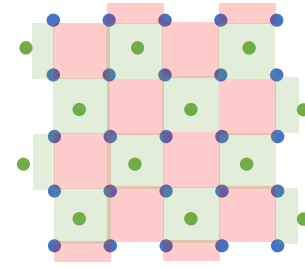
measure X stabilizers



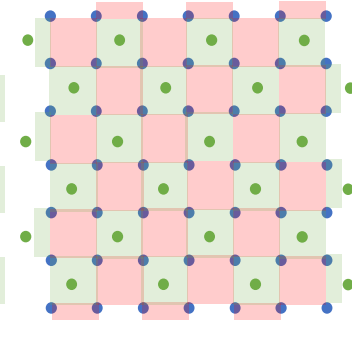
Surface code distance  $d$



$d = 5$



$d = 7$

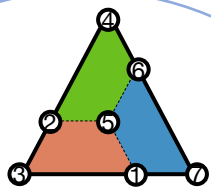


- Key QEC property: operations should improve with system size (code distance  $d$ )
- Here, state preparation non-Fault-Tolerant (nFT) beyond  $d=3$ , but still allows probing behavior of transversal CNOT



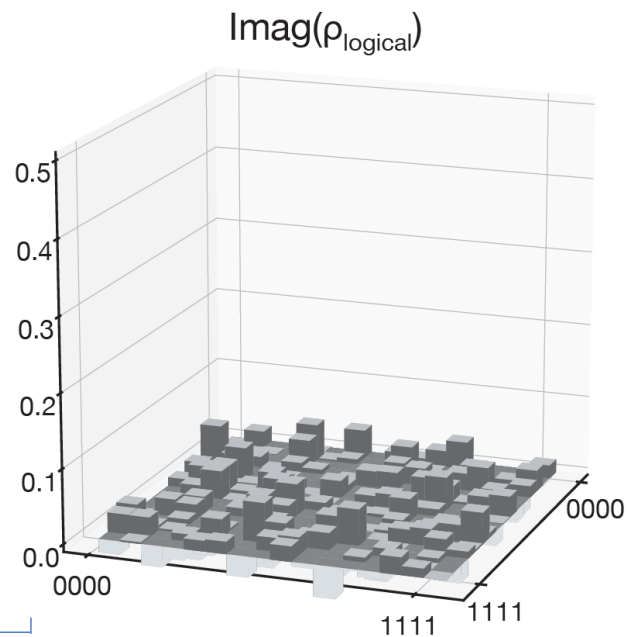
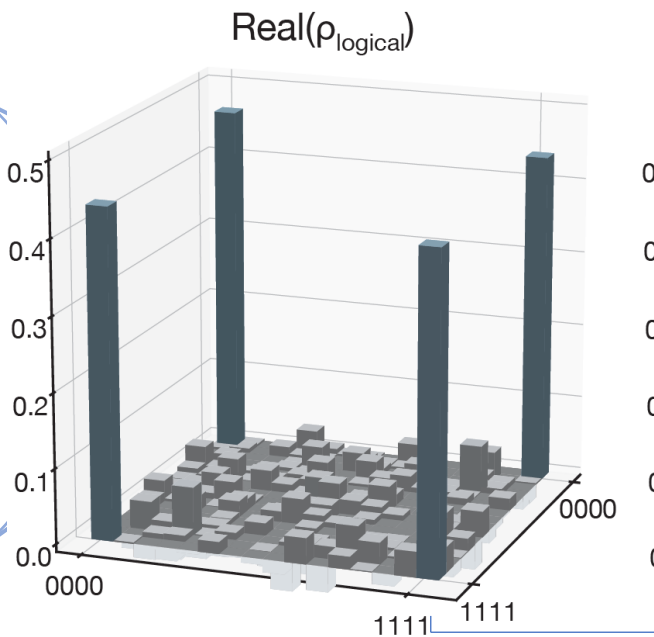
# Fault-tolerant algorithms: GHZ state

GHZ (or “cat”) state:  $|0_L 0_L 0_L 0_L\rangle + |1_L 1_L 1_L 1_L\rangle$



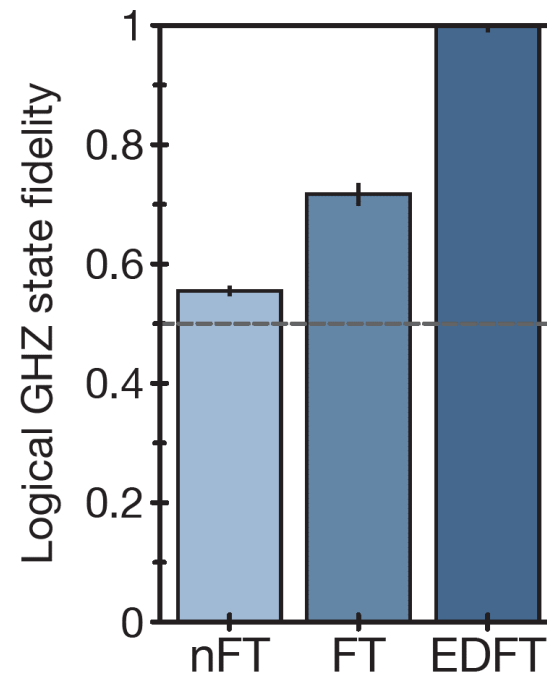
*Color code logical qubit*

*Similar to surface code, but more logical operations allowed*



“Steane error correction” – improves logical initialization to 99.9%

Measurements in all 81 logical bases



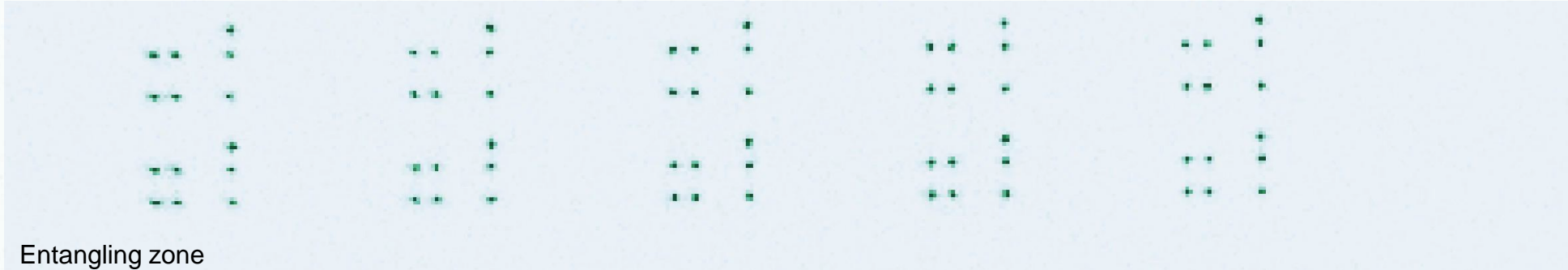
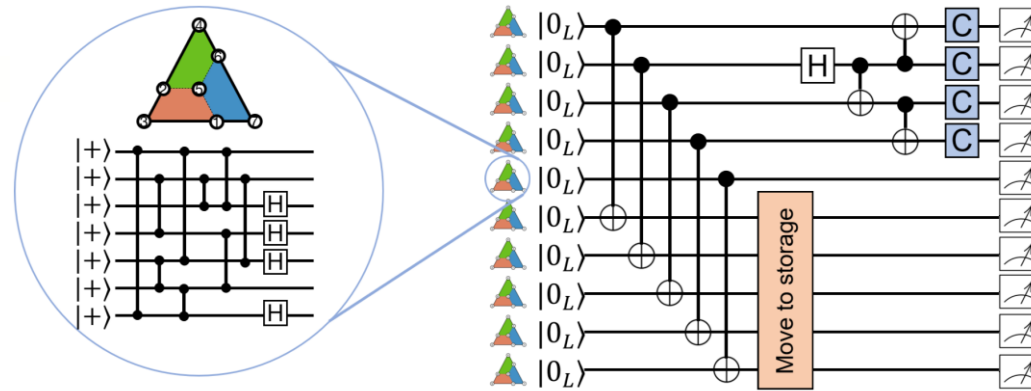
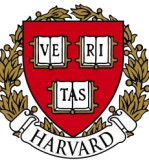
**nFT:** non-Fault-tolerant algorithm. Don’t postselect on ancilla logicals.

**FT:** Fault-tolerant algorithm. Postselect on ancilla logicals.

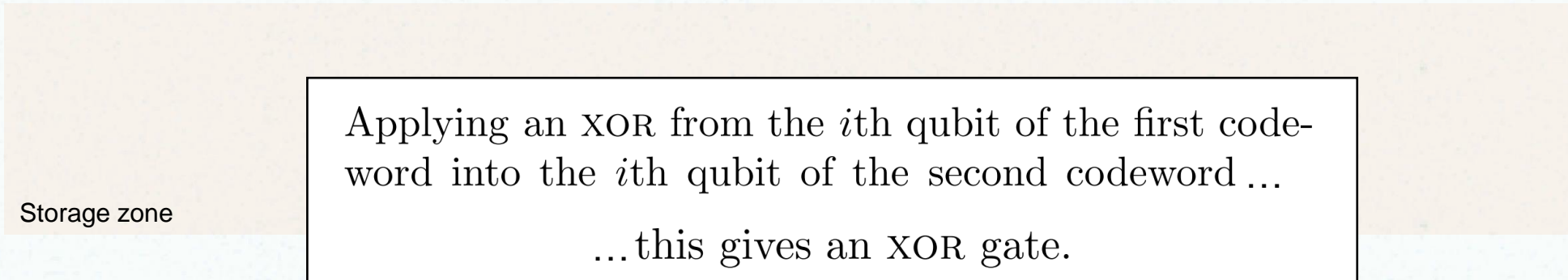
**EDFT:** Fault-tolerant algorithm with error detection. Postselect on all stabilizers correct.

*Exploring early fault-tolerant computations*

# GHZ circuit



Entangling zone



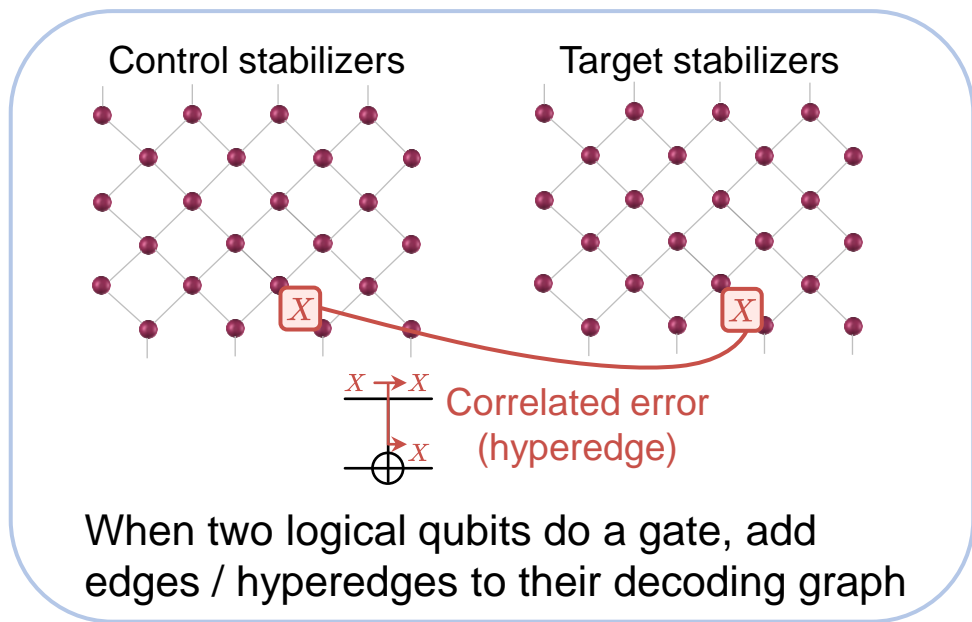
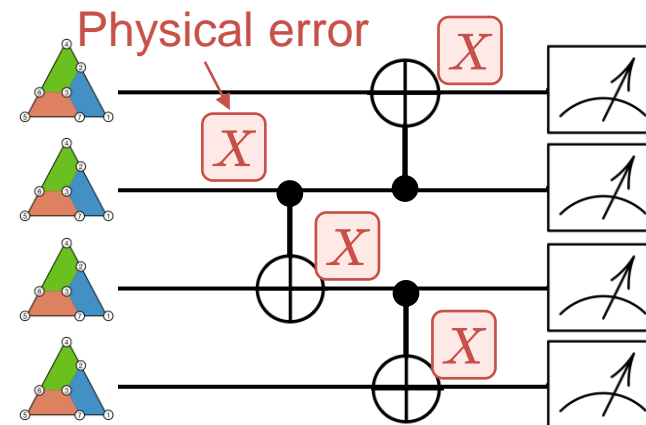
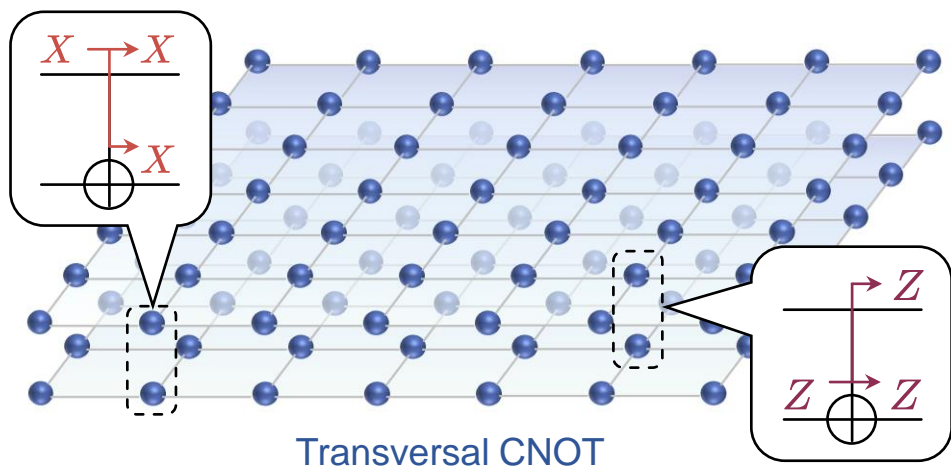
Storage zone

Applying an XOR from the  $i$ th qubit of the first code-word into the  $i$ th qubit of the second codeword ...  
 ...this gives an XOR gate.  
**Shor 1996 – *Fault-tolerant Quantum Computation***



# Correlated decoding

Madelyn Cain, C Zhao, H Zhou ... Jaffe, DB, Lukin – arXiv:2403.03272



**Correlated syndromes between multiple logical qubits provides significant information**

- improved decoding performance (eg, Steane QEC)
- “undoing” error propagation by tracking

**However, new errors build in time ... need to remove entropy**

see also  
 Gidney, C. Quantum 5, 497 (2021),  
 Delfosse, N. & Paetznick, A. arXiv:2304.05943v2 (2023),  
 McEwen, M., Bacon, D. Gidney, C. arXiv:2302.02192 (2023)

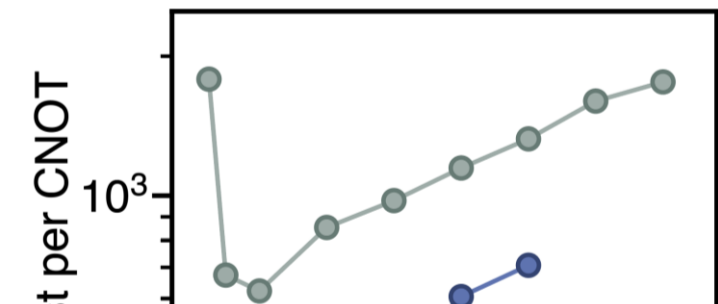
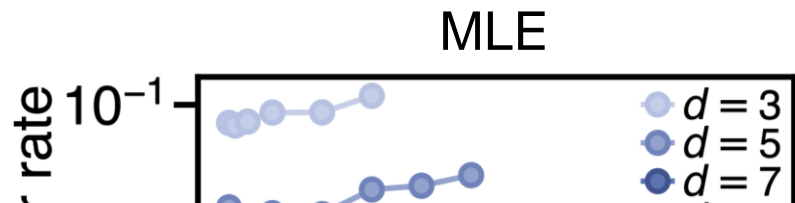
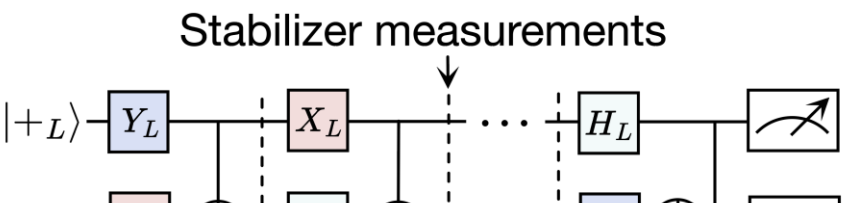


# Correlated decoding $\rightarrow \sim 1$ round per CNOT

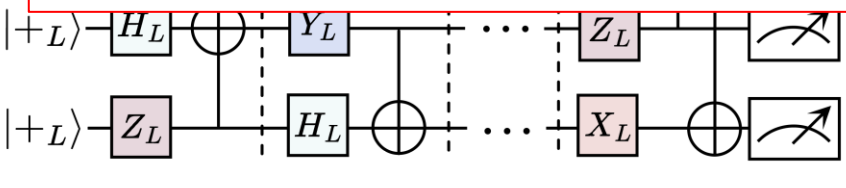
Madelyn Cain, C Zhao, H Zhou ... Jaffe, DB, Lukin – arXiv:2403.03272

## Key insights:

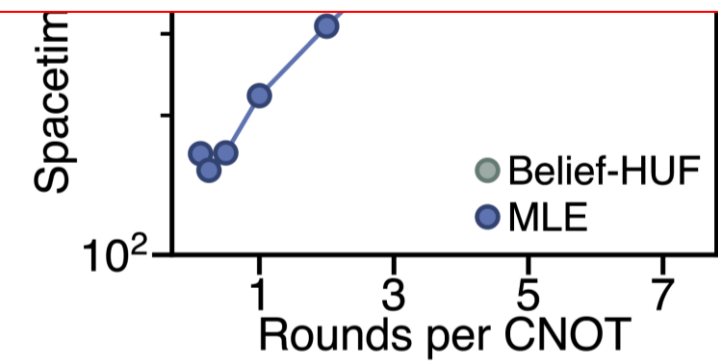
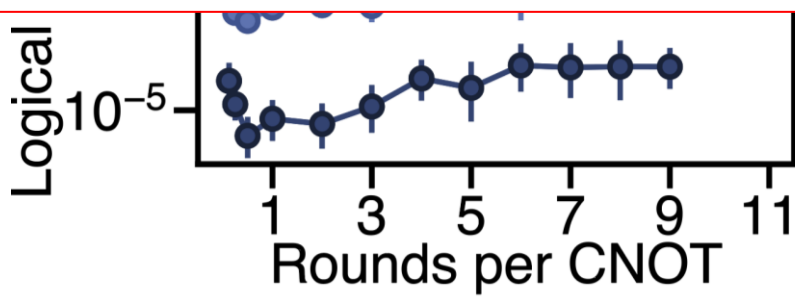
- Only need  $\sim 1$  round of stabilizer measurement per transversal CNOT to remove the newly created entropy
- “d rounds” is a “bug” from measurement errors. Since they propagate deterministically, can verify as circuit proceeds.



Lessons learned when experimenting with logical qubit algorithms in the lab



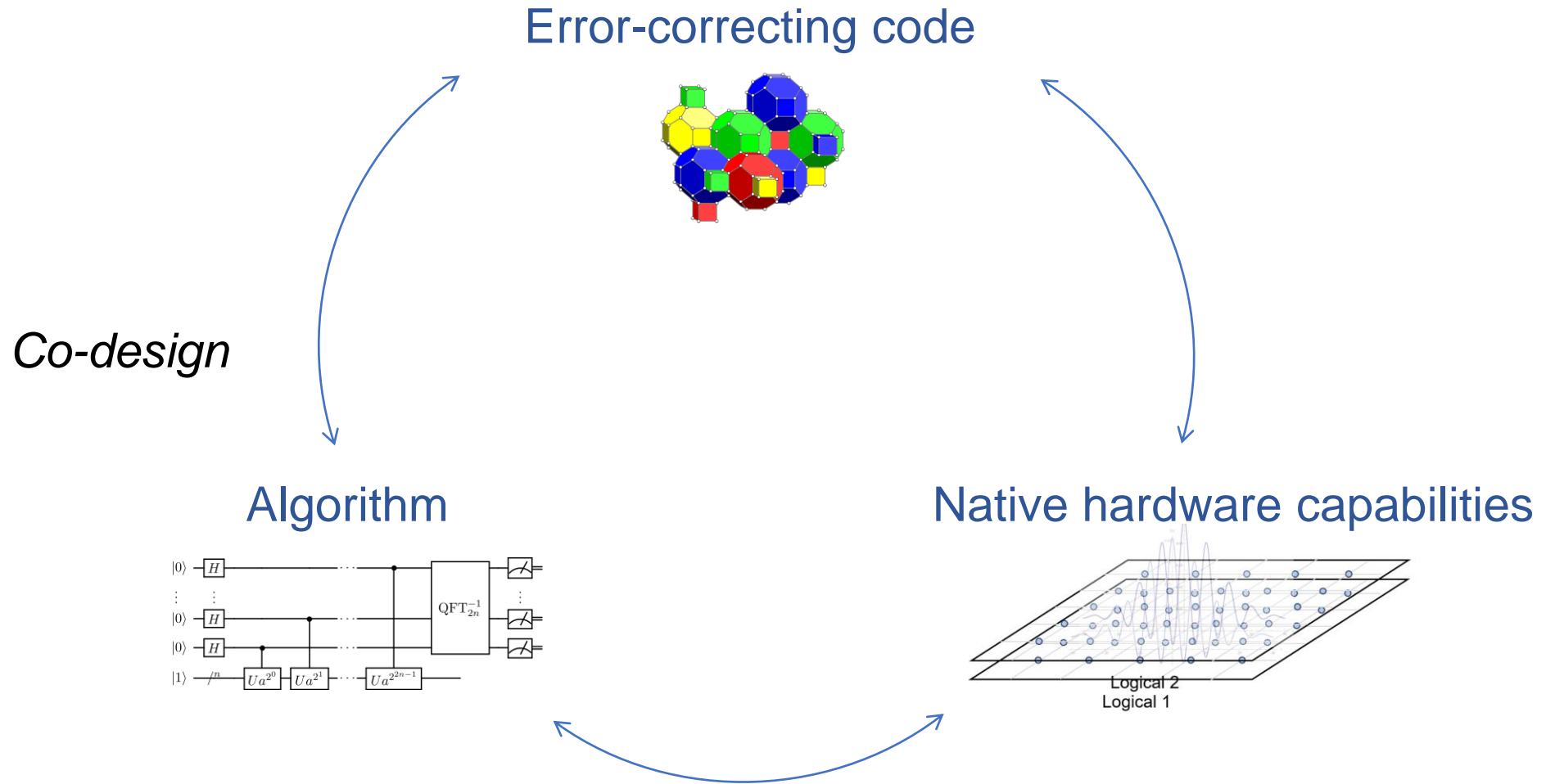
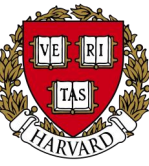
Variable number  $n_r$  syndrome extraction rounds per CNOT



Space-time reduction by factor of  $\sim d$



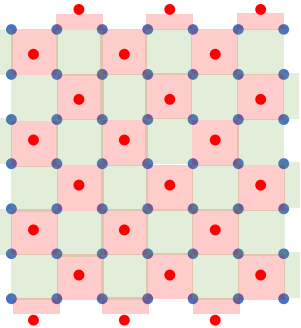
# Building logical processors in the lab



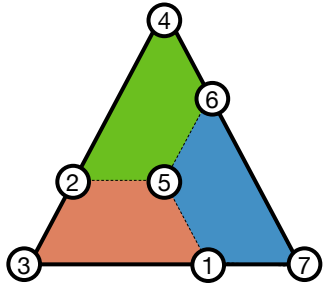
Entering the era of early fault-tolerant quantum computation ...  
Search for as many breakthroughs as possible ...

# Challenges with error-corrected computation

*non-Cliffords and universality*



Surface code



Color code

- QEC codes have a *discrete gate set* available
- 2D codes can do *Cliffords* {H, S, CNOT} easily, but cannot do *non-Cliffords* {T, CCZ} easily
- non-Clifford gate needed to complete *universal gate set*
  - *Actually, non-Clifford needed for any classically hard computation...*



Control-Control-Z gate (CCZ)  
*Non-Clifford gate*

Computational complexity grows exponentially with number of non-Cliffords applied<sup>1</sup>  
(State-of-the-art “Clifford + T” simulators<sup>2</sup> can handle ~16 CCZ’s)

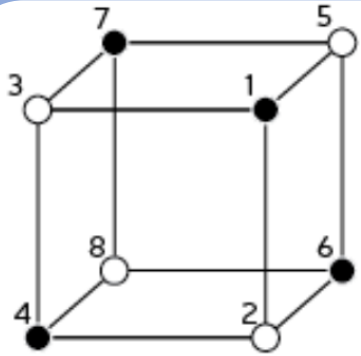
Amount of non-Clifford = “magic”

*See many works on magic from Jaffe group, e.g. Bu, Jaffe, Wei, arXiv:2402.05780*

1: Generically  
2: Simulators that take advantage of small number of non-Cliffords

# Fault-tolerant compiling: programming complex logical circuits

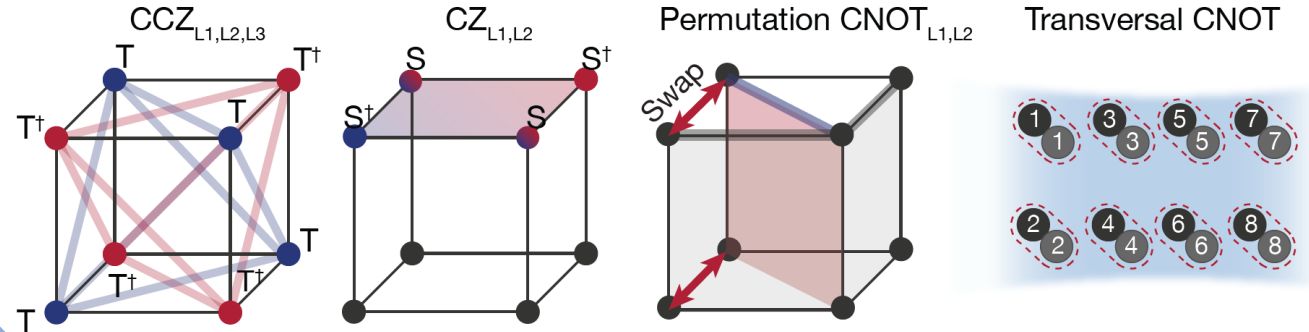
3D codes lose the transversal  $H$ , but gain transversal non-Cliffords



[[8,3,2]] code – small 3D code

“The smallest interesting colour code”, Earl Campbell blog

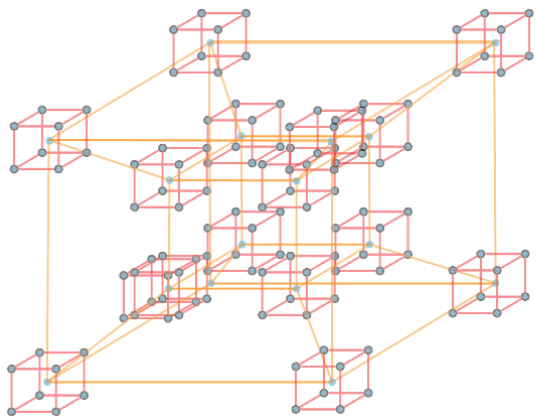
Vasmer, Kubica PRX Quantum 2022  
See also arXiv:2309.08663, arXv:2309.09893



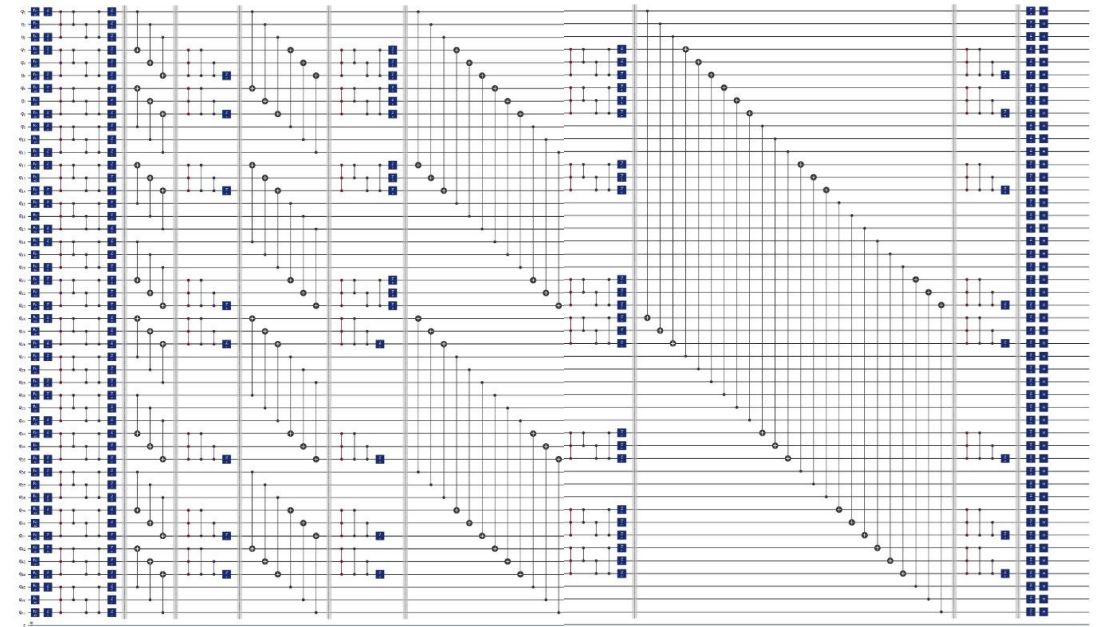
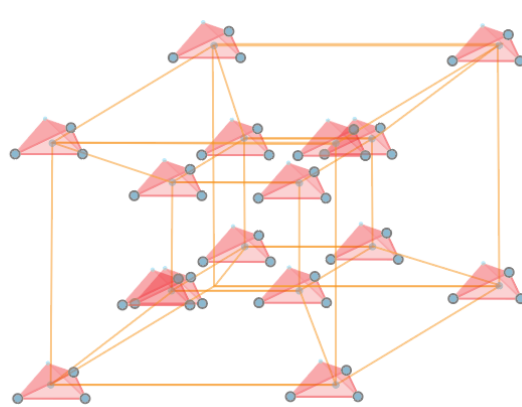
48 logical qubits, 228 logical two-qubit gates, 48 logical CCZs

Scrambling / supremacy circuits: utilize nonlocal connectivity of **logical qubits** and make hypercubes of **logical qubits**

Physical connectivity



Logical connectivity

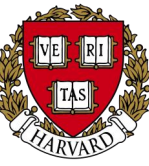


Logical circuit

IQP circuit (+CNOT) – Bremner et al arXiv:1005.1407 – see also Paletta et al arXiv: 2307.10729, Mezher et al arXiv: 2005.11539

D Hangleiter\*, M Kalinowski\*, DB\* ... Kubica, Lukin, Gullans, in prep – further analysis, extension, and connection to IQP Bluvstein et al Nature 2023

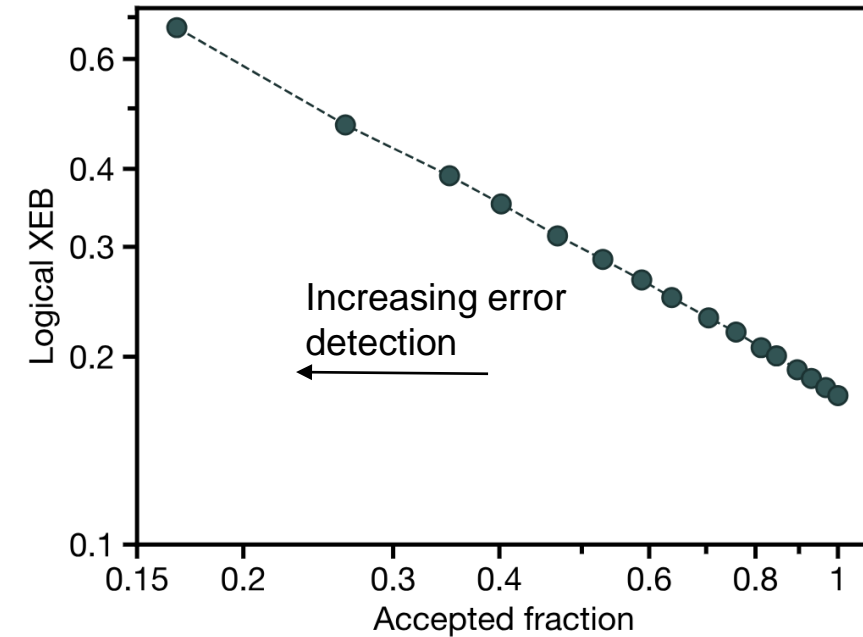
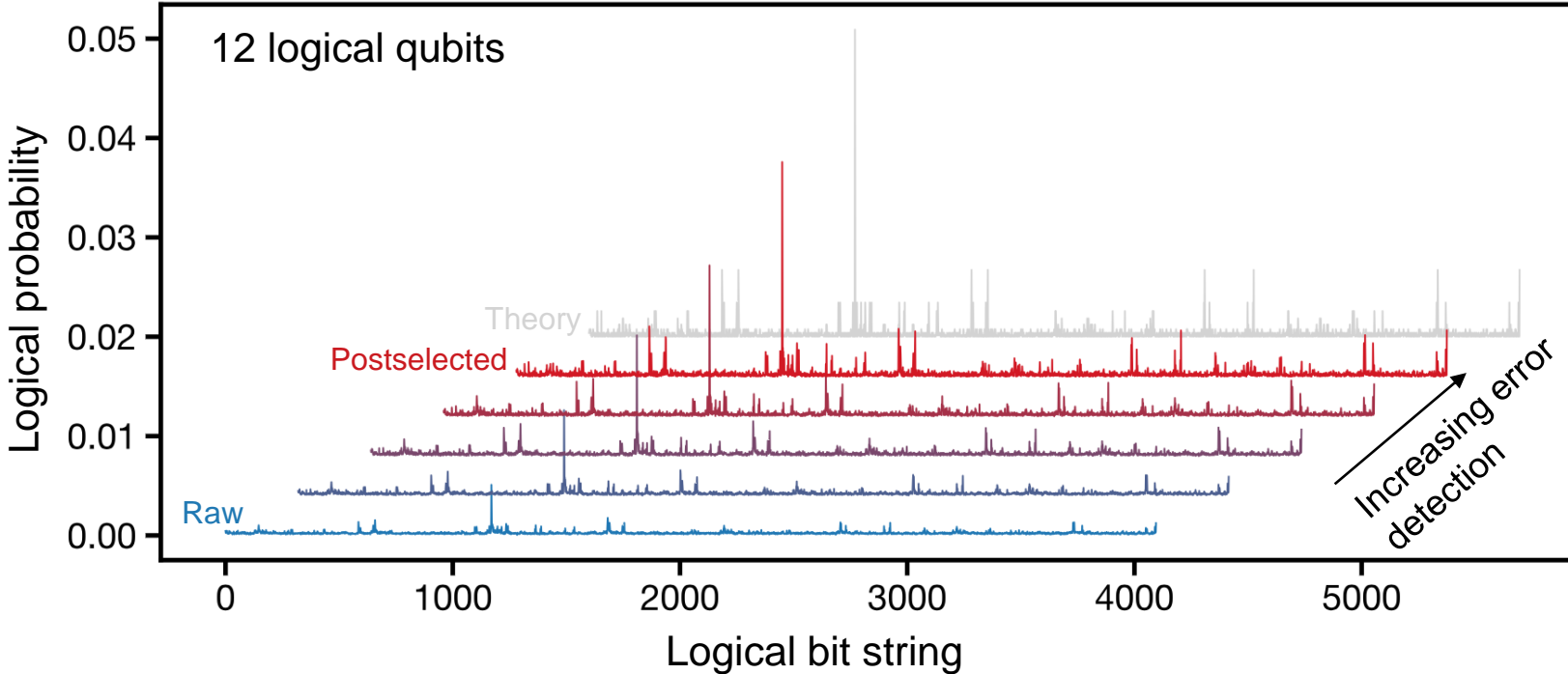
# Complex quantum circuits with logical qubits – sampling



$|\psi\rangle$

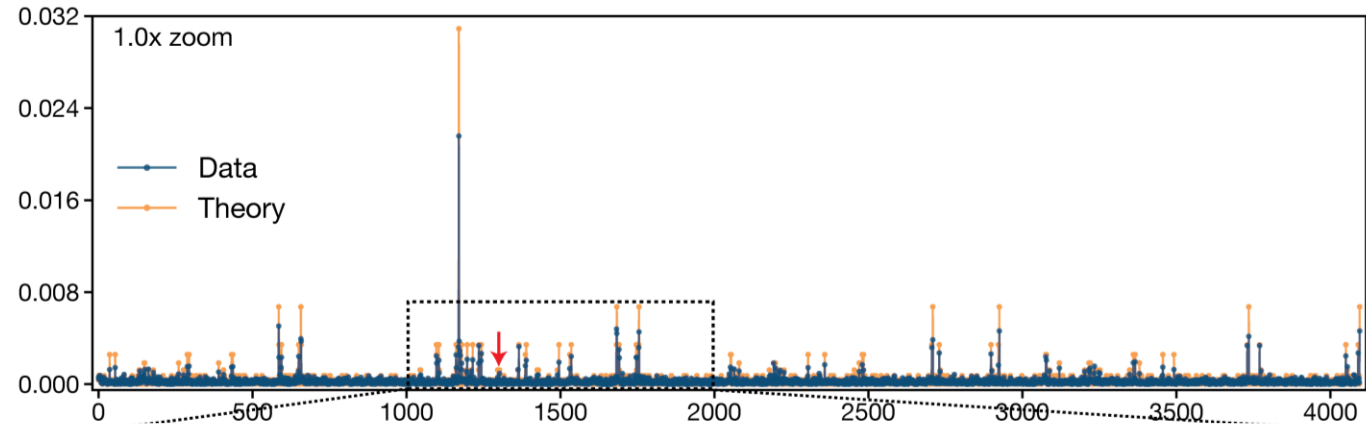
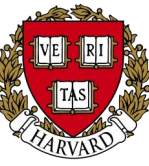


- Sampling: Take snapshots of many-body wavefunction and compare to expected distribution (simulations)
- XEB: sampling score (weighted sum normalized from 0 to 1).



Google Nature 2019  
Google arXiv: 2304.11119  
Pan group PRL 2021  
See also photon groups, USTC , Xanadu ..

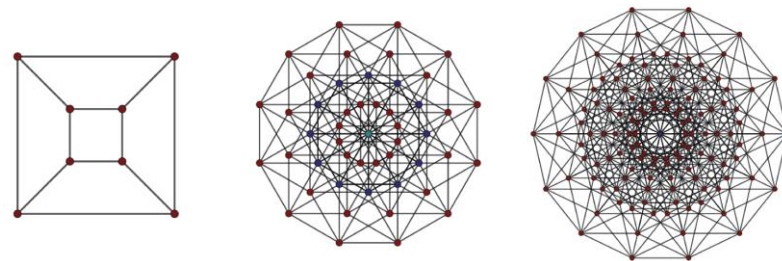
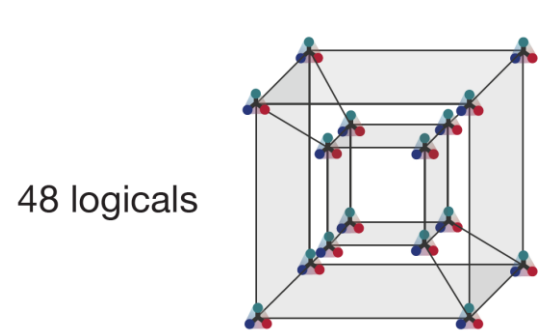
# Complex quantum circuits with logical qubits – sampling



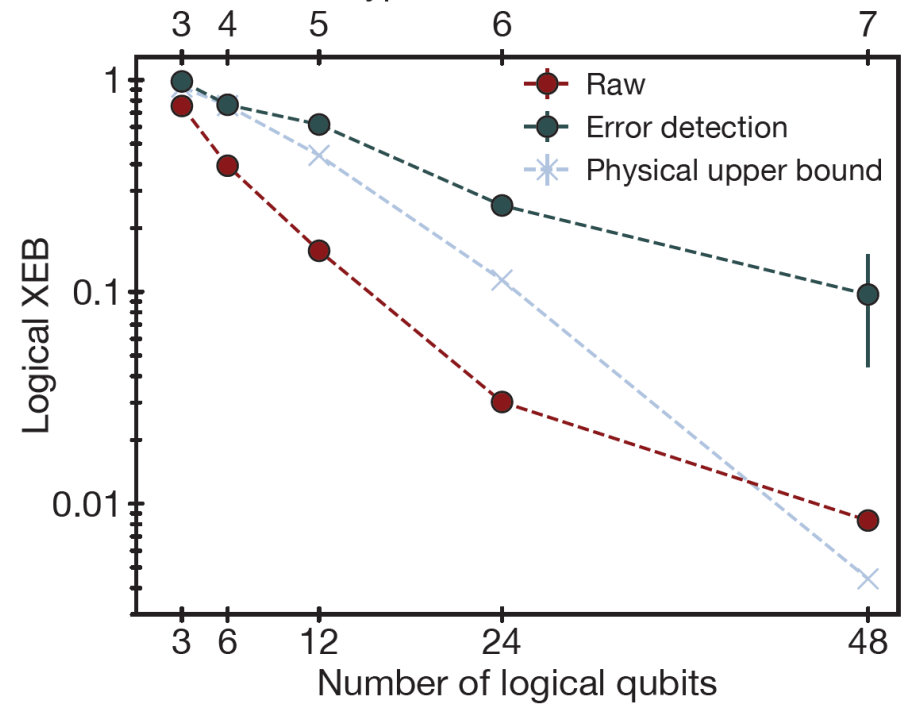
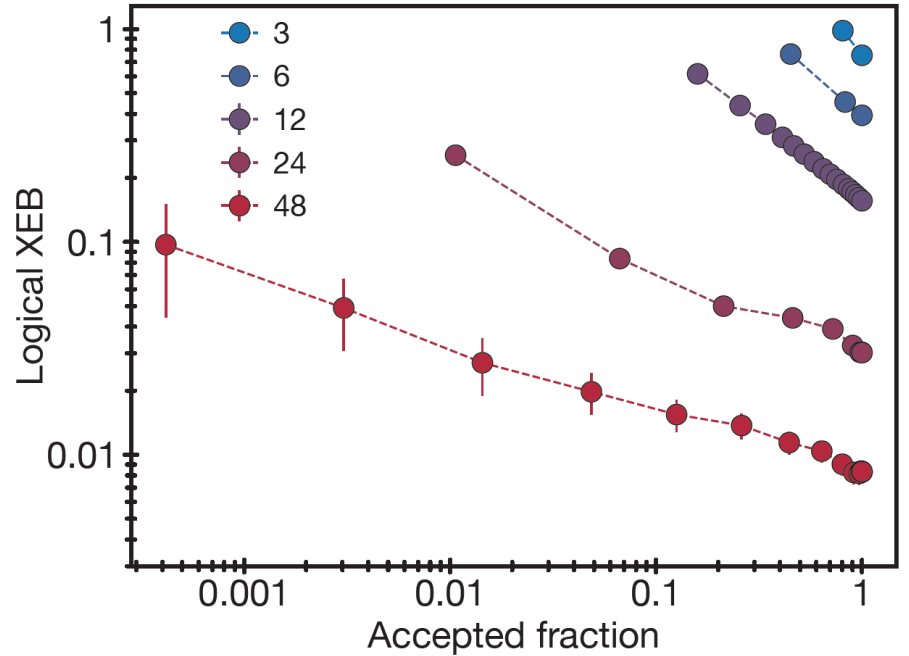


# Complex logical circuits – scaling to large sizes

*Up to 48 logical qubits, 228 logical two-qubit gates, 48 logical CCZs*

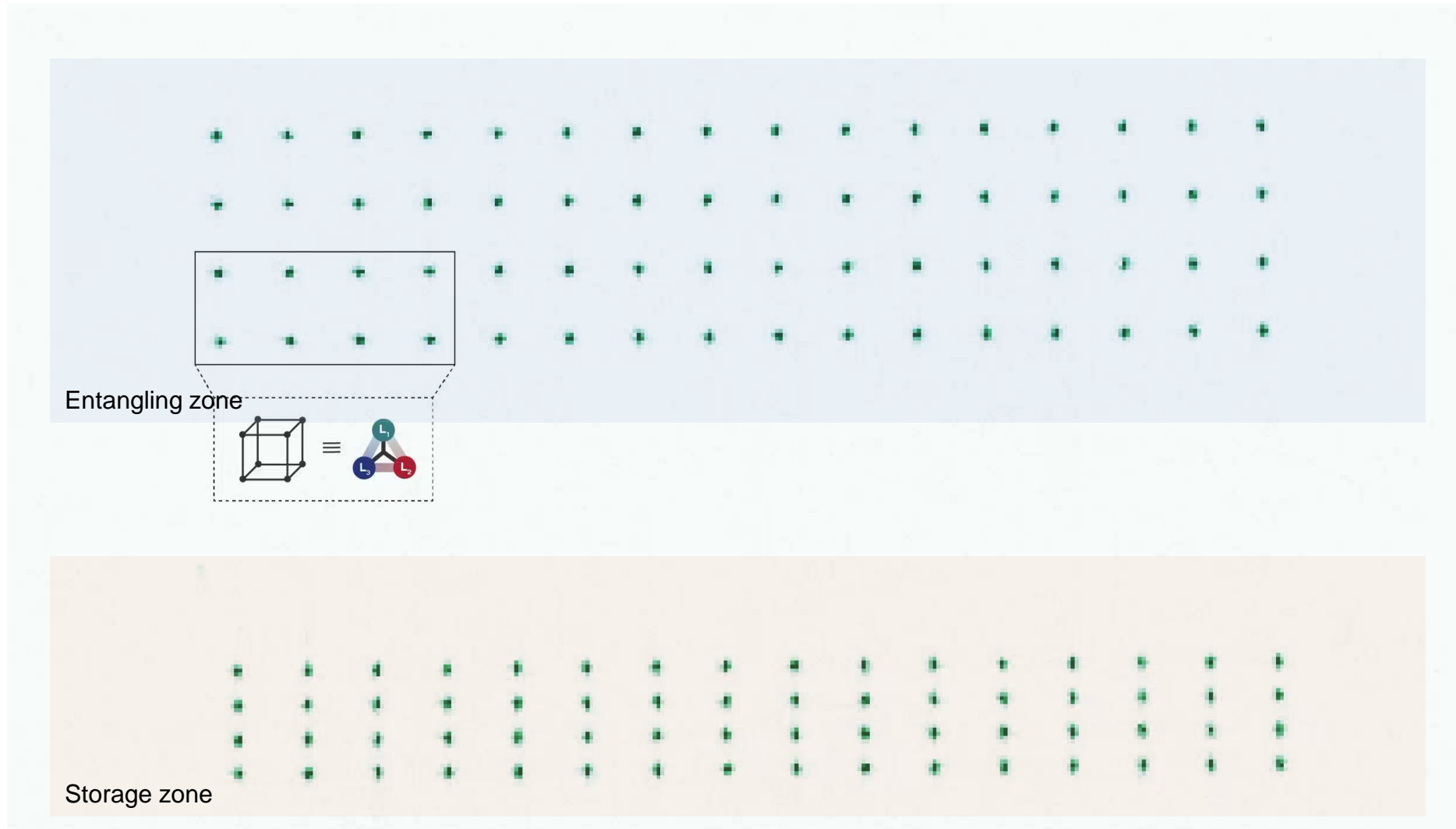


Hypercube dimension



- Finite XEB – successful sampling. XEB score improves with increased error detection
- Postselected logical XEB up to ~10x higher than previous physical implementations (at cost of measurement time)

# Seven-dimensional hypercube circuit



Inspired by conversations with Jason Cong and Daniel Tan  
See also Kuriyattil et al PRX Quantum 2023

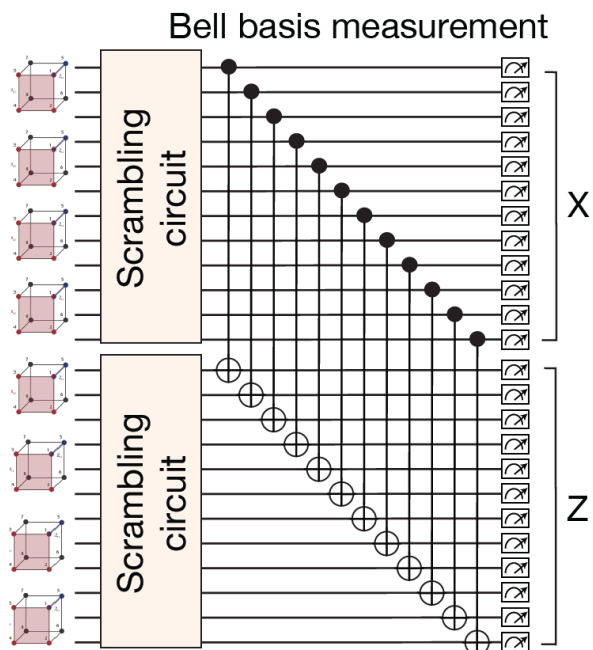


# “Fault-tolerant compiling” of quantum simulations

Bell basis measurement: extremely powerful tool in quantum information

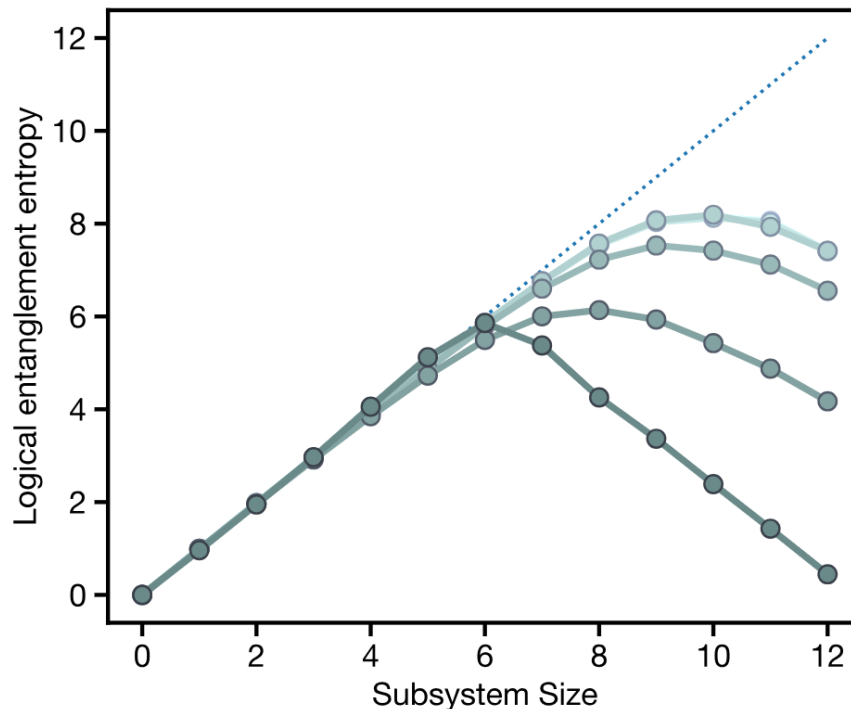
- Entanglement entropy – Daley, Pichler et al PRL 2012
- Simultaneous extraction of all  $4^N$  Pauli strings (absolute values) – Huang Science 2022

*Fully compatible with logical qubits and transversal CNOT!*



Two copies of 12 logical circuit, followed by transversal CNOTs for Bell basis measurement

*Scrambling and thermalization dynamics*



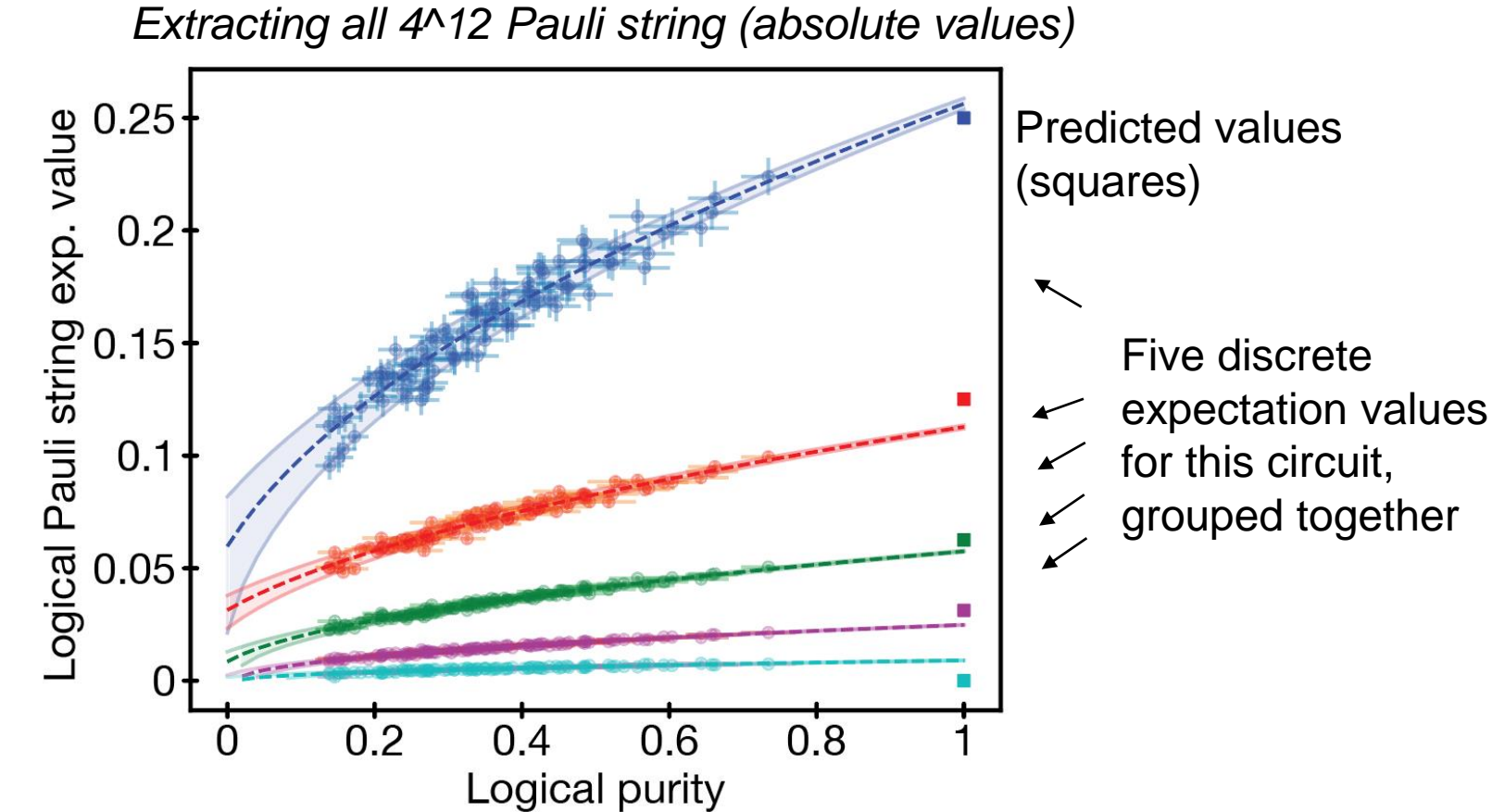
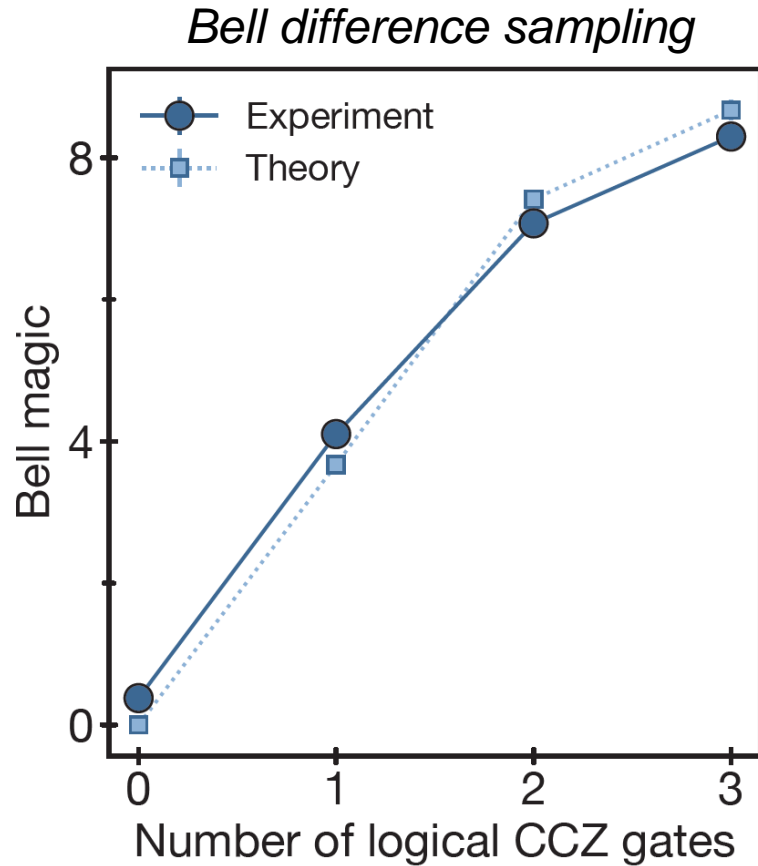
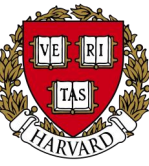
Increasing error detection  
↓

*Error-correction allows us to see clean Page curve behavior with limited decoherence*

*Logical qubits can already be used for various interesting physics and quantum information explorations*

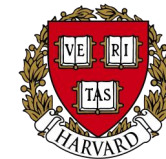


# Measuring Magic, Pauli extraction, zero-noise extrapolation



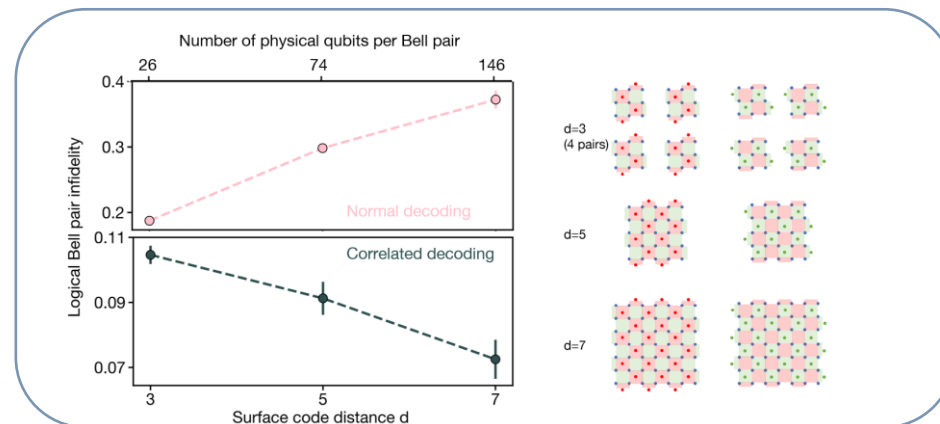
*Sliding-scale error detection ... zero-noise extrapolation*

Combination of encoded qubits with two-copy measurements offers interesting possibilities and error mitigation strategies in quantum simulation

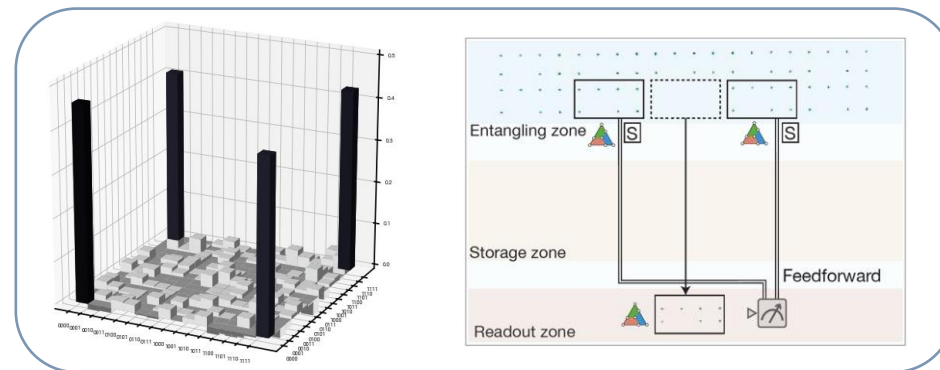


# Summary: Quantum information processing with logical qubits

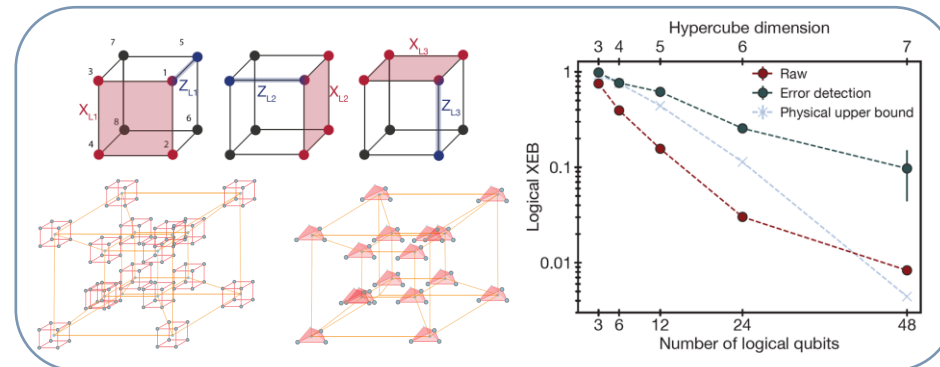
- Quantum operations improve with code size (surface codes as large as  $d=7$ )



- Fault-tolerant algorithms and characterizing zoned processor

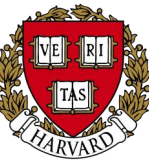


- Complex scrambling circuits with small 3D codes and hundreds of logical entangling gates

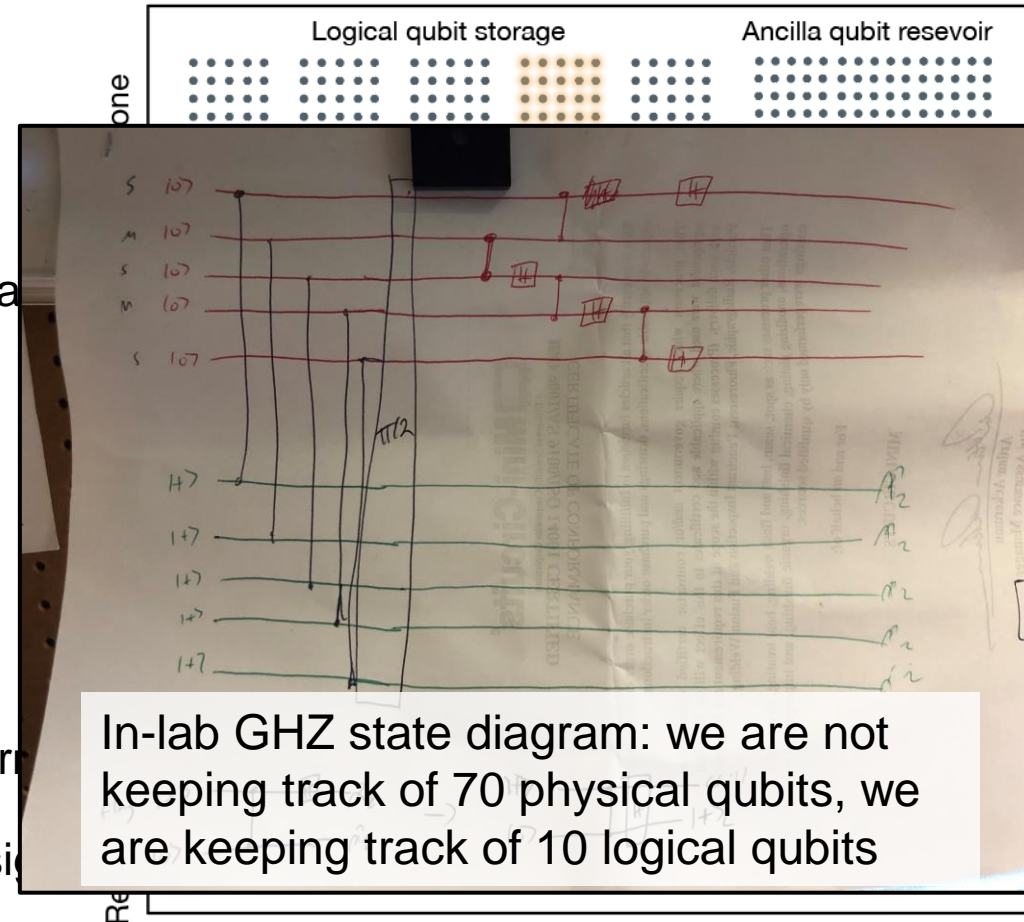


*So, what did we learn, and where do we go from here?*

# Lessons from our first-generation logical processor



- Control over single *logical* qubits as the fundamental units can dramatically reduce costs of a logical processor
- Instead of considering individual logical qubits, consider algorithms as a whole (e.g. correlated decoding)
- Logical qubits (many-body systems) are not literally qubits, and sometimes don't behave exactly the way you'd expect
  - 😞 Bell state fidelity  $\neq$  SPAM x gate fidelity
  - 😊 *Inherently digital operation*, coherent logical errors suppressed
- There are genuine examples where logical qubits can already outperform physical qubits at interesting problems
  - The discrete gate set is a curse, but also a blessing if you can design your problem around it
  - Many more opportunities for early-generation logical algorithms

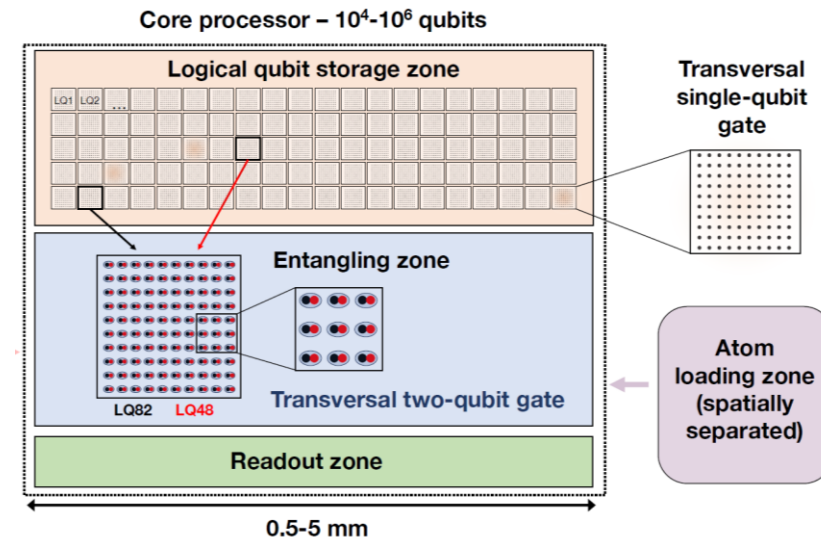




# Medium-scale logical processors: unique opportunity for atoms

## Open theory questions:

1. What algorithms should we study with 100 logical qubits and 1M logical gates?
2. How do we structure these algorithms for efficient QEC implementation? *Fault-tolerant compiling*



→ A path to 100 logical qubit device with  $\sim 10^{-5}$  logical error

Nonlocal connectivity, transversal logic gates, zoned architecture

Mid-circuit readout: Cavities, Shelving, Dual species, zones

Graham 2303.10051  
Deist PRL 2022  
Lis 2305.19266  
Also Schleier-Smith, Vuletic  
Singh Science 2023  
See also Ni, semeghini

Alkaline-earth; long T2, esp nuclear

Jenkins PRX 2022  
Ma PRX 2022  
Atom Computing, Endres, Covey ...  
Moving will work great for nuclear spins!

**A medium-scale QEC device:  
New tool for scientific discoveries  
Accelerating the path to large scale QC**

*Erasure conversion*  
Idea: Thompson group, Puri

Exciting results: Endres 2023 2305.03406,  
Thompson 2023 2305.05493  
Wu Nat Comm 2022  
Sahay 2302.03063  
Noise bias Cong et al PRX 2022

Various approaches to local addressing, lattices, clever optical technologies ...

Kaufman group  
Saffman Group

PIC: Englund group ...  
Reloading: Bernien group, atom computing...

Optical networking

Covey, Bernien groups ...  
Ocola 2210.12879  
Vuletic group  
Ramette, Sinclair 2302.01296

**In the coming years, such devices will unlock completely new studies of quantum mechanics**

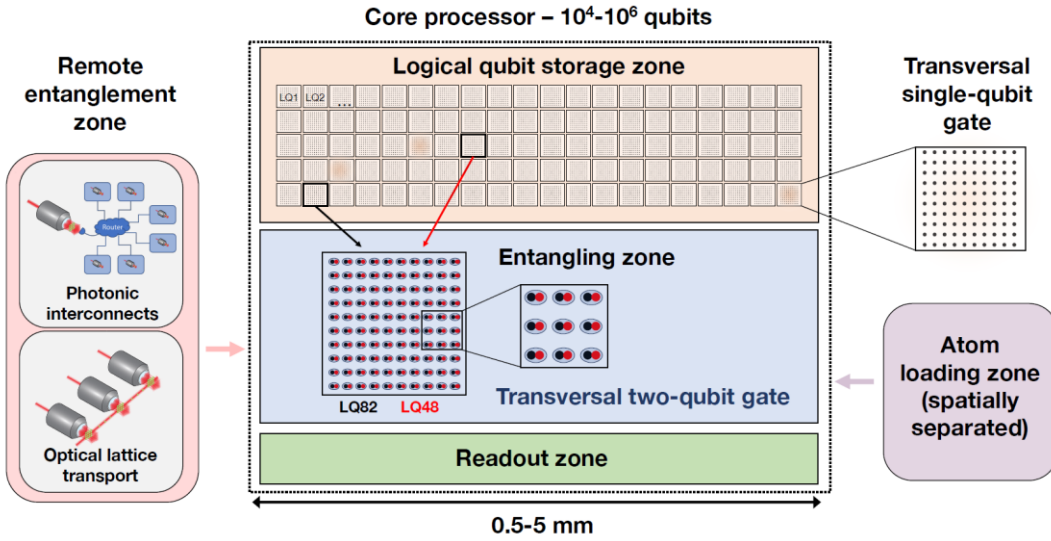
Very nice initial continuous reloading (2024): Atom computing 2401.16177, Zeiher 2402.04994

*Diverse opportunities with neutral atoms*



# How to get to large-scale processor, e.g. >1000 logicals?

- *Optimistic estimates*<sup>1</sup>: 1 million qubits,  $10^{-10}$  error rate, day-long computation
- *Possible avenues*:
  - LDPC<sup>2</sup> codes and other clever tricks: favorable scaling
  - Power-efficient trapping: e.g., 2D or 3D optical lattices for trapping 1 million atoms
  - Connected modules: photonic interconnects through cavities or free-space



*Should pursue all possible breakthroughs – hardware, code & algorithm design!*

**Experimentation with early generation logical devices will likely reshape the way we think these large-scale processors should be built (theory & hardware)**

1: Gidney Eker Quantum 2021  
2: Q Xu\*, P Bonilla\* et al., arXiv: 2308.08648 (collaboration of Lukin & Jiang groups + QuEra (H Zhou)) – see also Bravyi et al



Sophie Li

Tout Wang

Sasha Geim

Giulia Semeghini

Sepehr Ebadi

Simon Evered

Misha

Harry Zhou

DB

Tom Manovitz

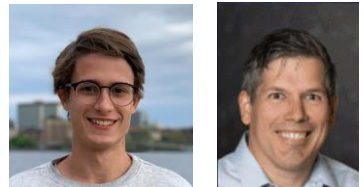


★ **Atom Array II:**

Tout Wang, Tim Guo, Allen Chiu, Pavel Stroganov, Mohamed Aboeih, Simon Hollerith, Sebastian Geier

★ **Collaboration with QuEra**

P Sales Rodriguez      T Karolyshyn



**PIs**

M. Lukin

V. Vuletic

M. Greiner



**Theory collaborators**

N. Maskara

M. Cain

I. Cong

JP Bonilla Ataides

M. Kalinowski

M. Gullans

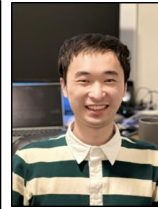
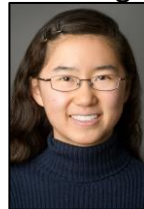
D. Hangleiter

X. Gao

C. Zhao

A. Jaffe

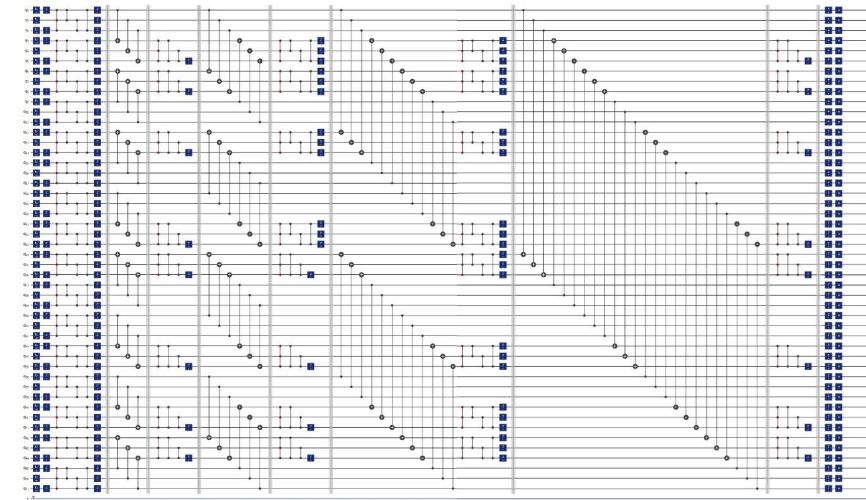
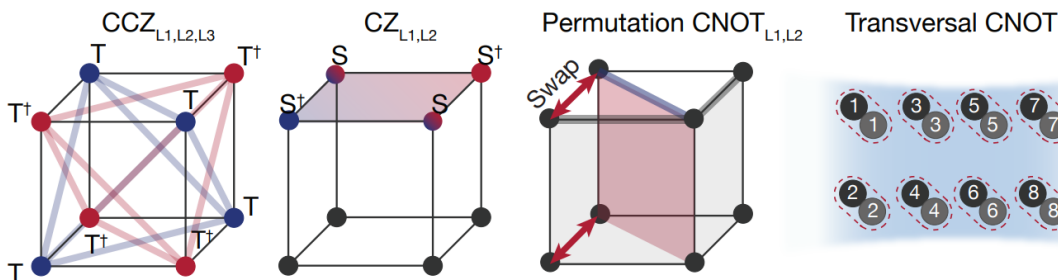
N. Meister



Funding: QuEra, NSF, CUA, Vannevar Bush, AFOSR MURI, DARPA, IARPA, NDSEG, Hertz

# Outlook: towards quantum advantage with logical qubits

- Present work: we intentionally structure our exact circuit so that they are susceptible to an “attack” using tensor contraction of two wavefunctions each with  $n/2$  qubits, partitioned by the final CNOT layer
  - Leads to  $2^{(n/2)}$  simulation cost
  - This “attack” is broken by applying additional CNOT layers, recovering  $2^n$  scaling
- Nice recent work by IBM / IonQ arXiv:2402.03211 finds another “attack” for our exact circuit by using “minimal covering sets” or “minimal vertex covers” of the CCZs
  - Leads to  $2^{(n/3)}$  simulation cost
  - This “attack” is broken by implementing permutation CNOTs, recovering  $2^n$  scaling
  - Permutation CNOT is fault-tolerant and directly achievable in our atom array system – is just a parallel reshuffling of qubits



Logical circuit

