Certifying almost all quantum states with few single-qubit measurements

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Motivation

- Quantum systems with **intricate entanglement** are pivotal in quantum information science.

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- To understand if we have created the desired quantum system in the lab, we need to perform certification.

What is Certification?

- We have a desired n-qubit state $|\psi\rangle$, which is our target state.

- We have an n-qubit state ρ created in the experimental lab.

- **Task:** Test if ρ is close to $|\psi\rangle\langle\psi|$ or not from data?

  ($\langle\psi|\rho|\psi\rangle$ is close to 1)

- A **fundamental task** in data science for quantum.

Motivation

- Many techniques based on statistics & learning theory have been proposed for performing certification.
- However, it remains **experimentally challenging** to certify highly-entangled quantum many-body systems.

How to Certify?

- **Approach 1:** Random Clifford measurements (classical shadow)

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- **Advantage:**

  Only needs **depth-n** random Clifford circuits on ρ

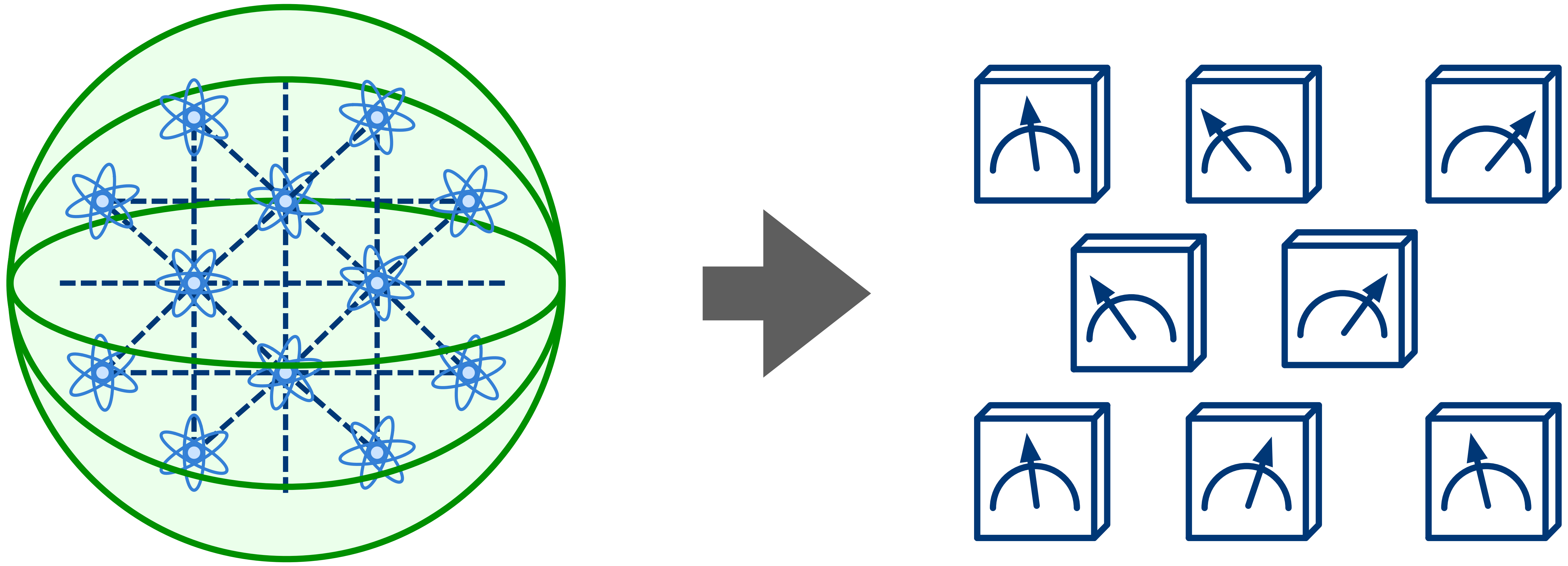
- **Challenge:**

  Implementing depth-n random Clifford circuits

  is still experimentally challenging.

How to Certify?

- **Approach 2:** Random Pauli measurements (classical shadow)



Quantum state

Single-qubit Measurement

How to Certify?

- **Approach 2:** Random Pauli measurements (classical shadow)
- **Advantage:**

  Only needs **single-qubit** measurements on ρ

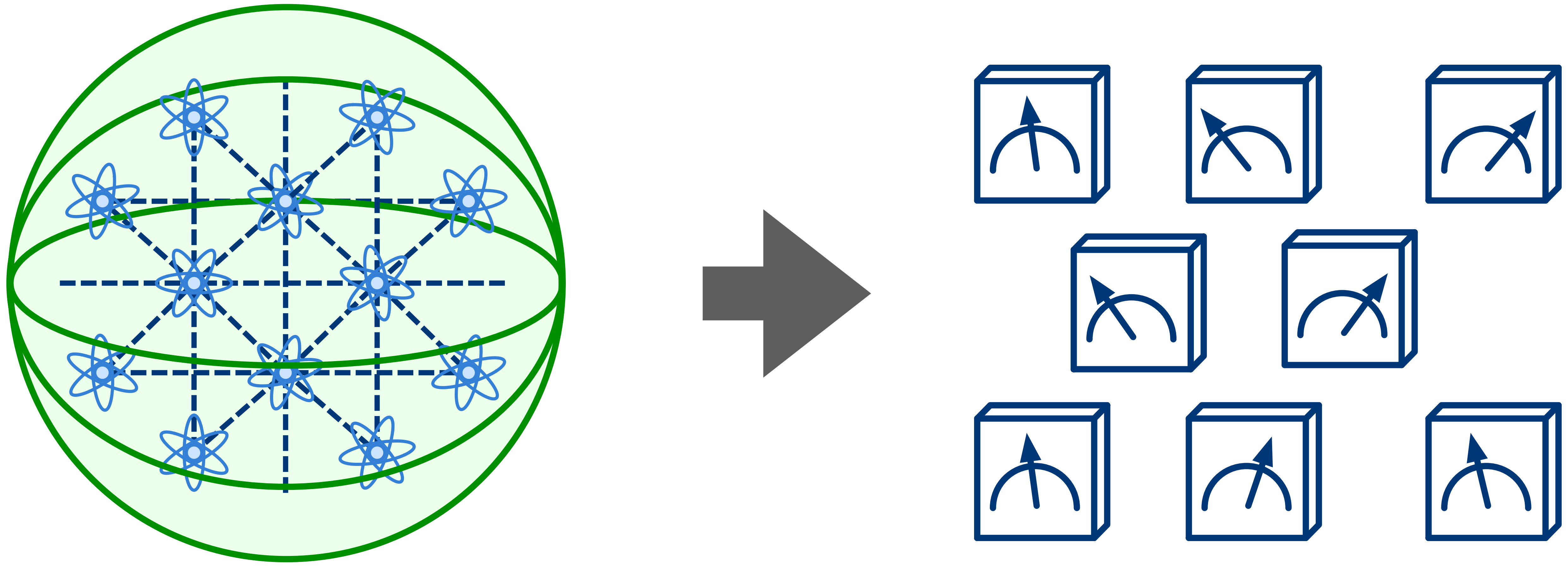
- **Challenge:**

  Requires **exp(n)** measurements for most target $|\psi\rangle$

  especially when $|\psi\rangle$ is highly entangled.

How to Certify?

- **Approach 3:** Cross-entropy benchmark (XEB)



Quantum state

Single-qubit
Measurement
(all Z bases)

How to Certify?

- **Approach 3:** Cross-entropy benchmark (XEB)
- **Advantage:**

  Only needs **single-qubit** measurements (Z-basis) on ρ

- **Challenge:**

  Does not rigorously address the certification task.

  ρ can be **far** from $|\psi\rangle\langle\psi|$ despite perfect XEB score.

Existing Challenges

- All existing certification protocols either

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- All existing certification protocols either

  a. Require **deep** quantum circuits before measurements

  b. Use **exponentially** many measurements

  c. Apply only for **low-entanglement** state $|\psi\rangle$

  d. Lack rigorous guarantees

Question

Can we rigorously certify **highly-entangled** quantum states from performing **few single-qubit** measurements?

Question

Can we rigorously certify **almost all** quantum states from performing **few single-qubit** measurements?

Outline

- Theorem
- Protocol
- Applications

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Certification

Theorem 1

For almost all n-qubit state $|\psi\rangle$, we can certify that ρ is close to $|\psi\rangle\langle\psi|$ using only $\mathcal{O}(n^2)$ single-qubit measurements.

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Certification

Theorem 1

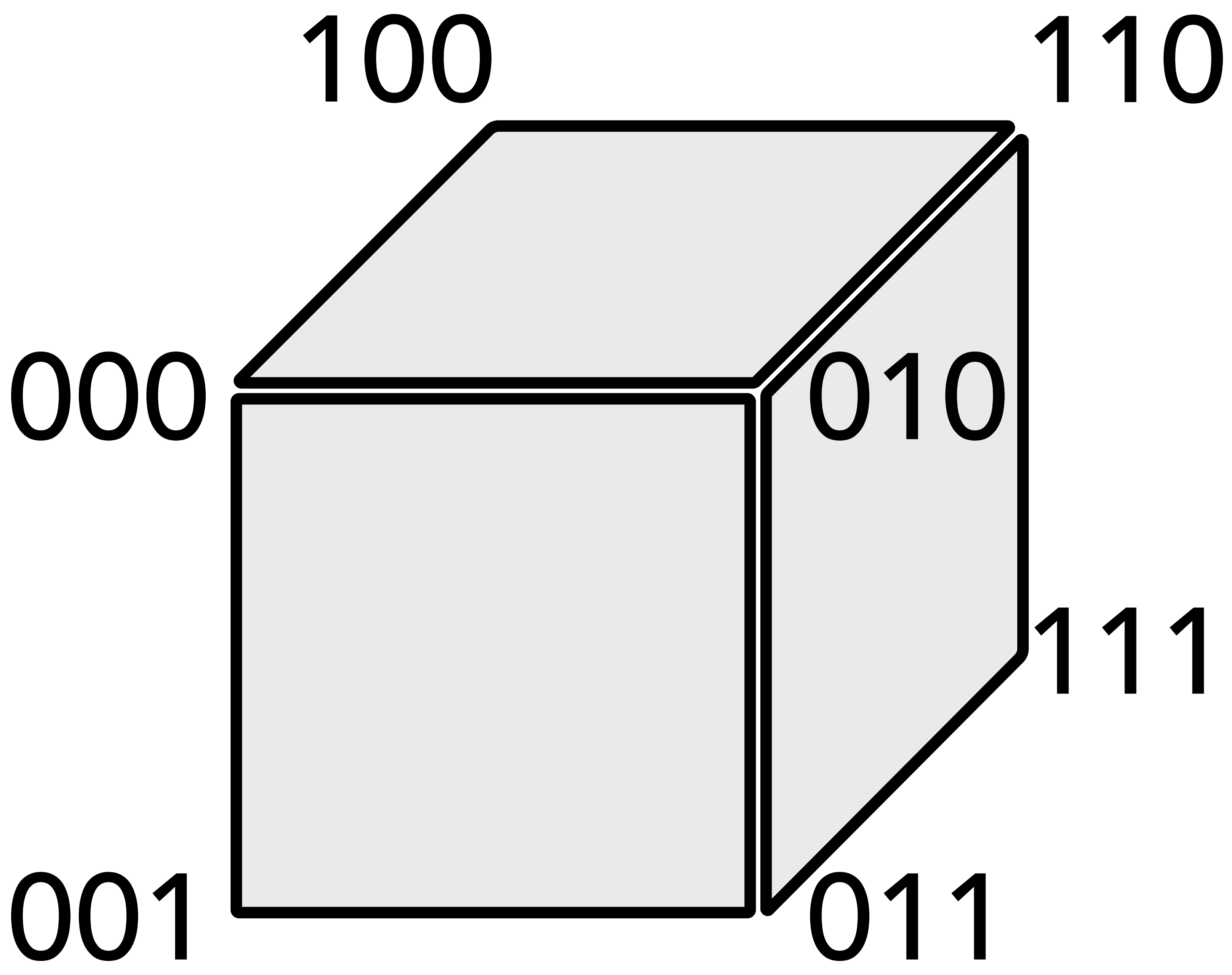
For almost all n-qubit state $|\psi\rangle$, we can certify that ρ is close to $|\psi\rangle\langle\psi|$ using only $\mathcal{O}(n^2)$ single-qubit measurements.

- The certification procedure applies to any ρ.
- $\mathcal{O}(n^2)$ is enough even when $|\psi\rangle$ has $\exp(n)$ circuit complexity.

Relaxation Time

- Consider an n-qubit target state $|\psi\rangle$.
- Choose a basis $|b\rangle$, where $b \in \{0,1\}^n$ is a bitstring.
- Let $\pi(b) = |\langle b|\psi\rangle|^2$ be the **measurement distribution**.

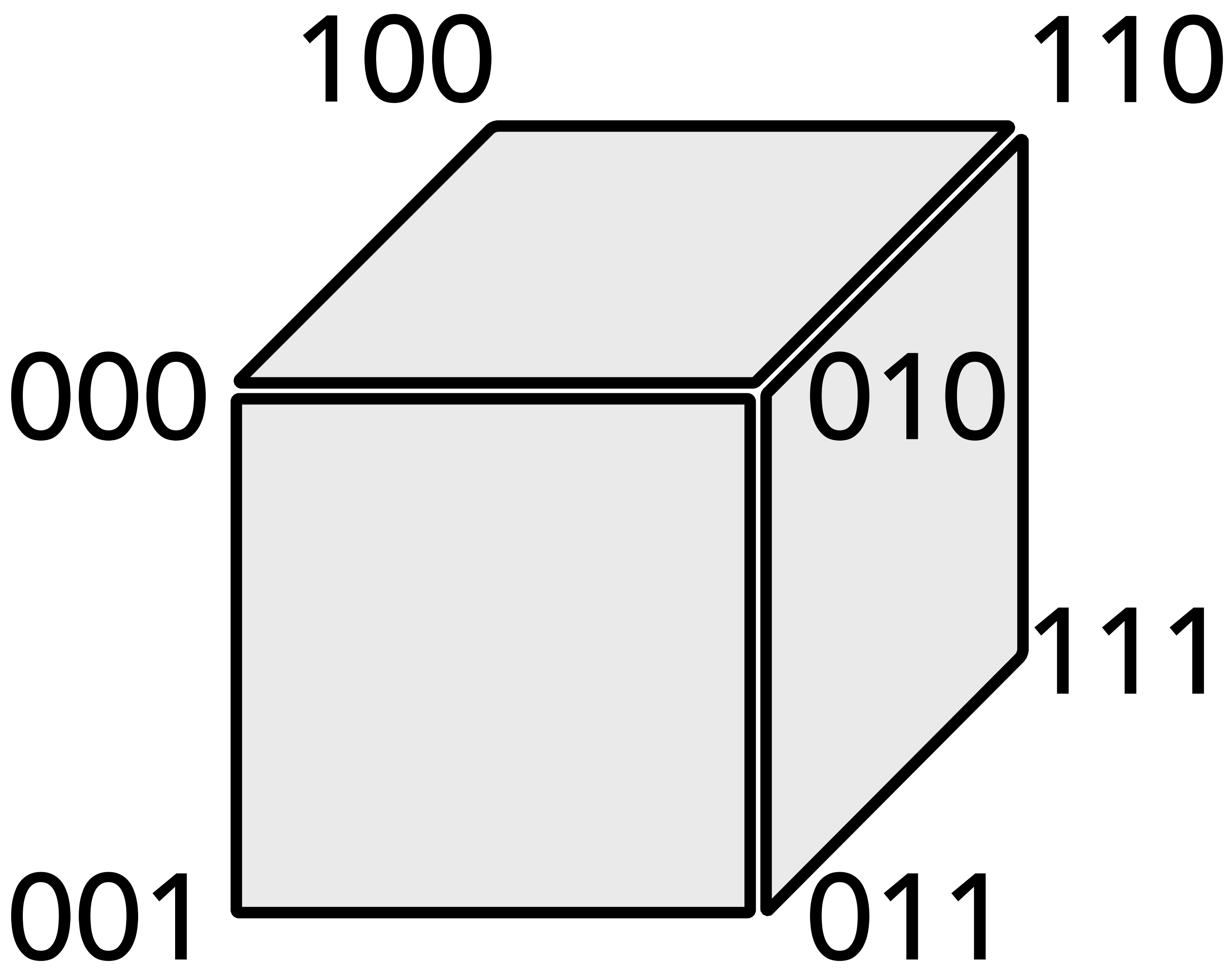
Boolean Hypercube



Relaxation Time

- Let $\pi(b) = |\langle b|\psi\rangle|^2$ be the **measurement distribution**.
- Consider a random walk on n-bit Boolean hypercube.

Boolean
Hypercube



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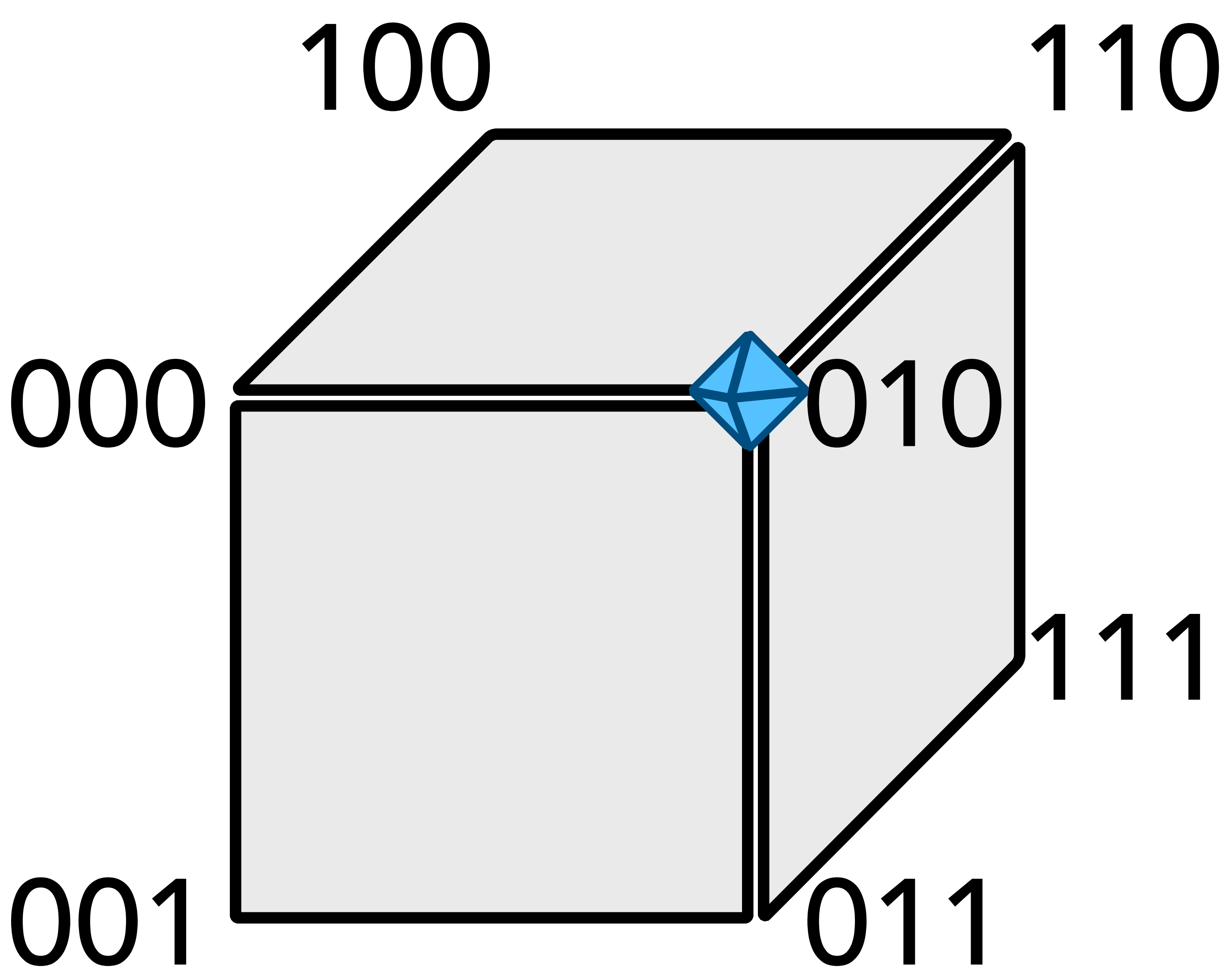

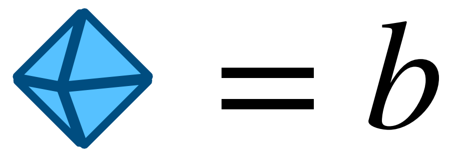

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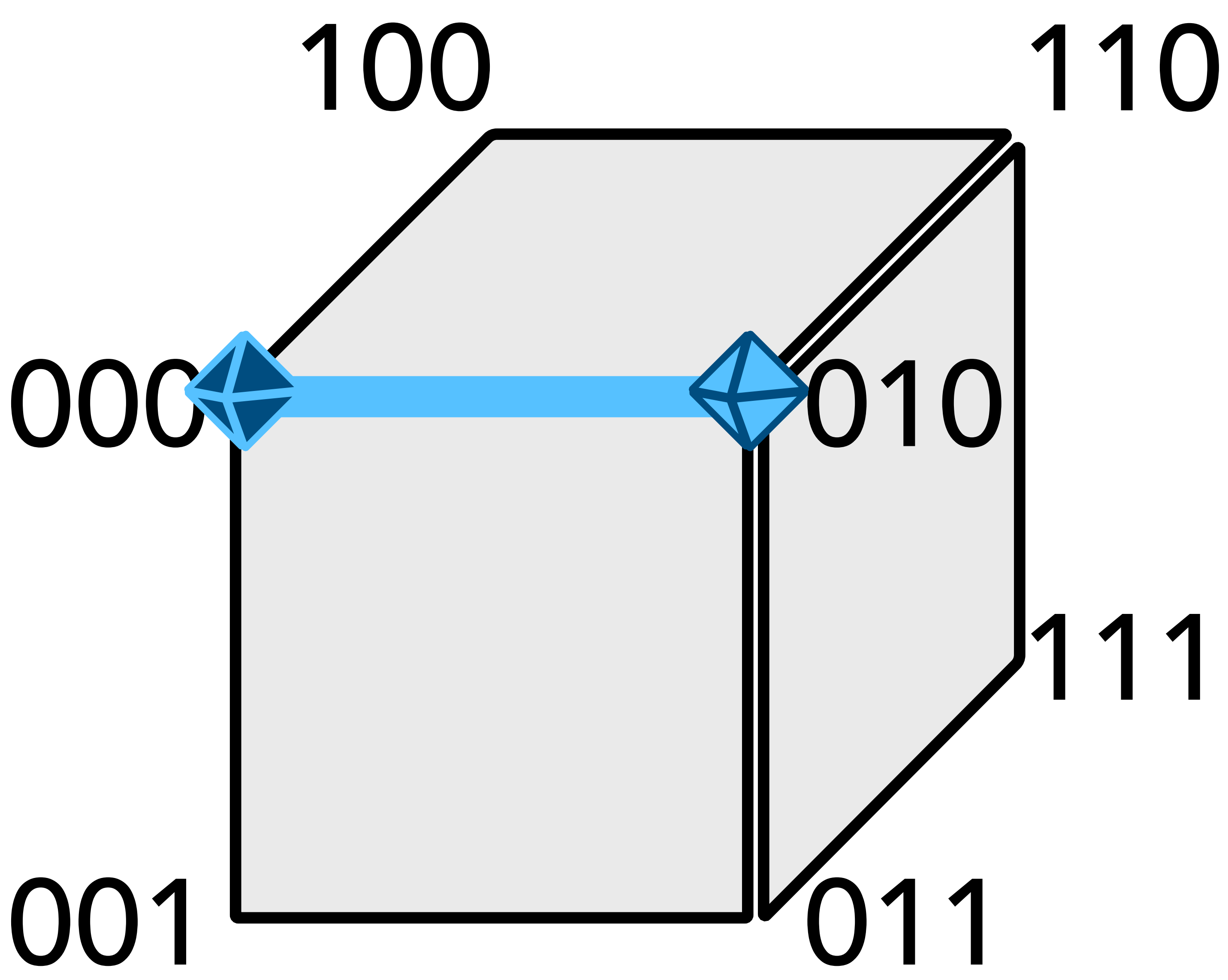
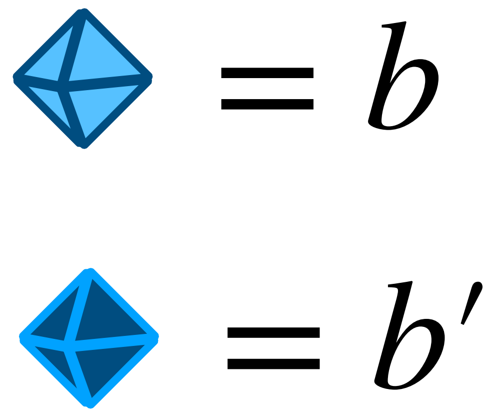


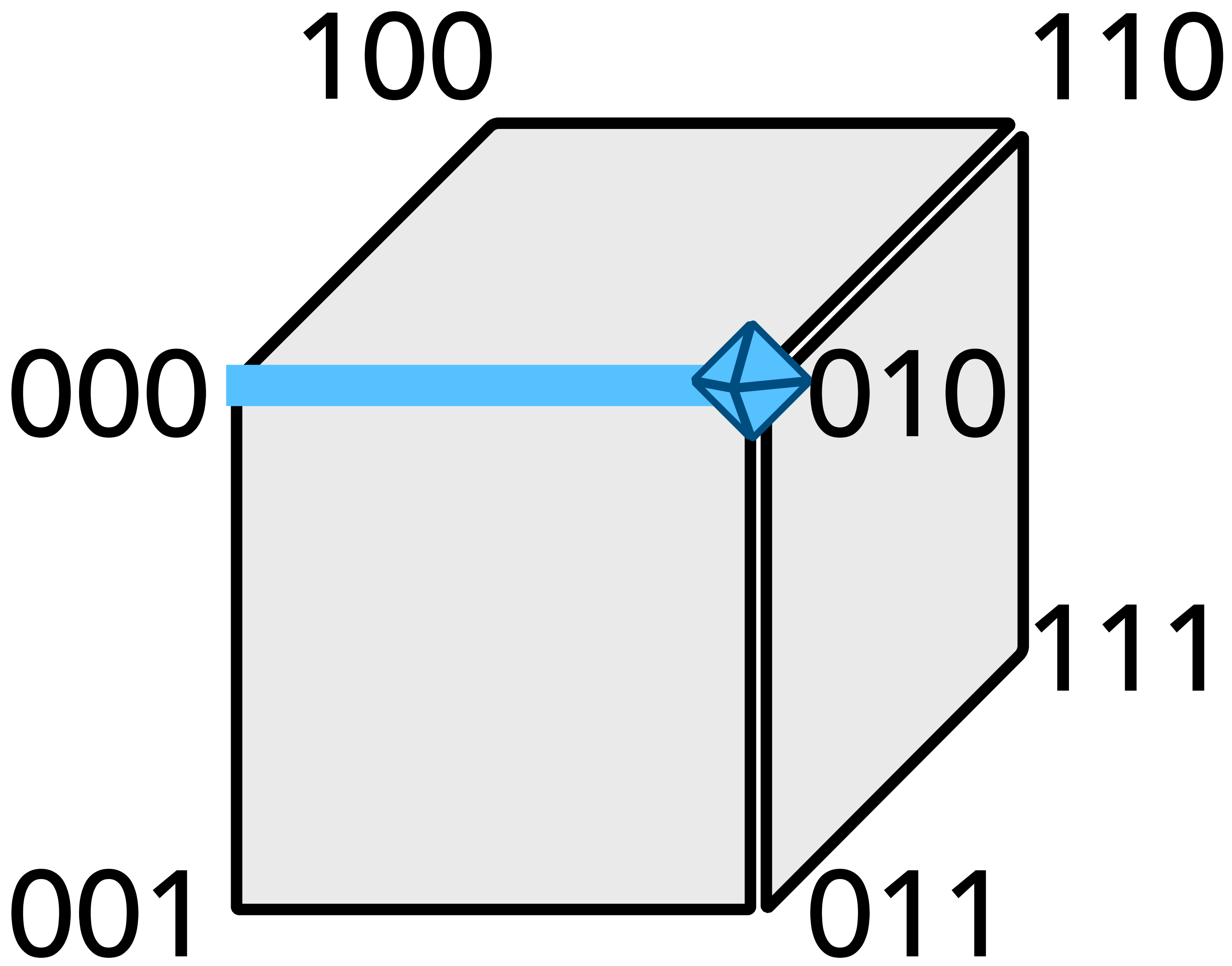


Relaxation Time

- Let $\pi(b) = |\langle b|\psi\rangle|^2$ be the **measurement distribution**.
- Consider a random walk on n-bit Boolean hypercube.

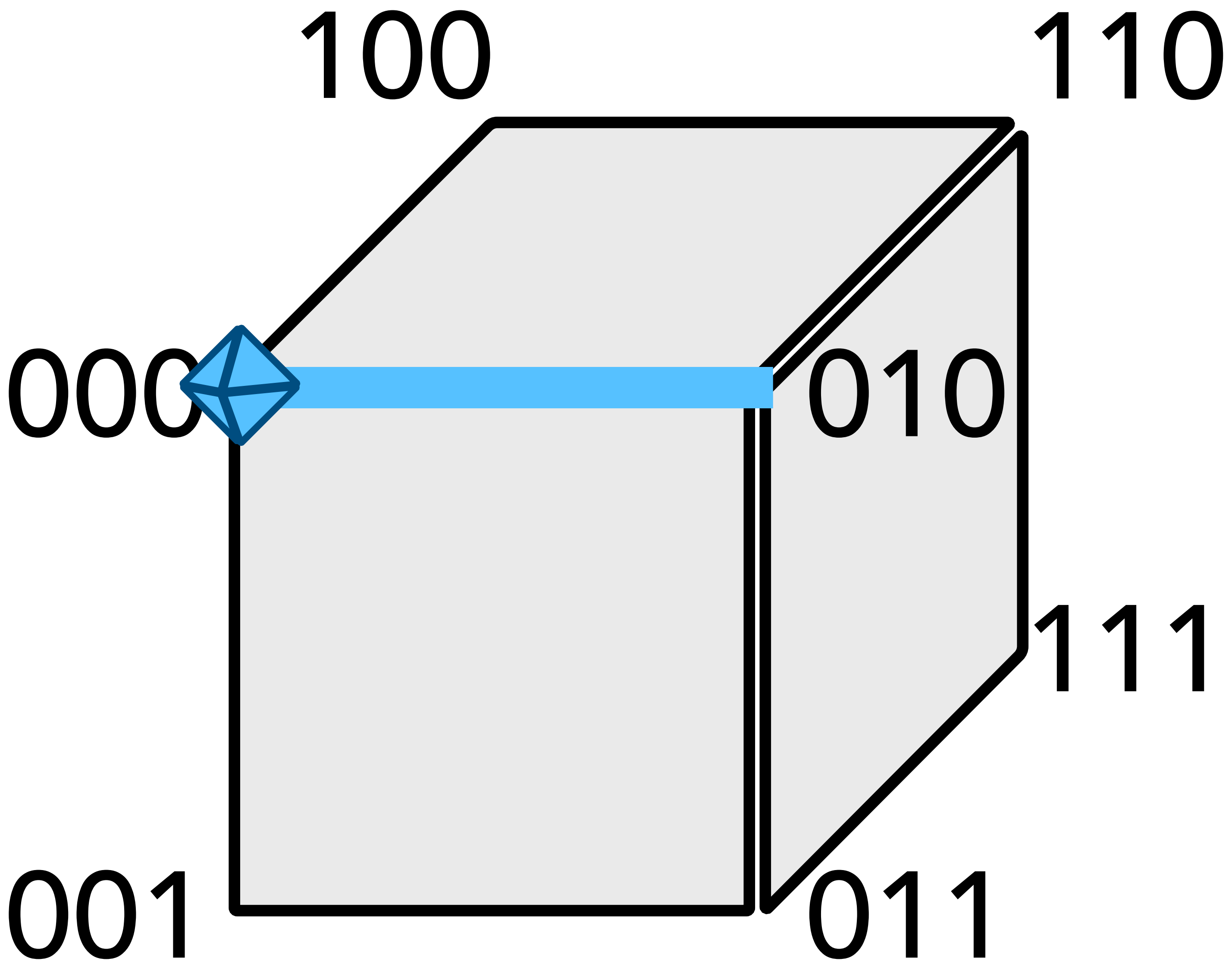
With prob.

$$\frac{\pi(b)}{\pi(b) + \pi(b')}$$



With prob.

$$\frac{\pi(b')}{\pi(b) + \pi(b')}$$

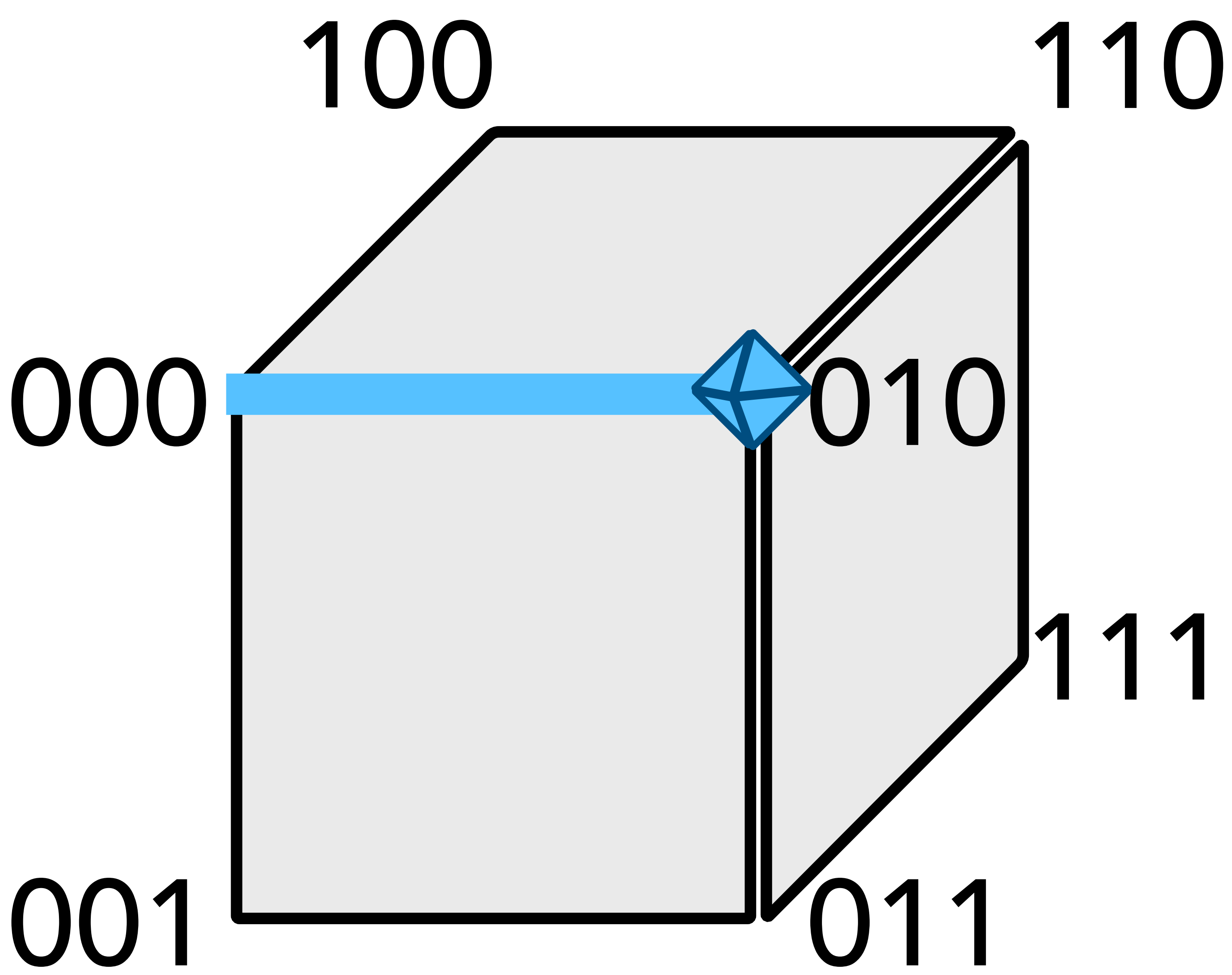


Relaxation Time

- Let $\pi(b) = |\langle b|\psi\rangle|^2$ be the **measurement distribution**.
- Let τ be the time the random talk takes to relax to stationary π.

With prob.

$$\frac{\pi(b)}{\pi(b) + \pi(b')}$$



With prob.

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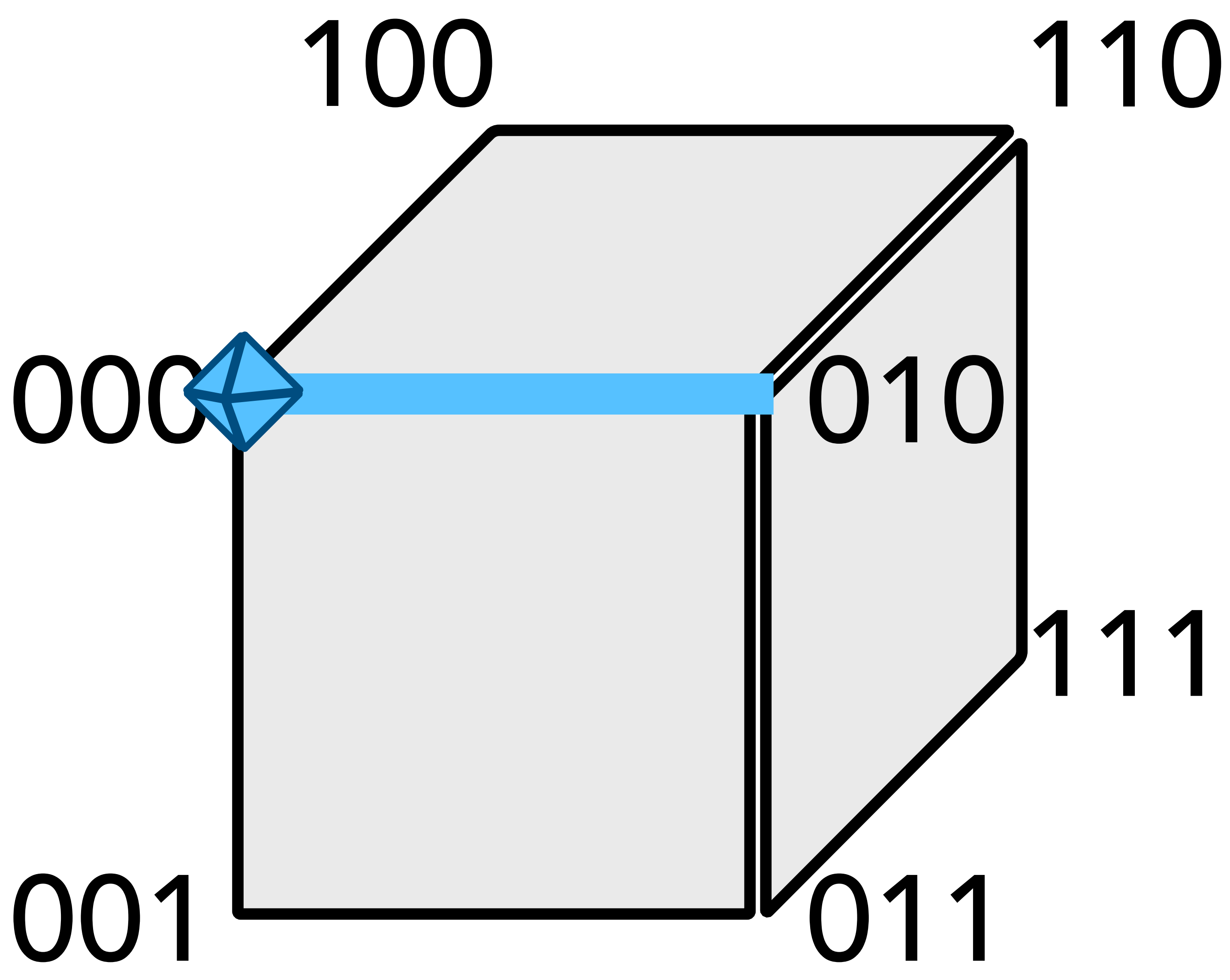

Certification

Theorem 2

For an n-qubit state $|\psi\rangle$ with relax. time τ, we can certify that ρ is close to $|\psi\rangle\langle\psi|$ with $\mathcal{O}(\tau)$ single-qubit measurements.

- When restricted to independent Pauli-basis measurements, we need $\mathcal{O}(\tau^2)$ single-qubit measurements.

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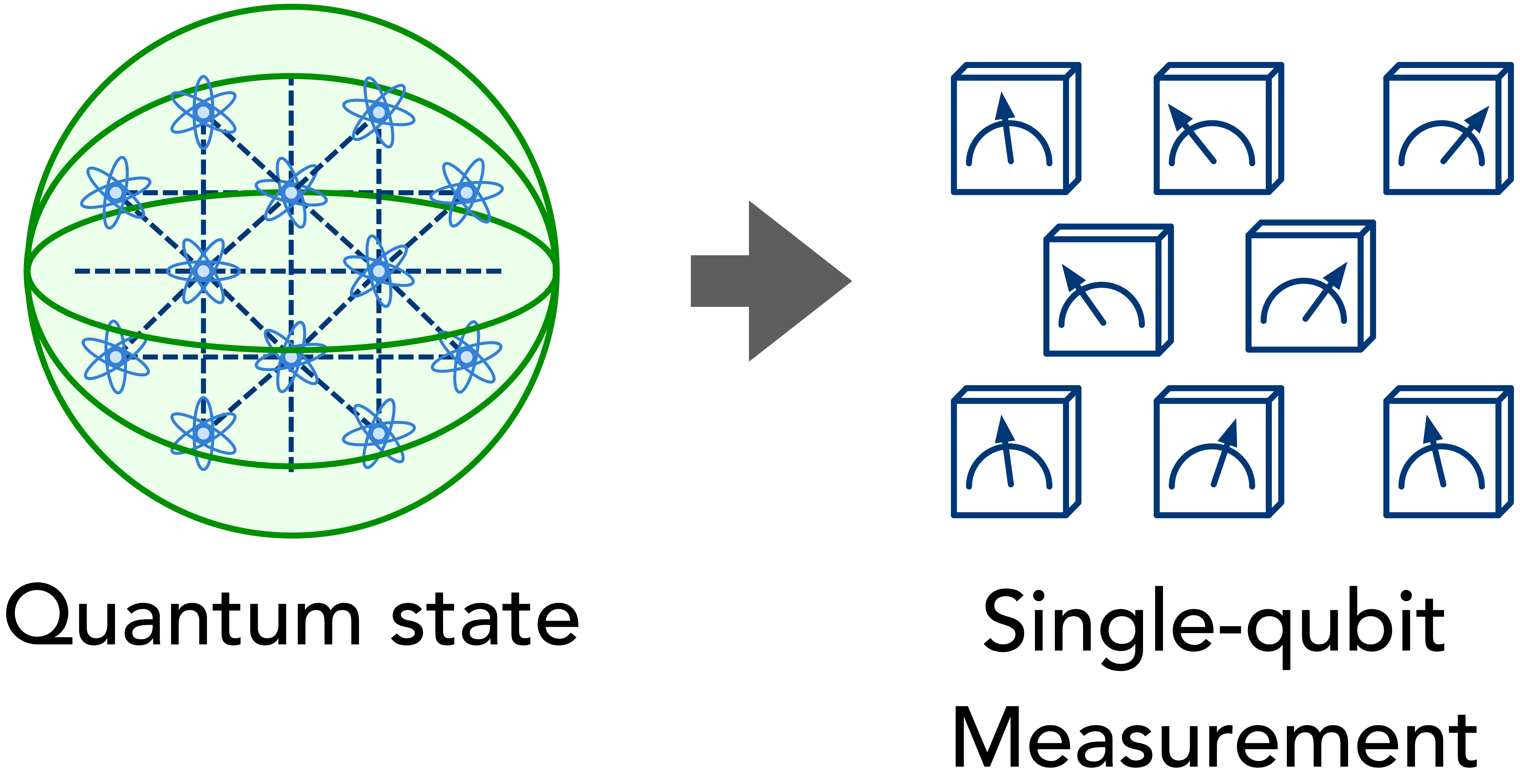
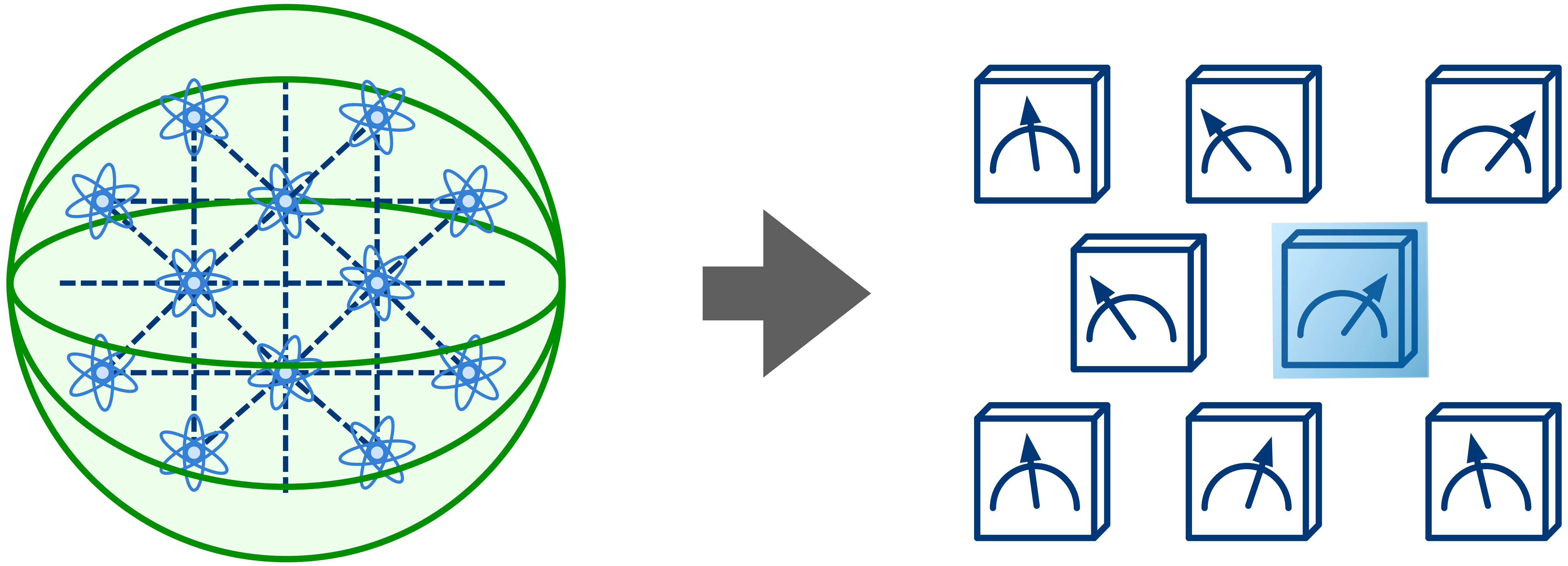
Measurement Protocol

- Repeat the following measurement a few times.



Measurement Protocol

- Pick a random qubit x.

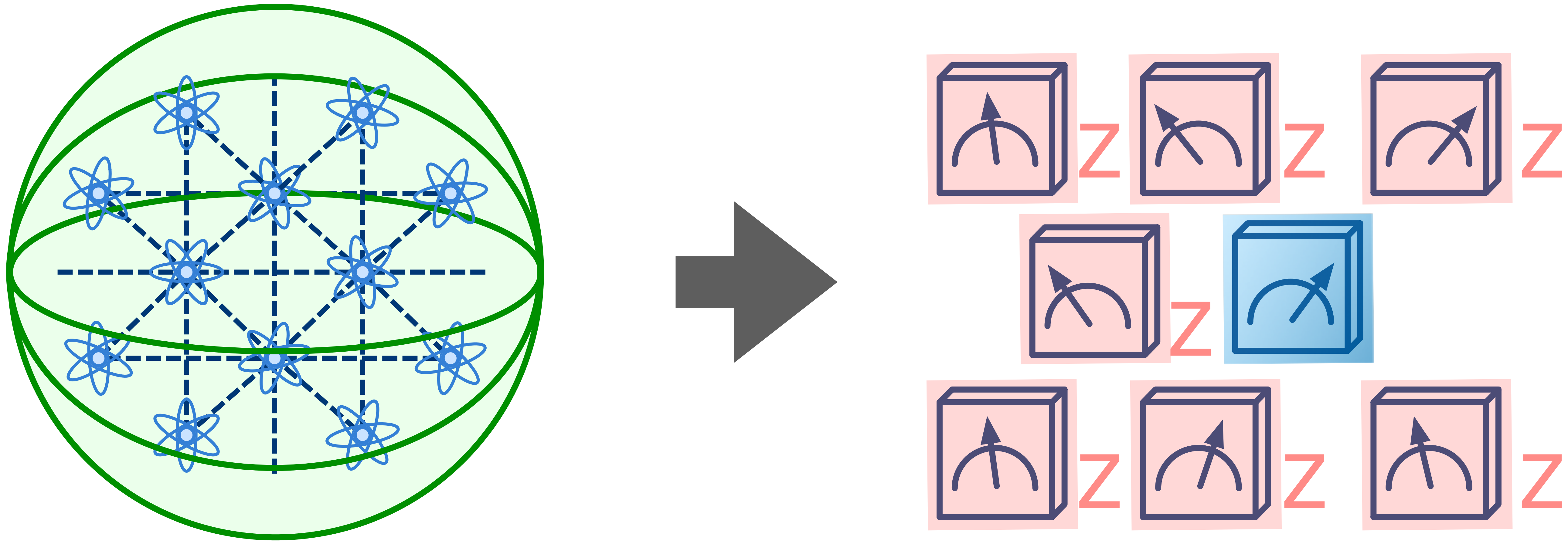


Quantum state

Single-qubit Measurement

Measurement Protocol

- Pick a random qubit x. Measure all except qubit x in Z basis.

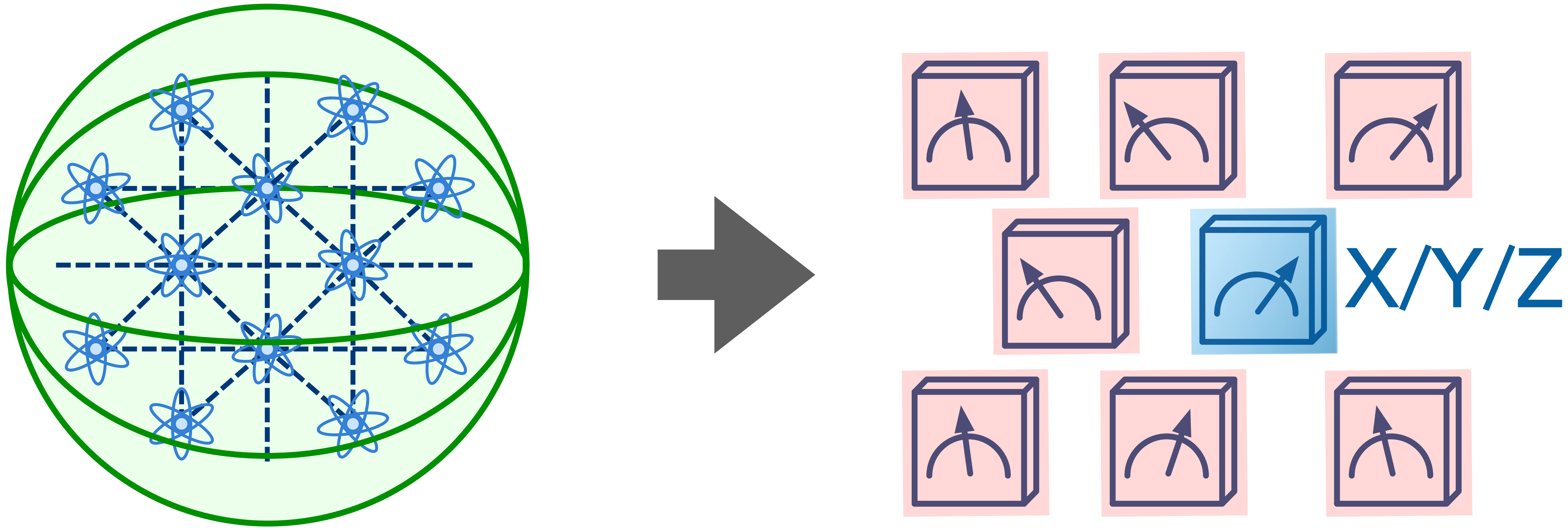


Quantum state

Single-qubit Measurement

Measurement Protocol

- Pick a random qubit x. Measure x in random X/Y/Z basis.

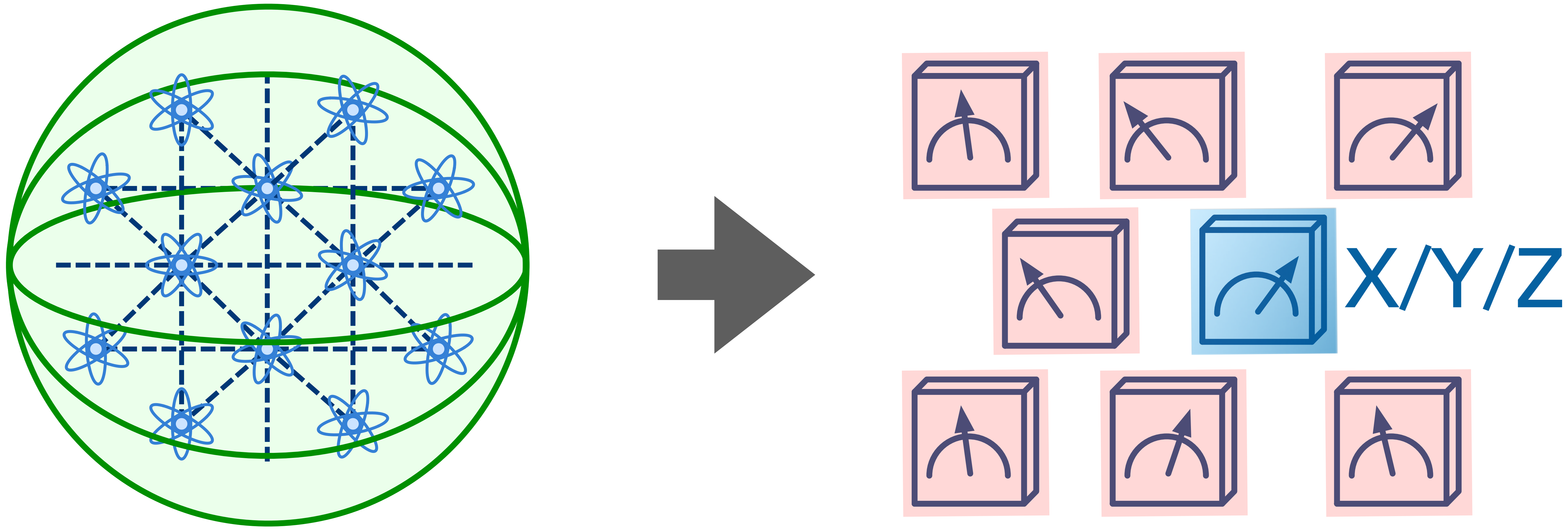


Quantum state

Single-qubit Measurement

Measurement Protocol

- That's it.

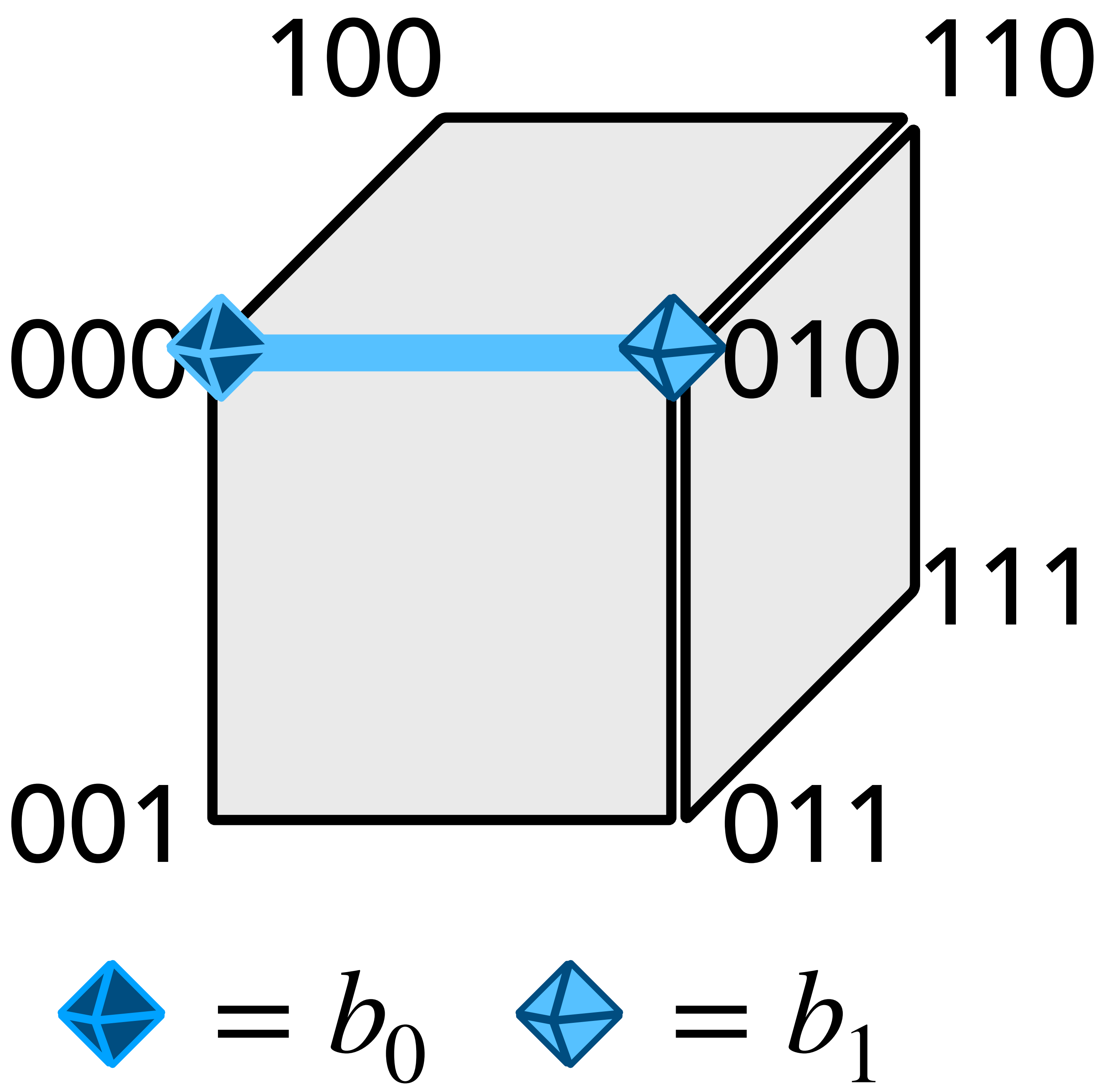


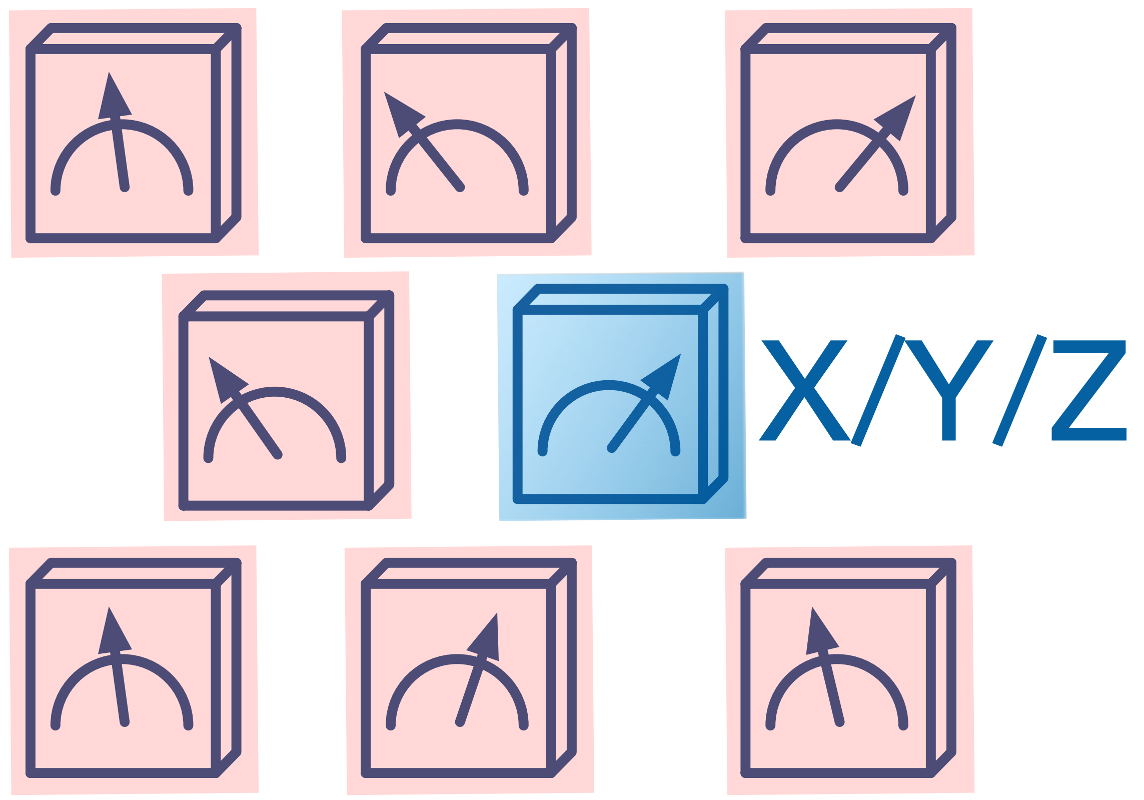
Quantum state

Single-qubit Measurement

Postprocessing

- The measurement outcomes on  specifies an edge (b_0, b_1) on the Boolean hypercube.

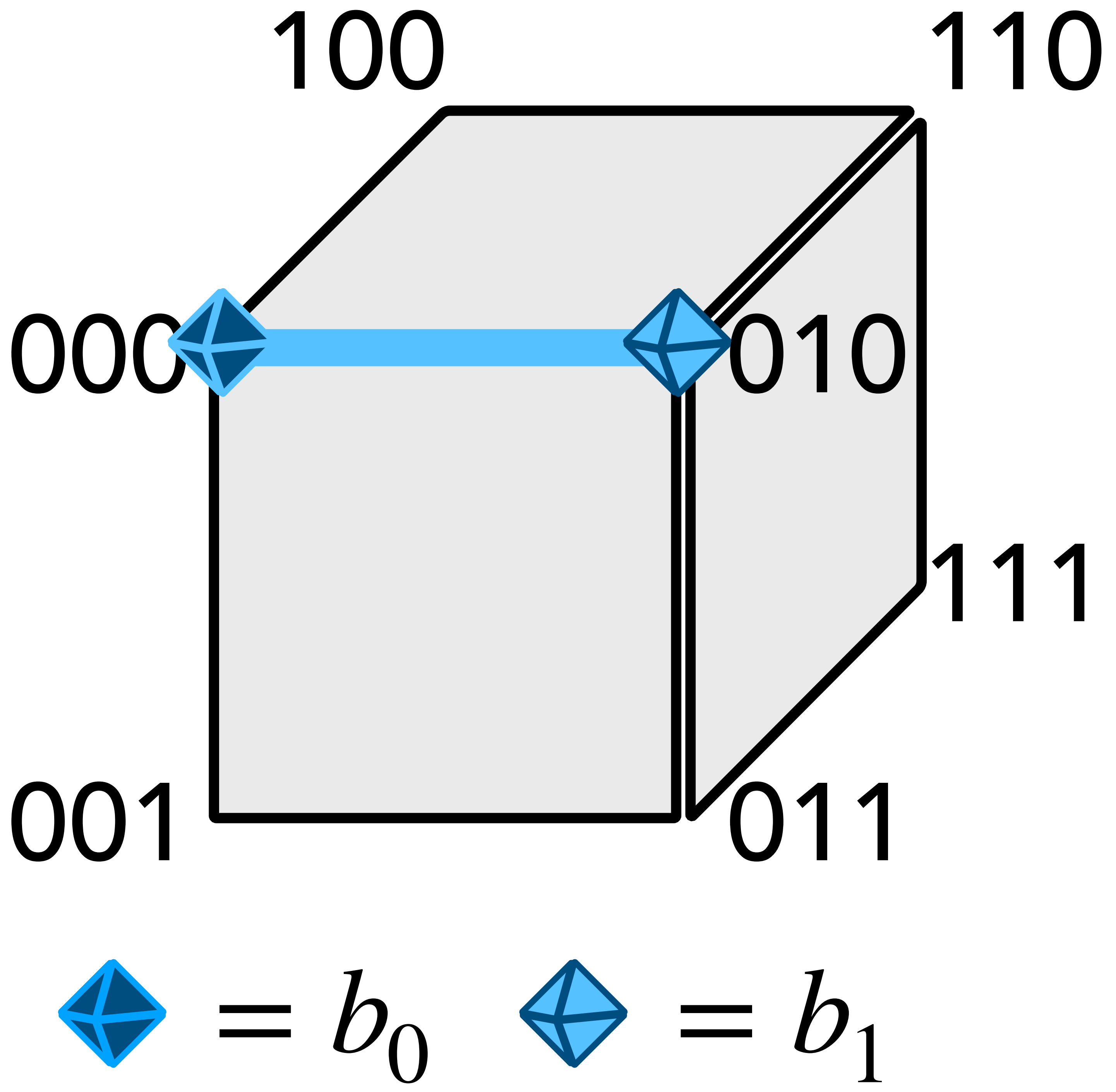


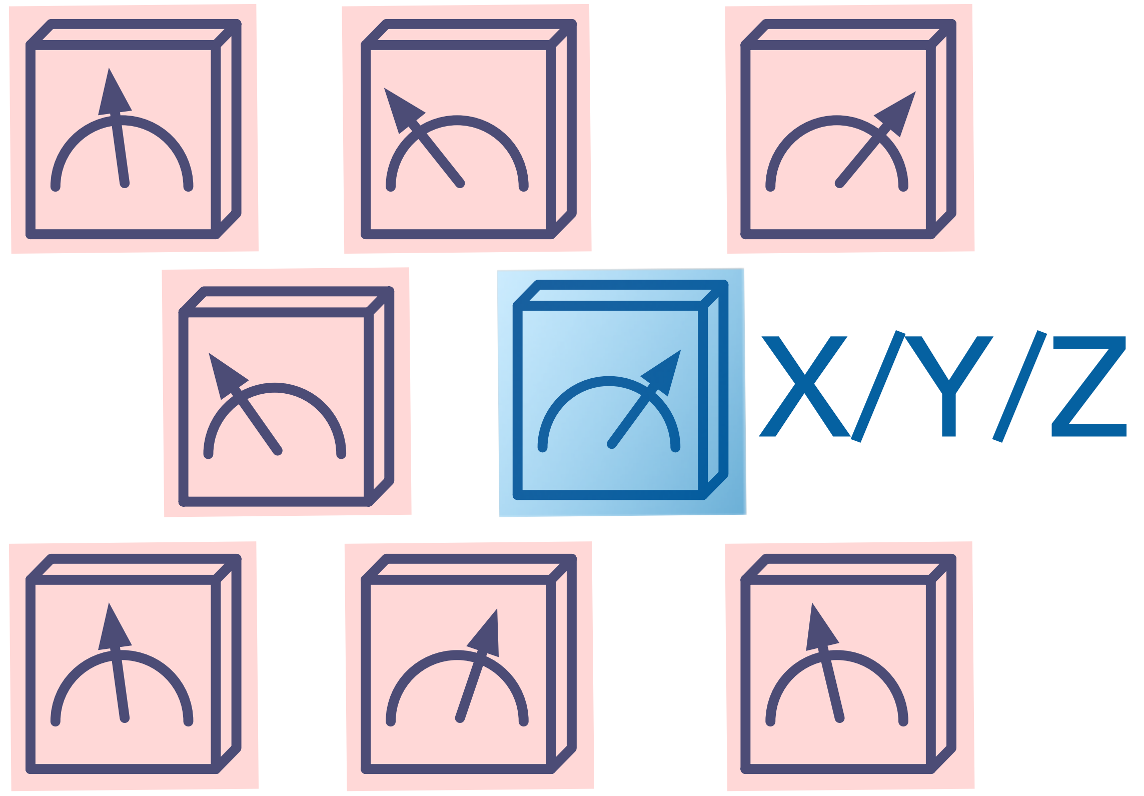


Single-qubit Measurement

Postprocessing

- The **ideal** post-measurement 1-qubit state $|\psi_{b_0,b_1}\rangle$ on **qubit** x is proportional to $\langle b_0|\psi\rangle|0\rangle + \langle b_1|\psi\rangle|1\rangle$.

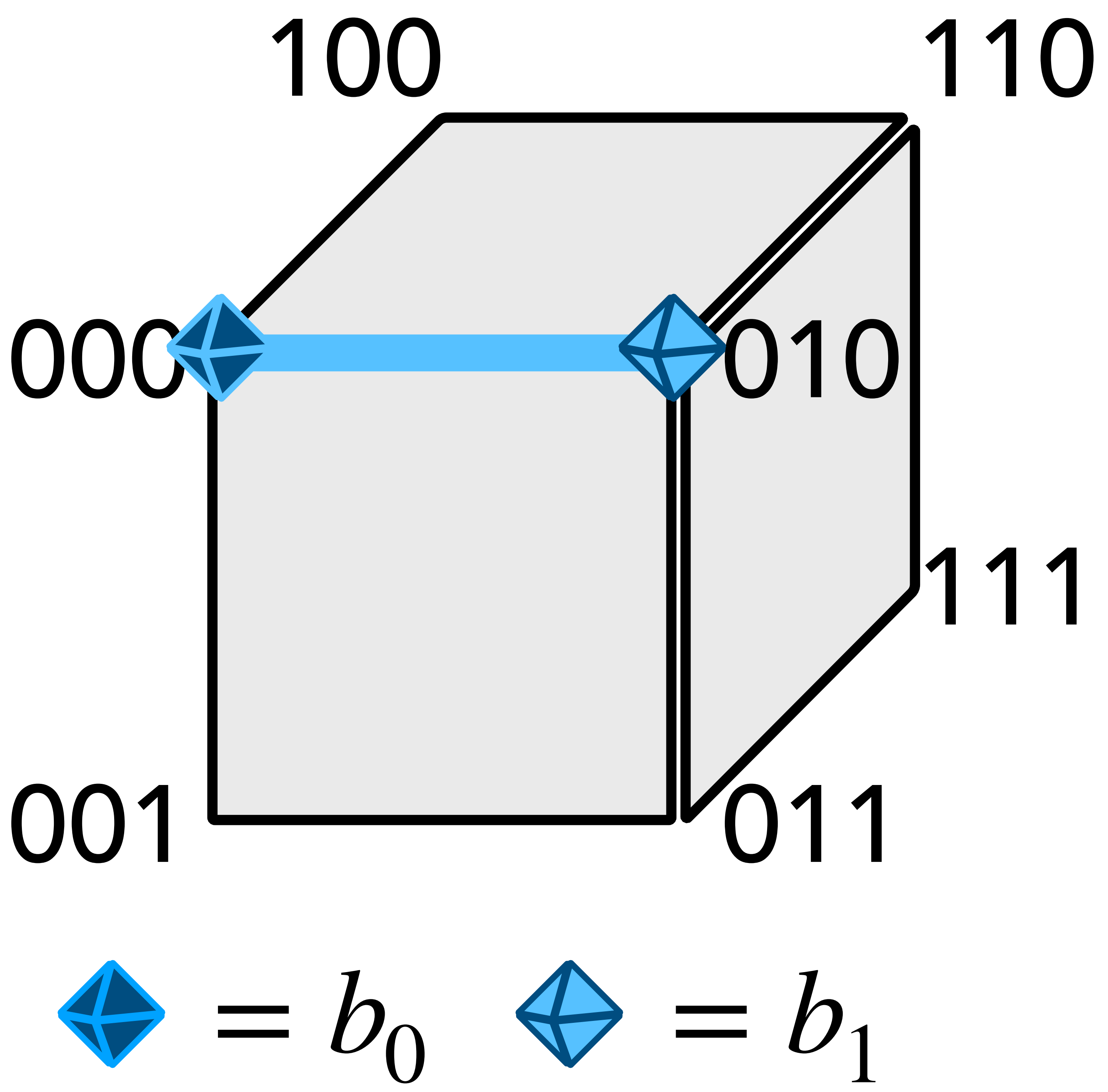


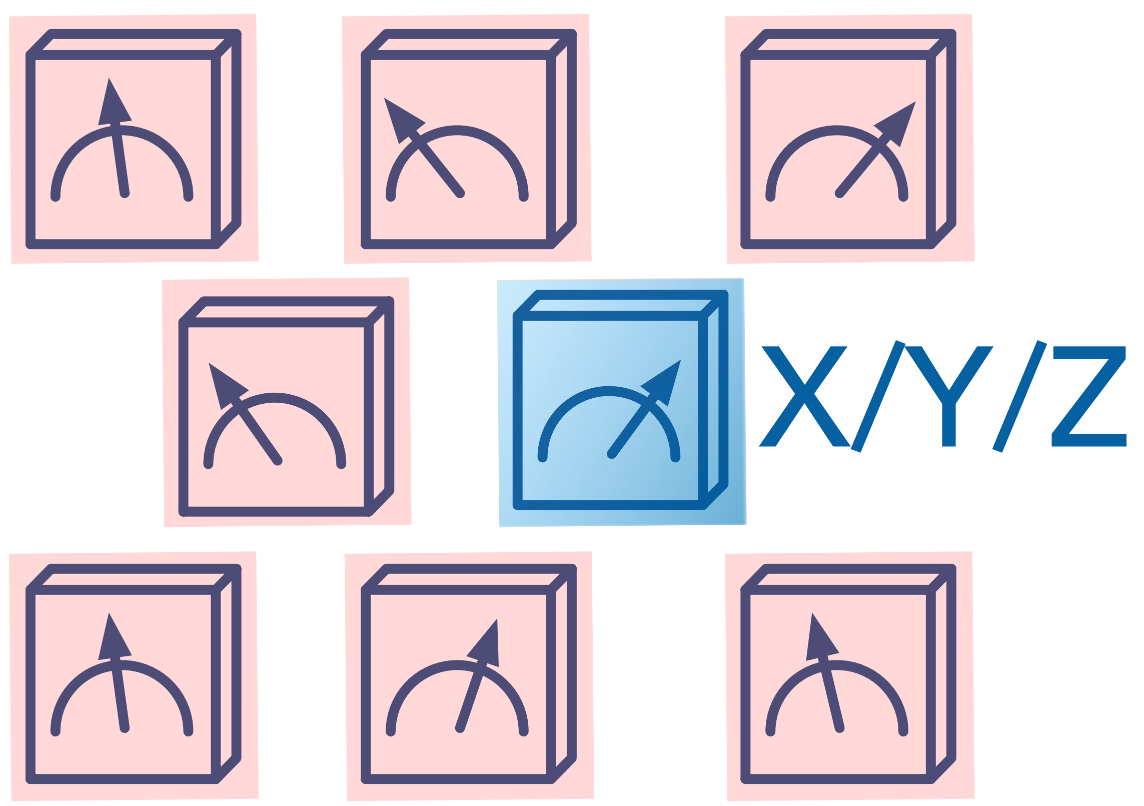


Single-qubit Measurement

Postprocessing

- Use randomized Pauli measurement (classical shadow) on **qubit x** to predict the fidelity ω with the **ideal** 1-qubit state $|\psi_{b_0,b_1}\rangle$.





Single-qubit Measurement

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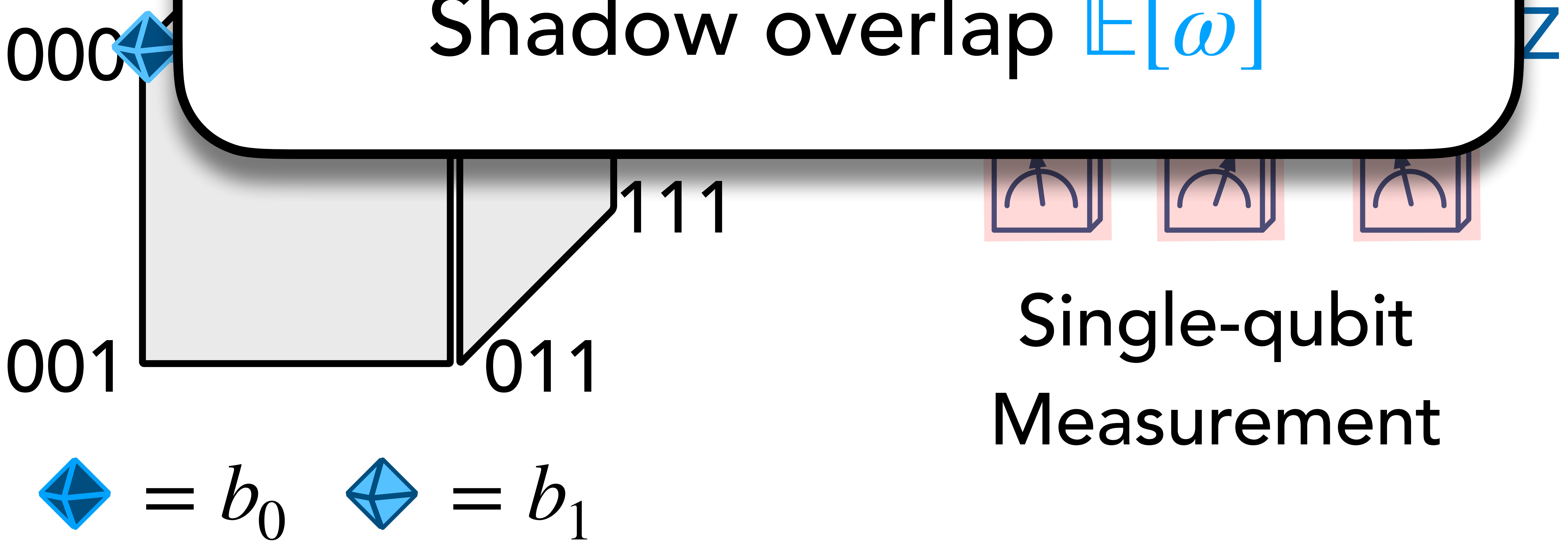
Average over ω to get
Shadow overlap $\mathbb{E}[\omega]$



Key Feature

Shadow overlap $\mathbb{E}[\omega]$ accurately tracks the fidelity $\langle\psi|\rho|\psi\rangle$.

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$$\mathbb{E}[\omega] \geq 1 - \epsilon \text{ implies } \langle \psi | \rho | \psi \rangle \geq 1 - \tau\epsilon$$

$$\langle \psi | \rho | \psi \rangle \geq 1 - \epsilon \text{ implies } \mathbb{E}[\omega] \geq 1 - \epsilon$$

τ is the time the random talk takes to relax to stationary π

Physical Intuition

Shadow overlap $\mathbb{E}[\omega] = \frac{1}{n} \sum_{i=1}^{n} \sum_{b_{\neq i} \in \{0,1\}^{n-1}} \mathrm{Tr}\left(\langle b_{\neq i} | \rho | b_{\neq i} \rangle \frac{\langle b_{\neq i} | \psi \rangle\langle \psi | b_{\neq i} \rangle}{\mathrm{Tr} \langle b_{\neq i} | \psi \rangle\langle \psi | b_{\neq i} \rangle} \right)$

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- $|+\ldots+\rangle\langle+\ldots+|$ and $|-\ldots-\rangle\langle-\ldots-|$ has fidelity 0.
- $|+\ldots+\rangle\langle+\ldots+|$ and $|-\ldots-\rangle\langle-\ldots-|$ has $\mathbb{E}[\omega]$= 0.

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- $|+\ldots+\rangle\langle+\ldots+|$ and $|+\ldots+-\rangle\langle+\ldots+-|$ has $\mathbb{E}[\omega]= \frac{n-1}{n}$.
- Shadow overlap has a Hamming distance nature.

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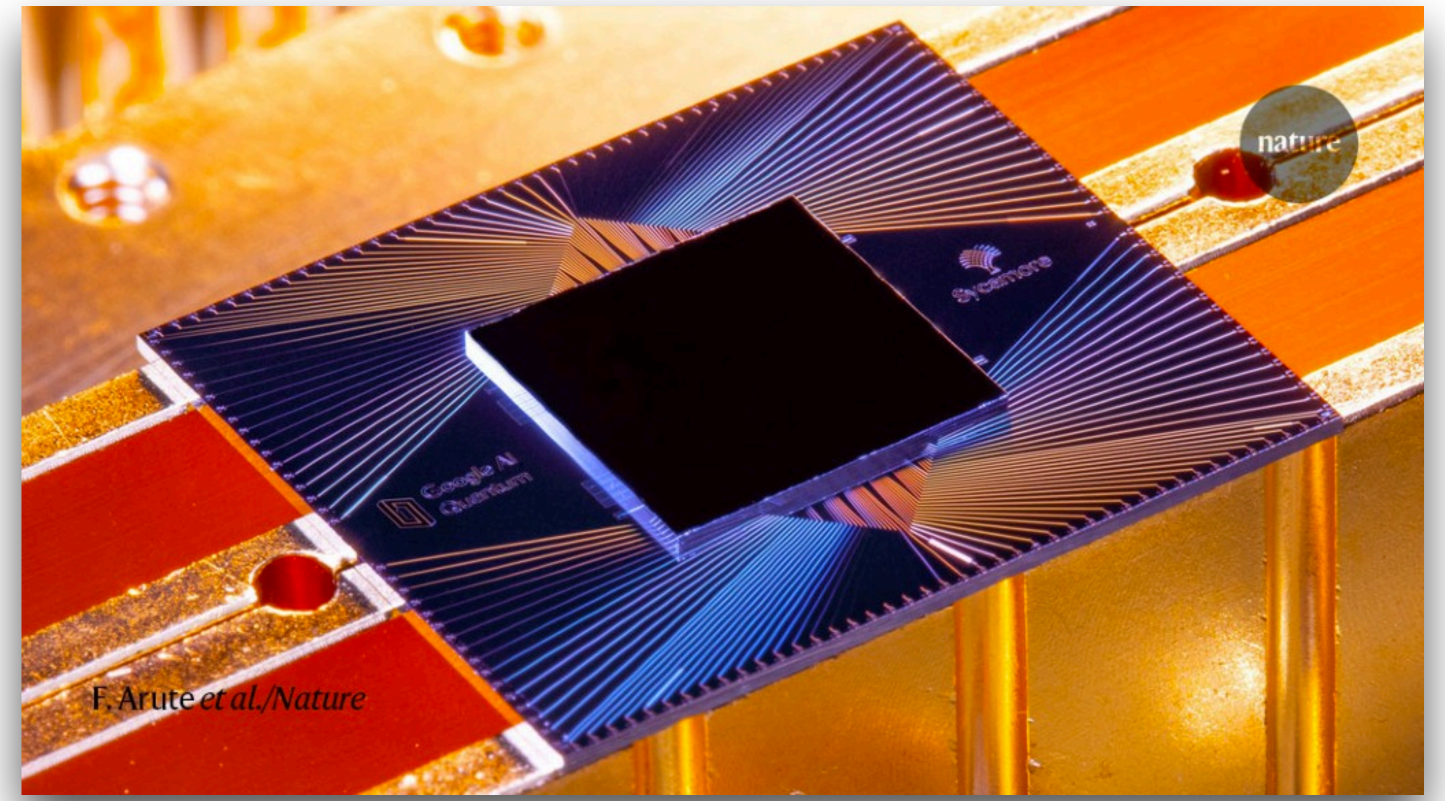
Example 1

Benchmarking

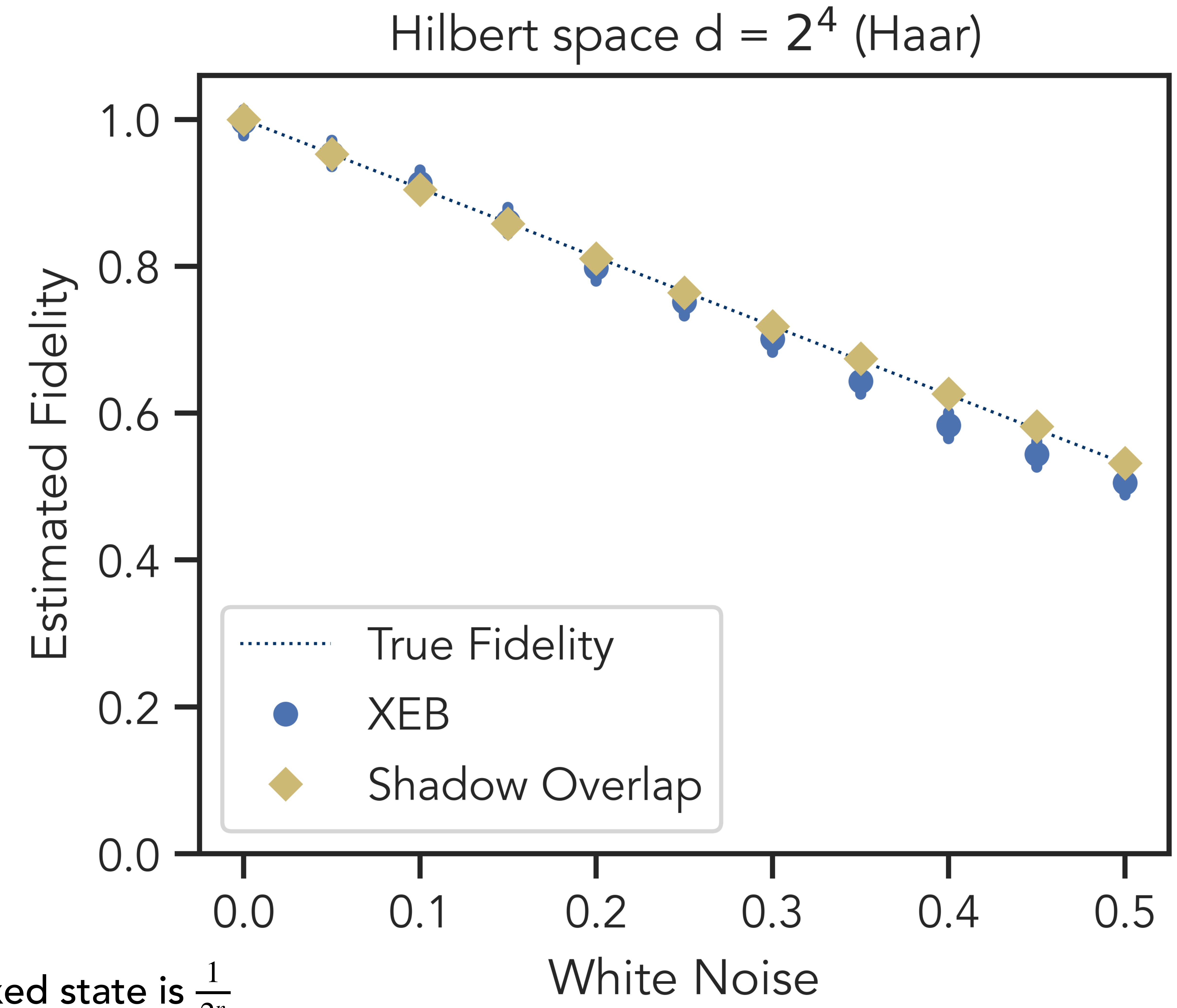
Shadow overlap $\mathbb{E}[\omega]$ certifies if the state has a high fidelity



Benchmarking quantum devices

4-qubit Haar random state

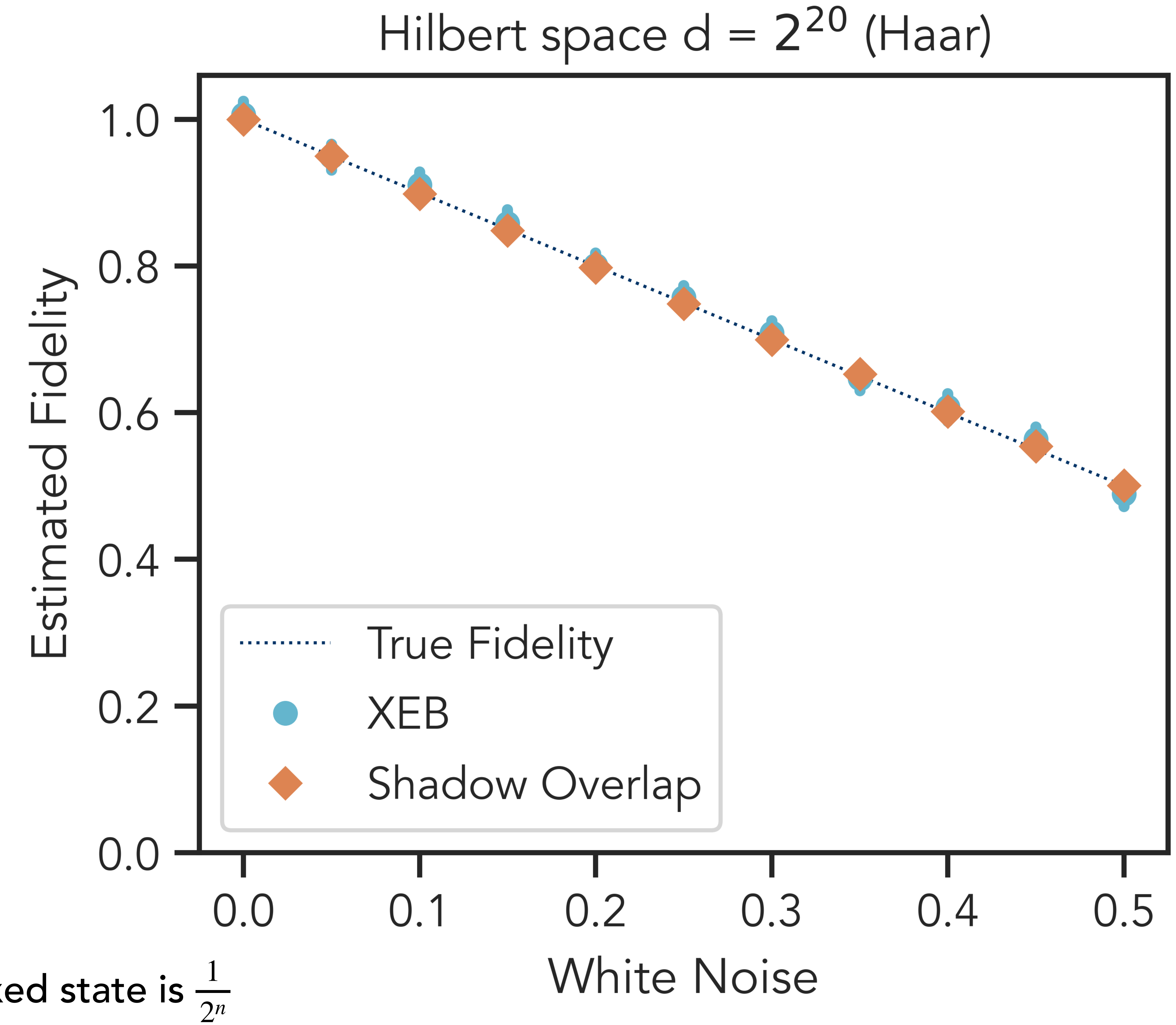
White Noise



*Shadow overlap normalized s.t., target state is 1, maximally mixed state is $\frac{1}{2^n}$

Benchmarking quantum devices

20-qubit Haar random state
White Noise



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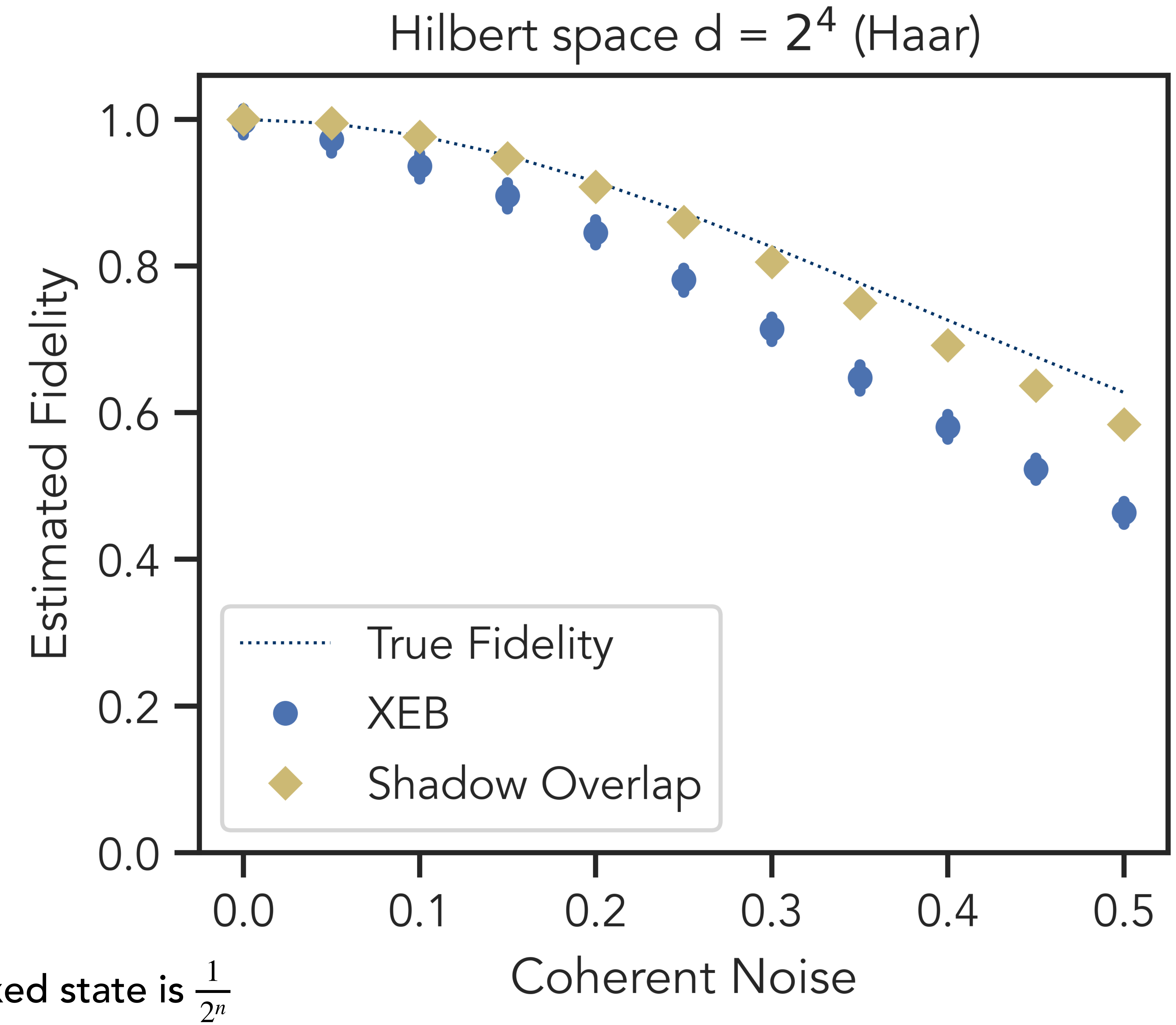
Benchmarking quantum devices

4-qubit Haar random state

Coherent Noise



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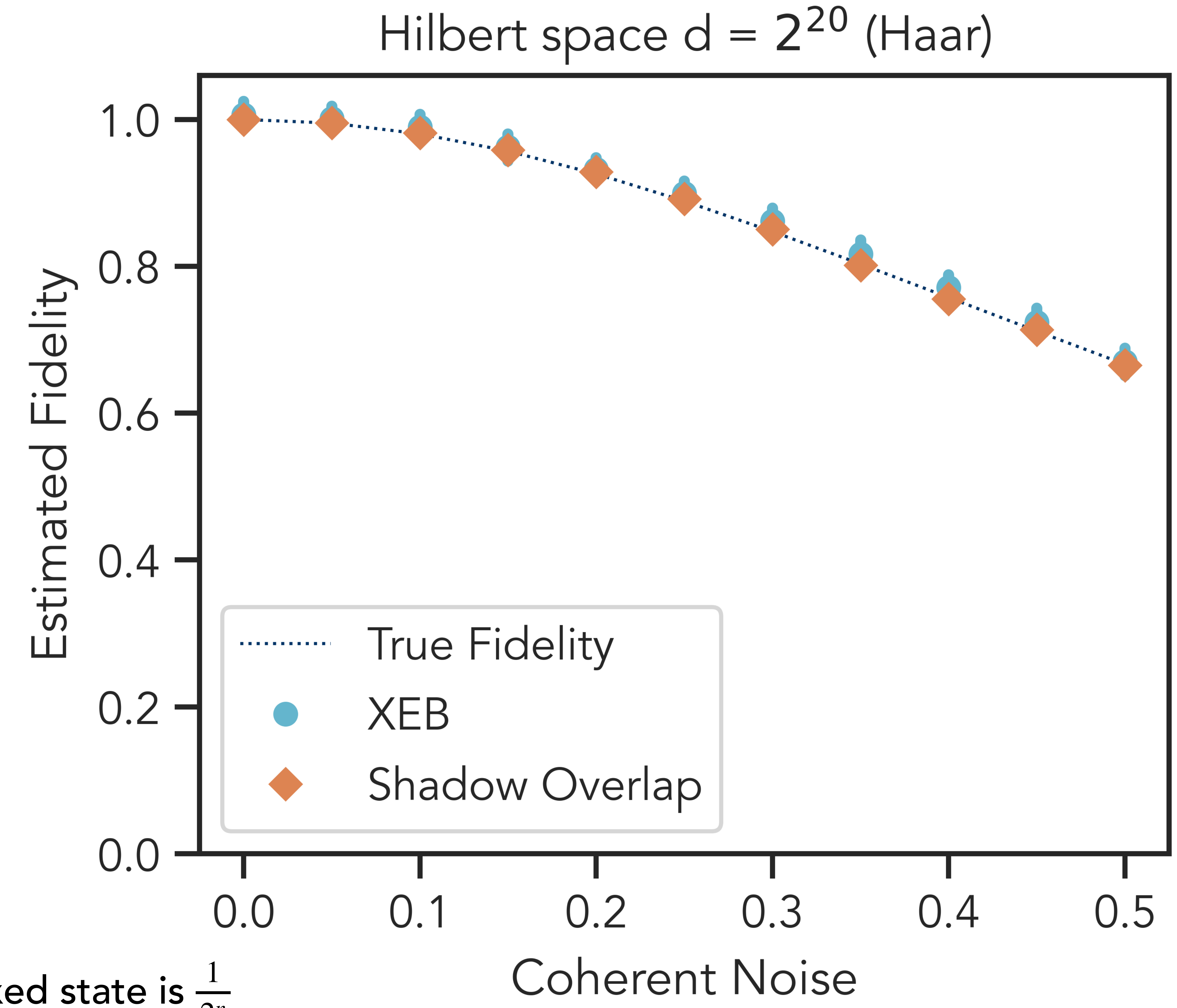

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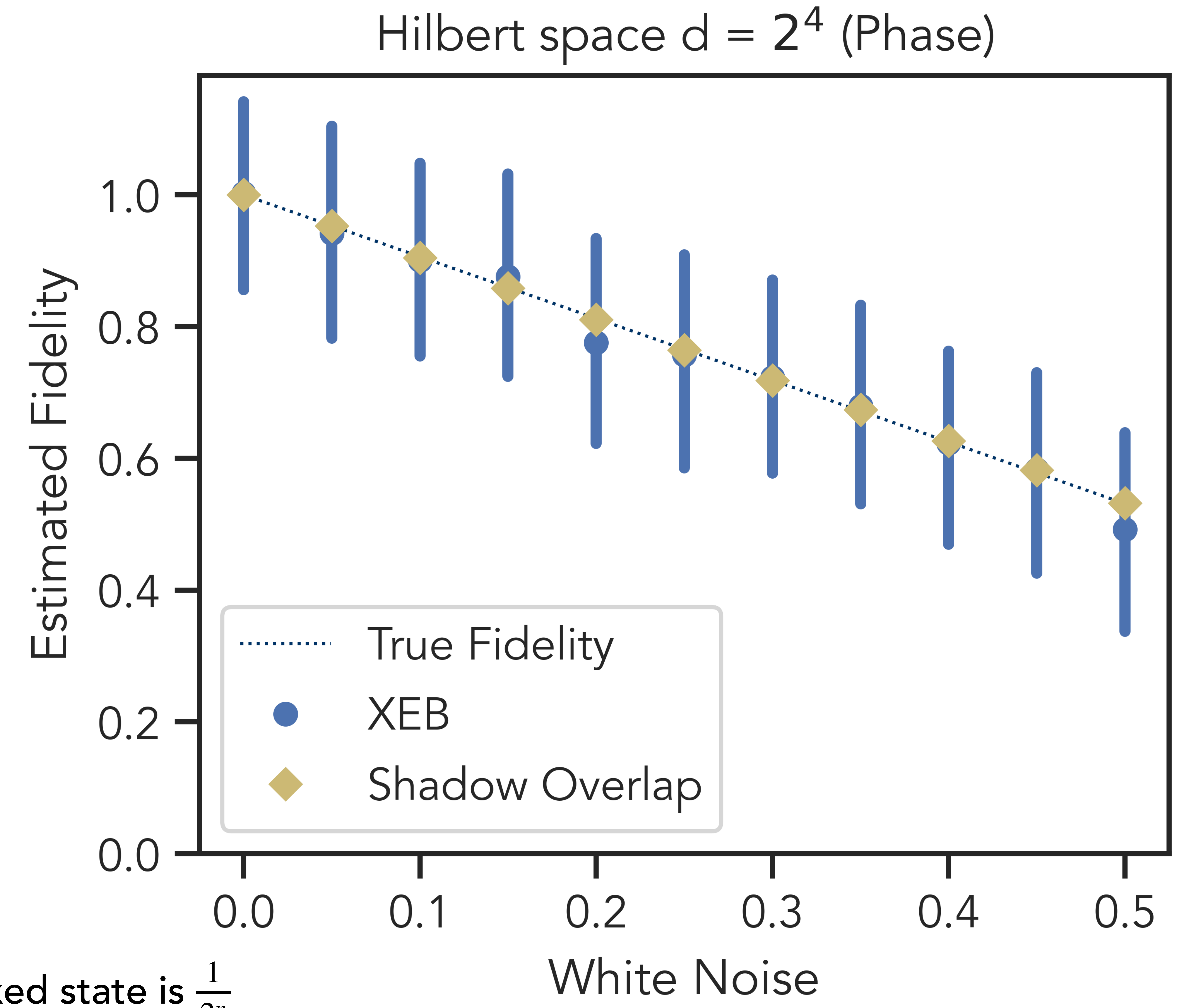
Benchmarking quantum devices

4-qubit random structured state

White Noise

$$|\psi\rangle = U_{\text{phase}} \bigotimes_{i=1}^{4} |\psi_i\rangle$$



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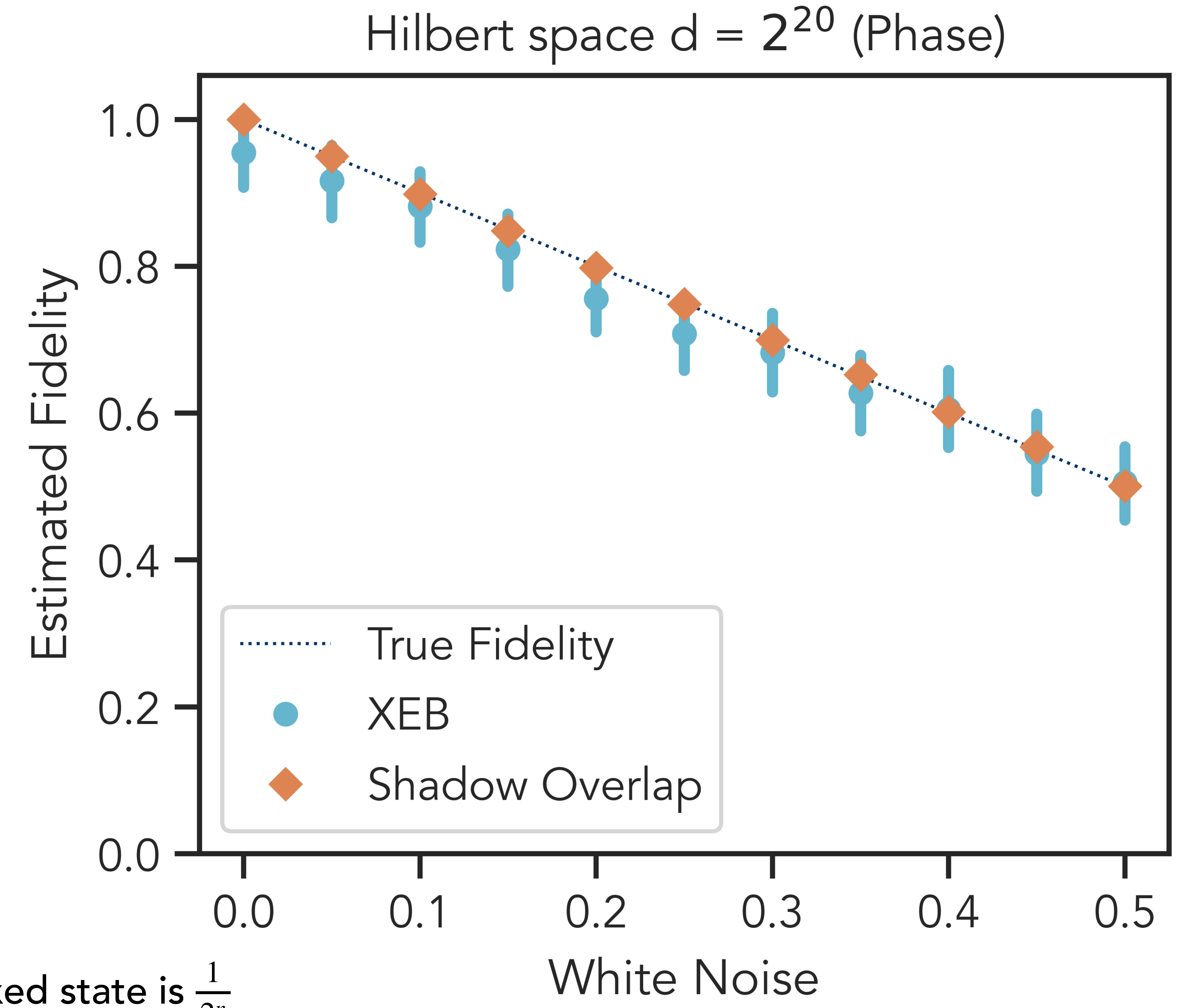
Benchmarking quantum devices

20-qubit random structured state
White Noise

$$|\psi\rangle = U_{\text{phase}} \bigotimes_{i=1}^{20} |\psi_i\rangle$$



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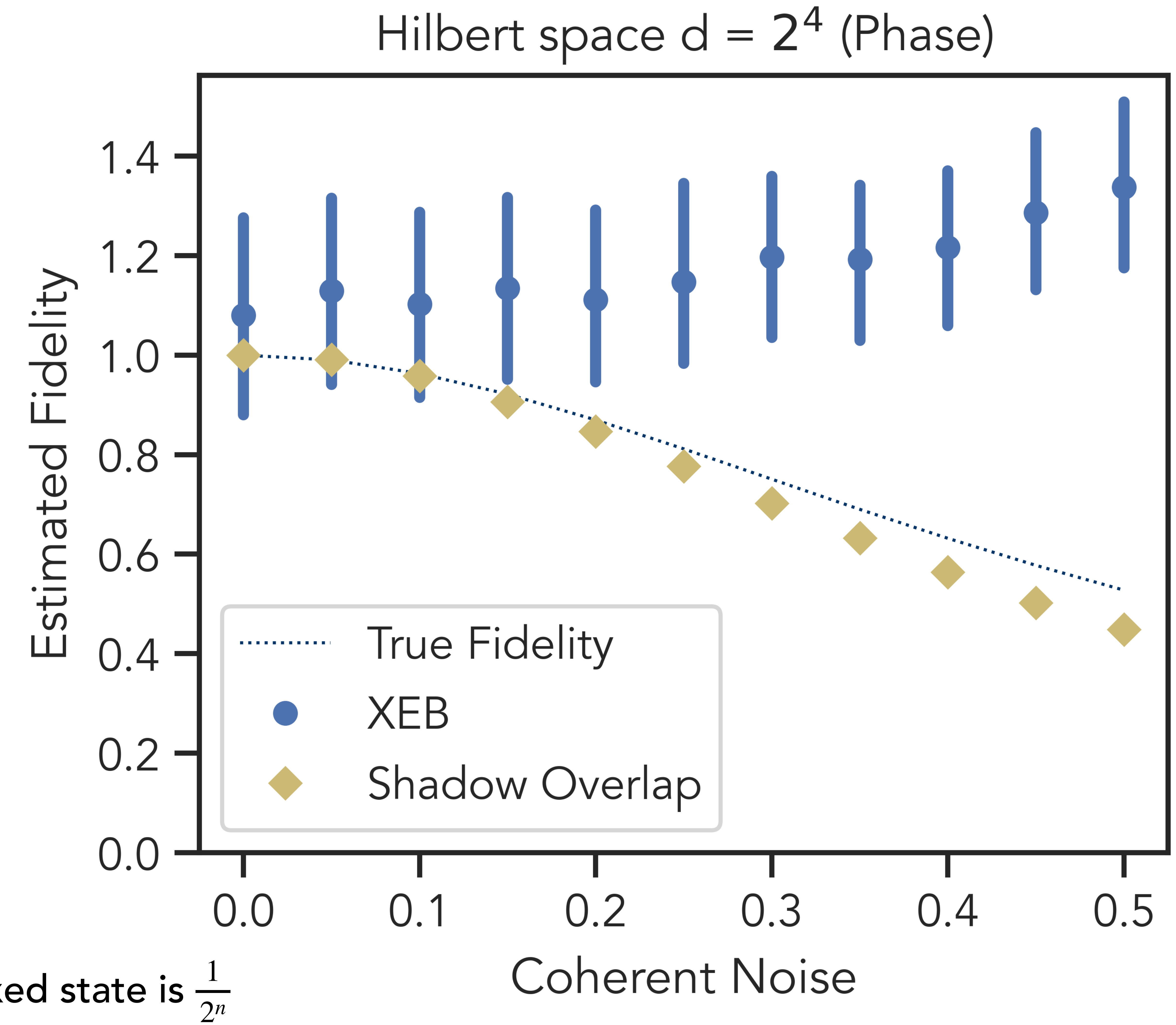
Benchmarking quantum devices

4-qubit random structured state

Coherent Noise

$$|\psi\rangle = U_{\text{phase}} \bigotimes_{i=1}^{4} |\psi_i\rangle$$



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Benchmarking quantum devices

20-qubit random structured state

Coherent Noise

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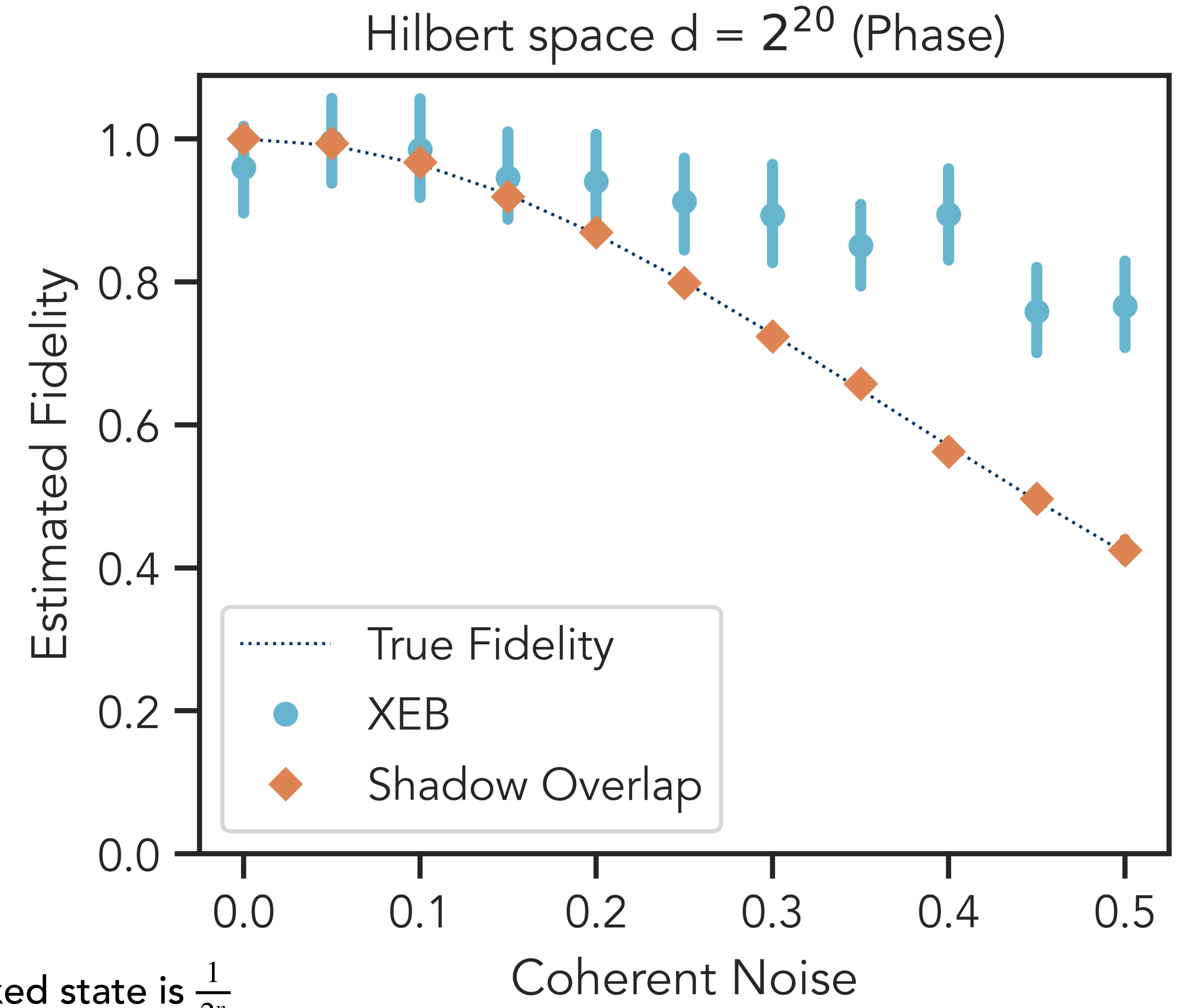

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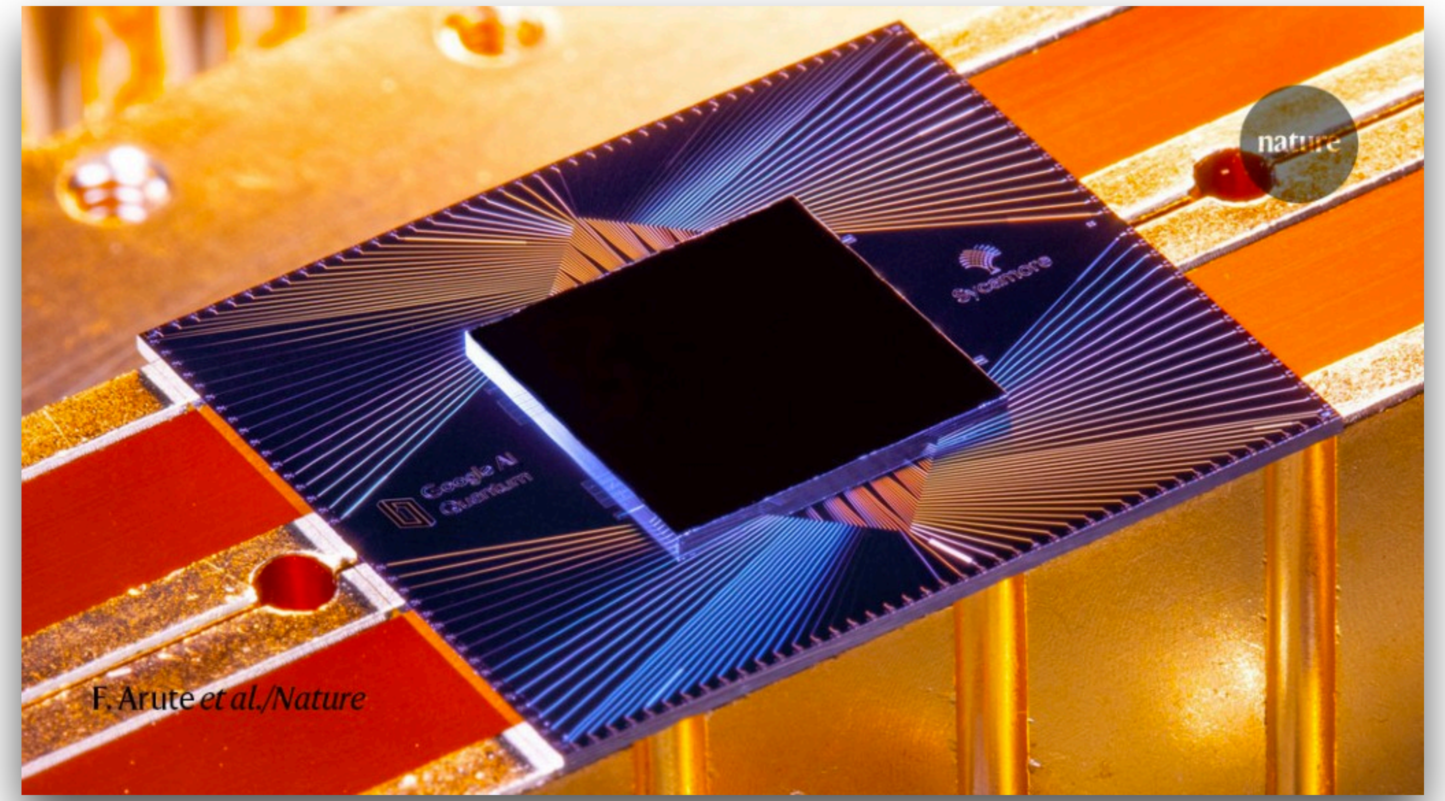
Applications

What can we use this new certification protocol for?

Example 1

Benchmarking

Shadow overlap $\mathbb{E}[\omega]$ certifies if the state has a high fidelity



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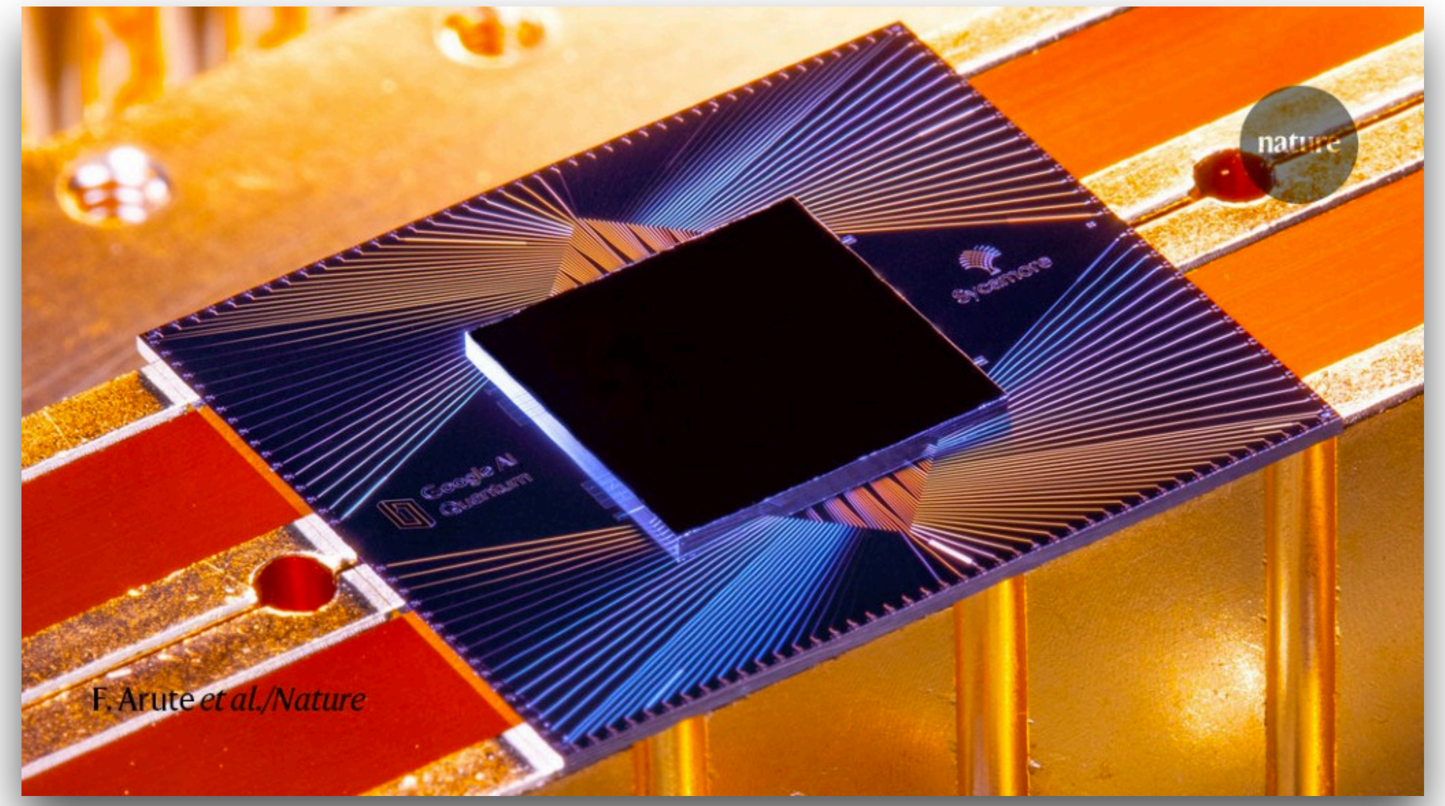
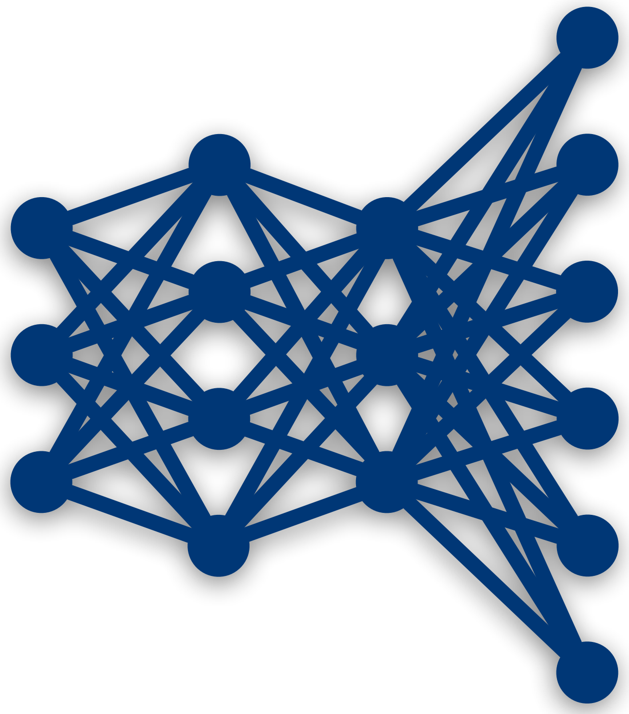


Example 2

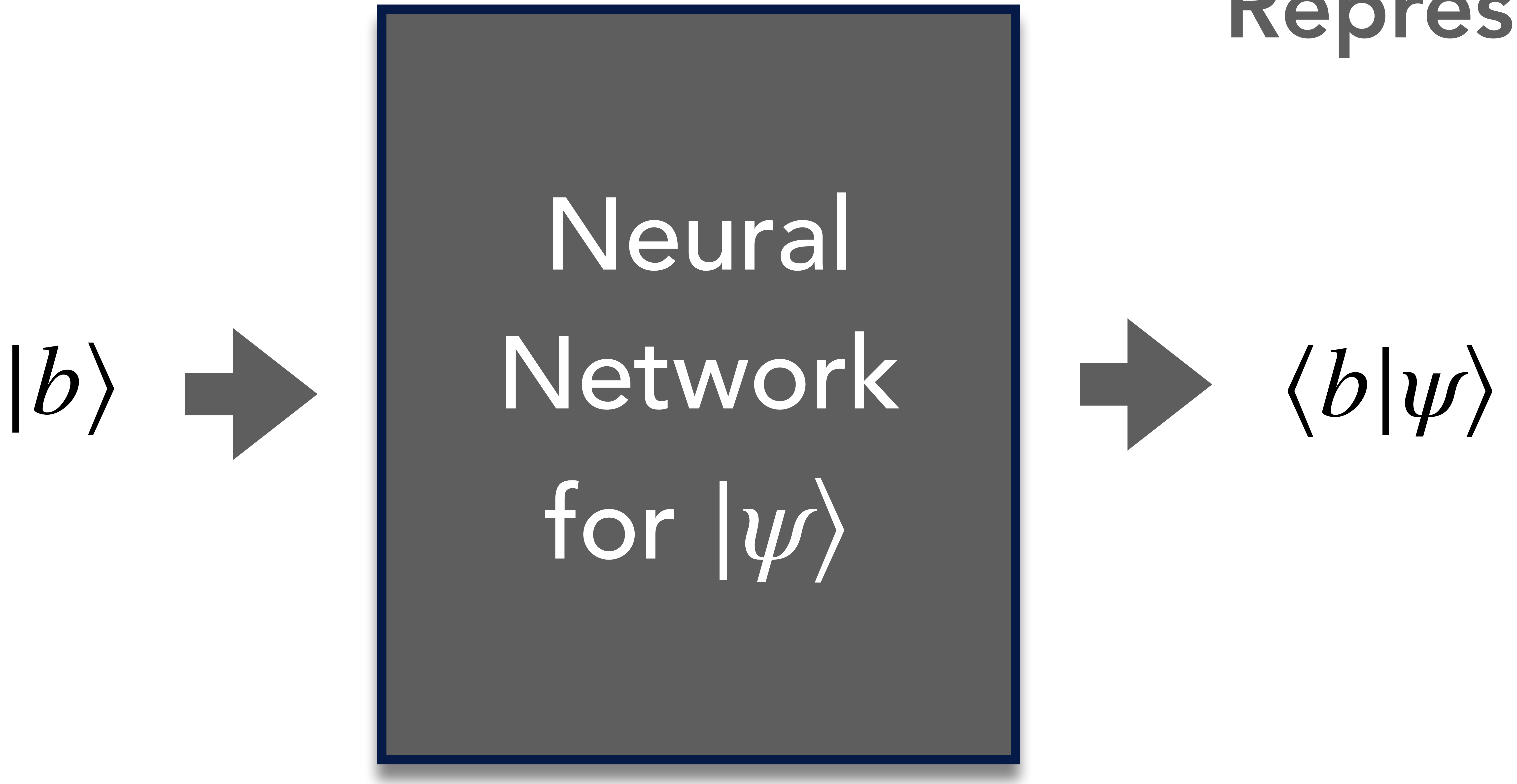
ML tomography

Train/certify ML models, such as neural quantum states, using shadow overlap $\mathbb{E}[\omega]$



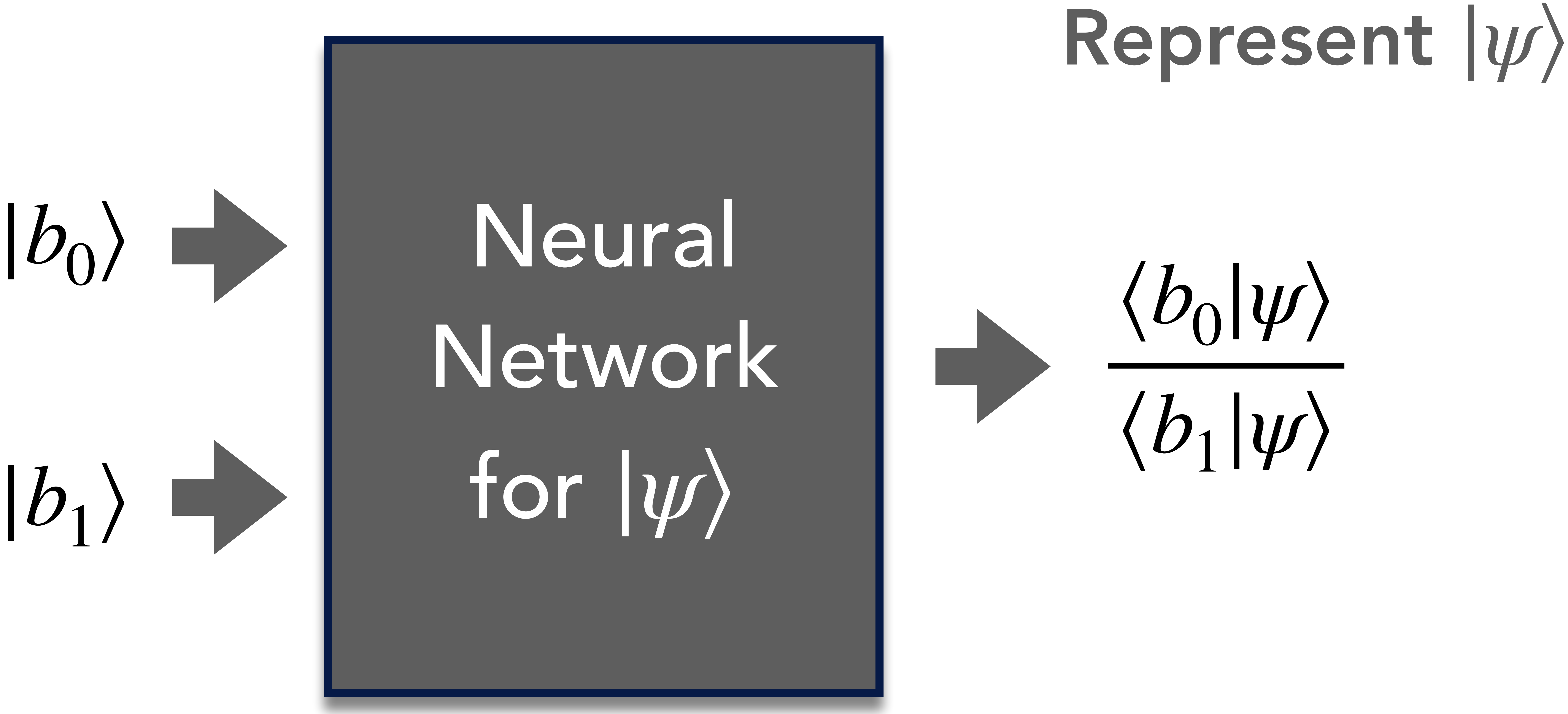
Training/Certifying NN tomography

Represent $|\psi\rangle$



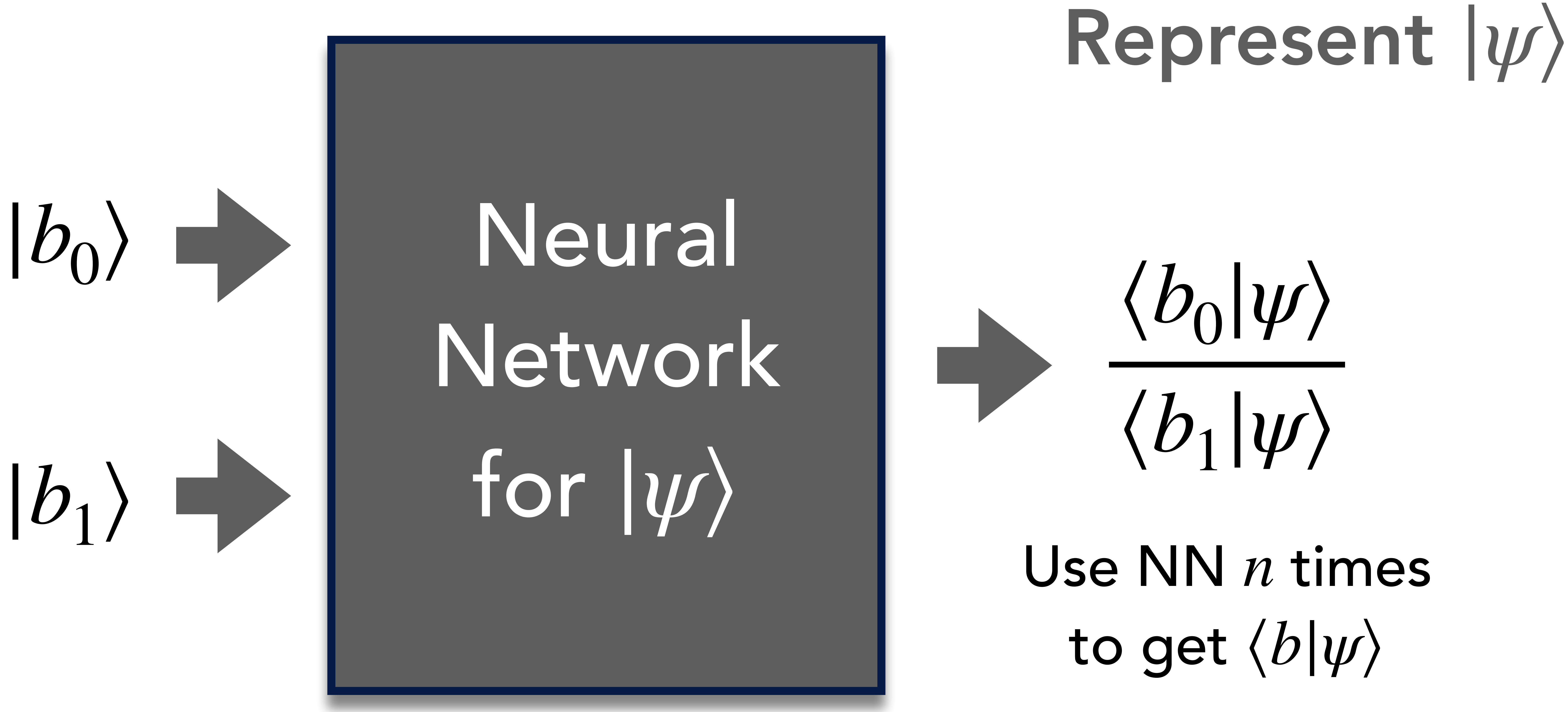
Standard Neural Quantum State

Training/Certifying NN tomography



Relative Neural Quantum State

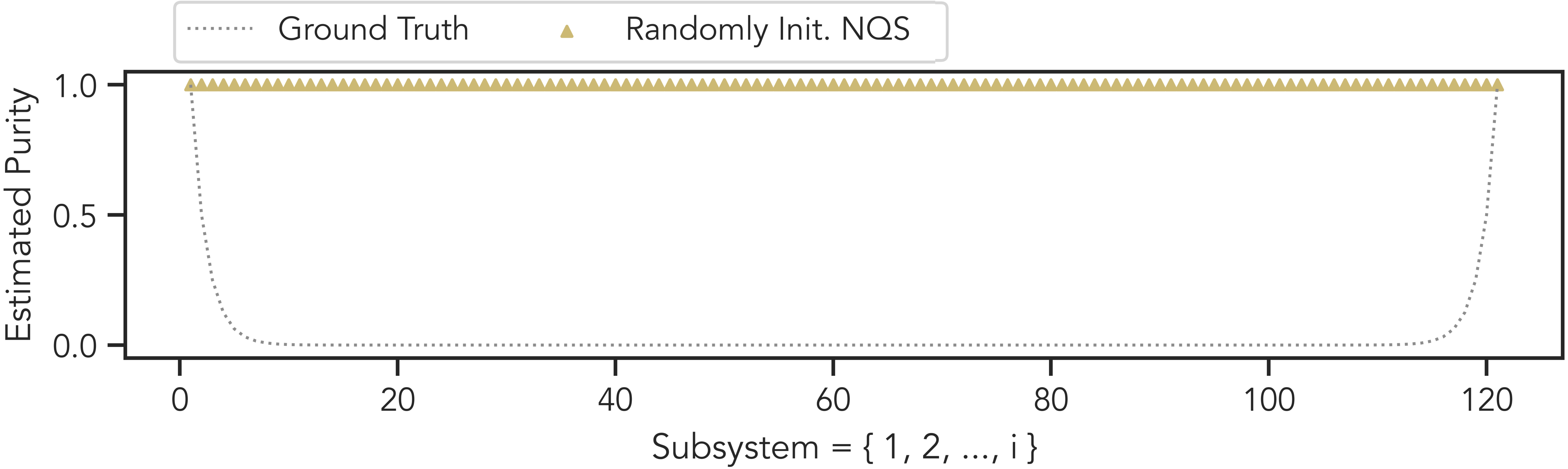
Training/Certifying NN tomography



Relative Neural Quantum State

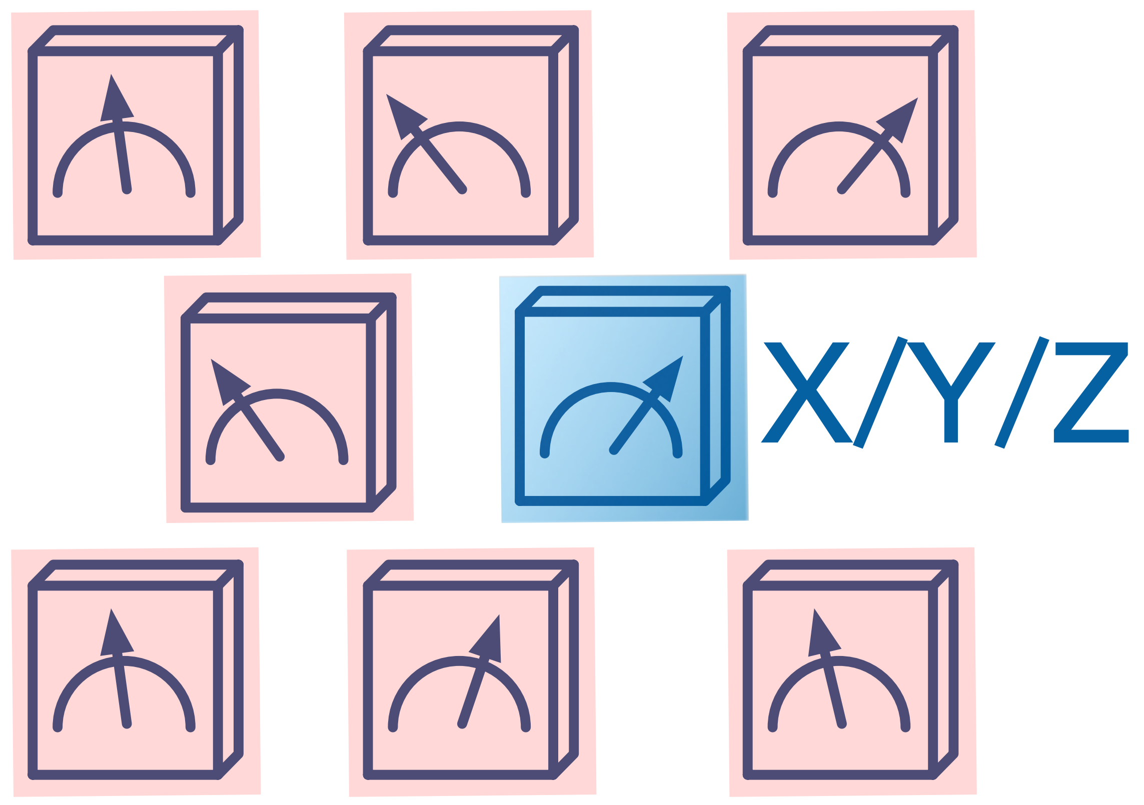
Training/Certifying NN tomography

We consider learning a class of 120-qubit states with exponentially high circuit complexity.

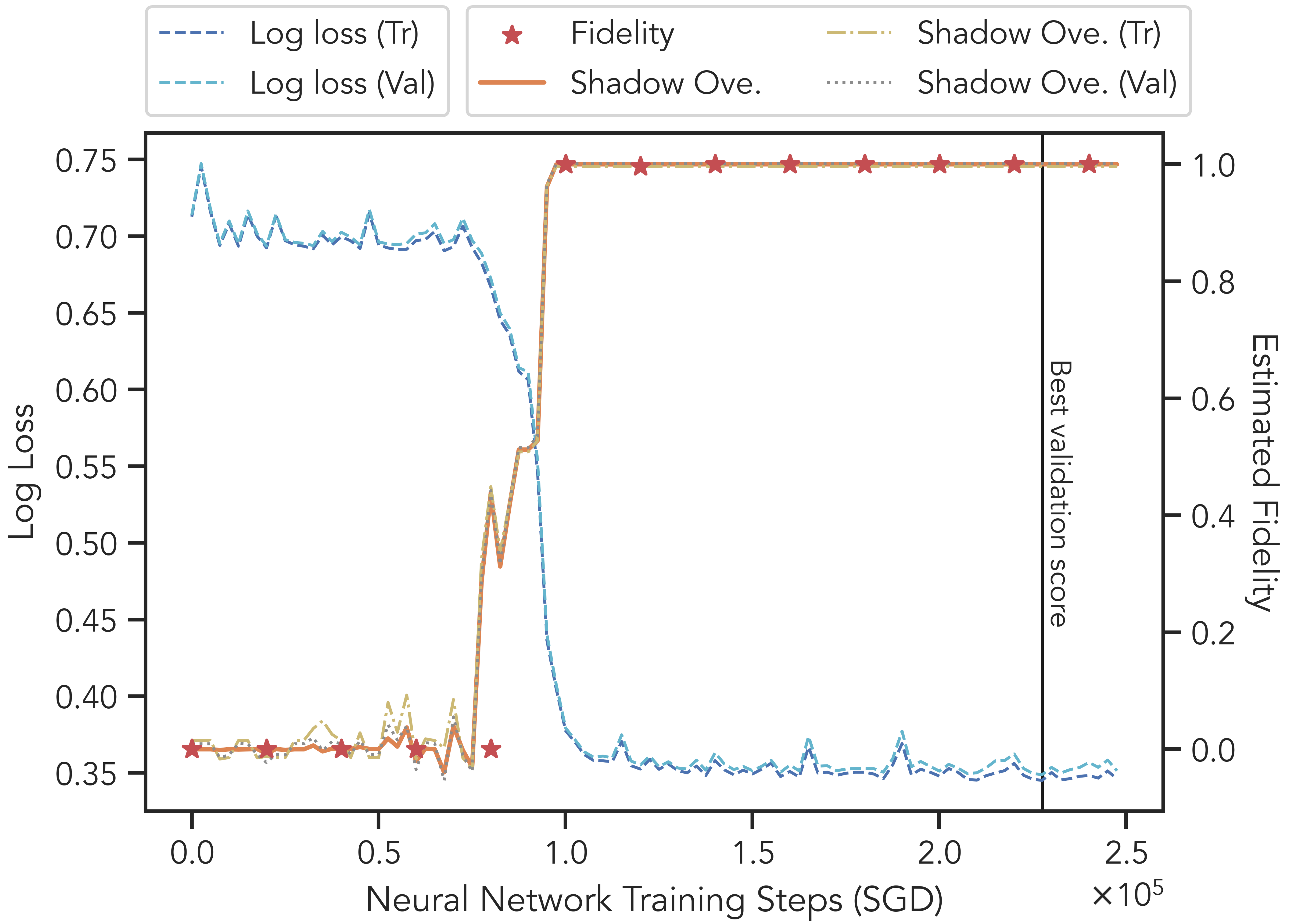


Training/Certifying NN tomography

Trained using
shadow-overlap-based loss

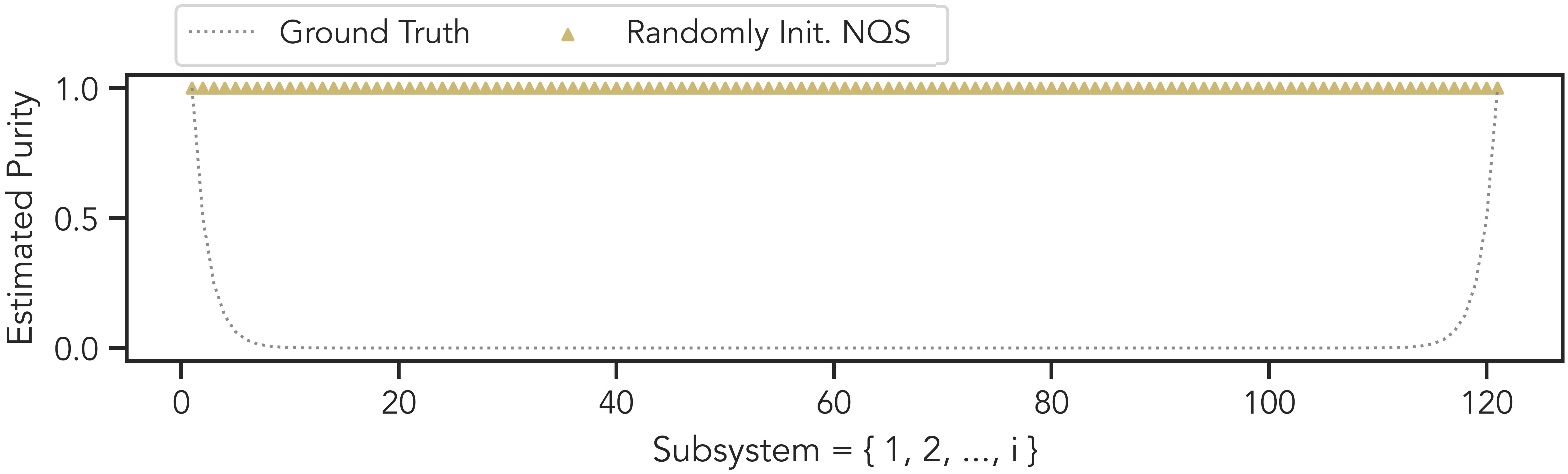


Certified using
shadow overlap



Training/Certifying NN tomography

We consider learning a class of 120-qubit states with exponentially high circuit complexity.



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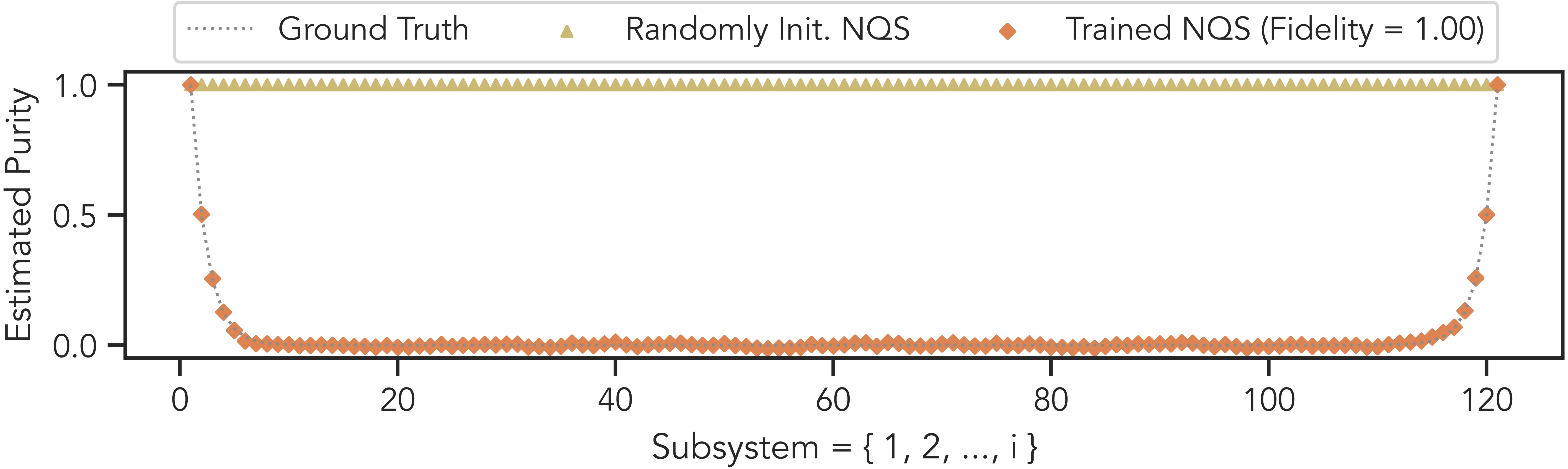


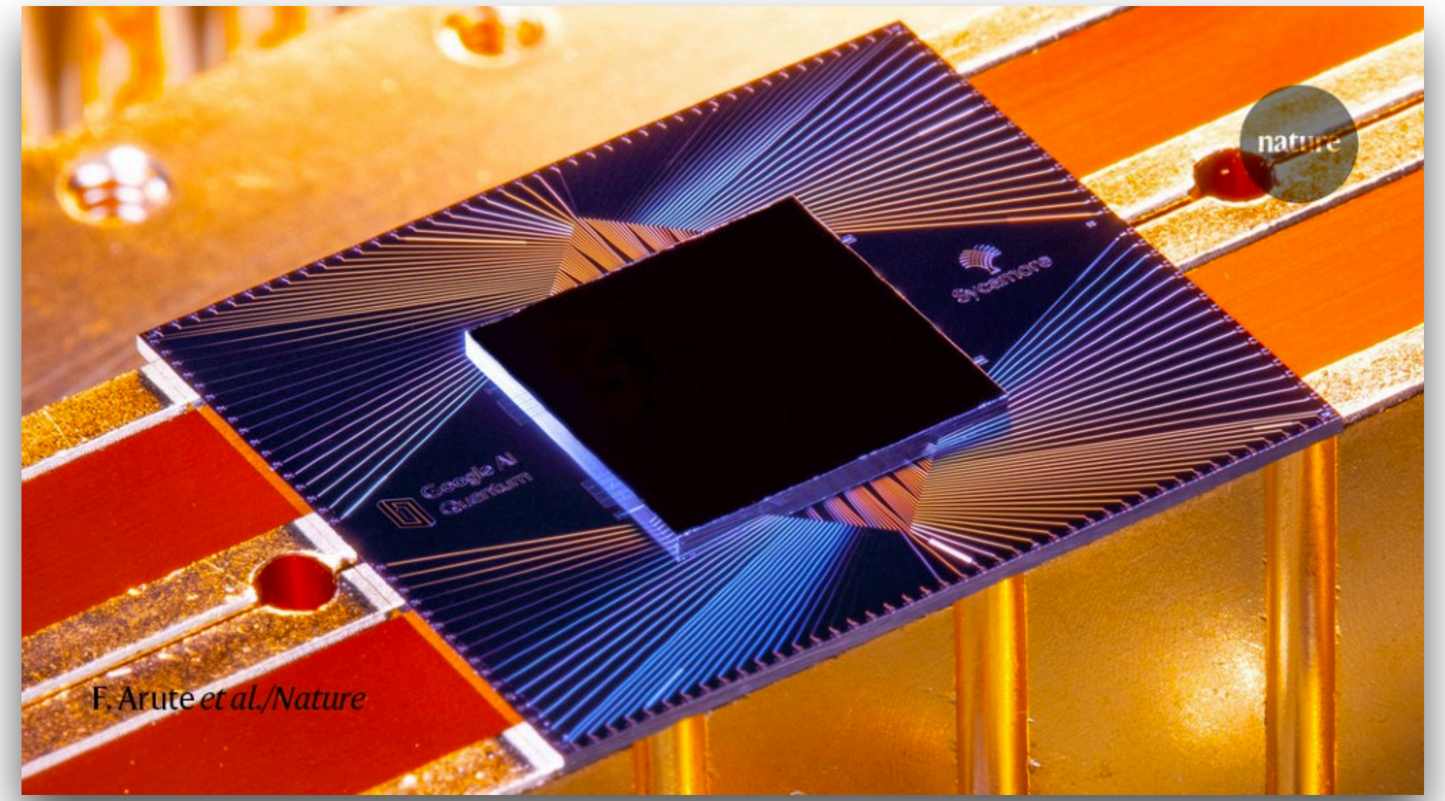
Applications

What can we use this new certification protocol for?

Example 1

Benchmarking

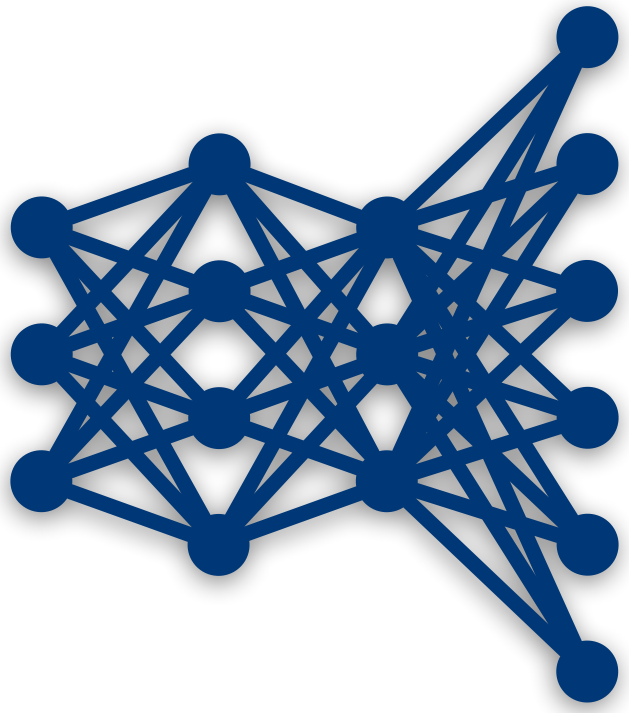
Shadow overlap $\mathbb{E}[\omega]$ certifies if the state has a high fidelity



Example 2

ML tomography

Train/certify ML models, such as neural quantum states, using shadow overlap $\mathbb{E}[\omega]$



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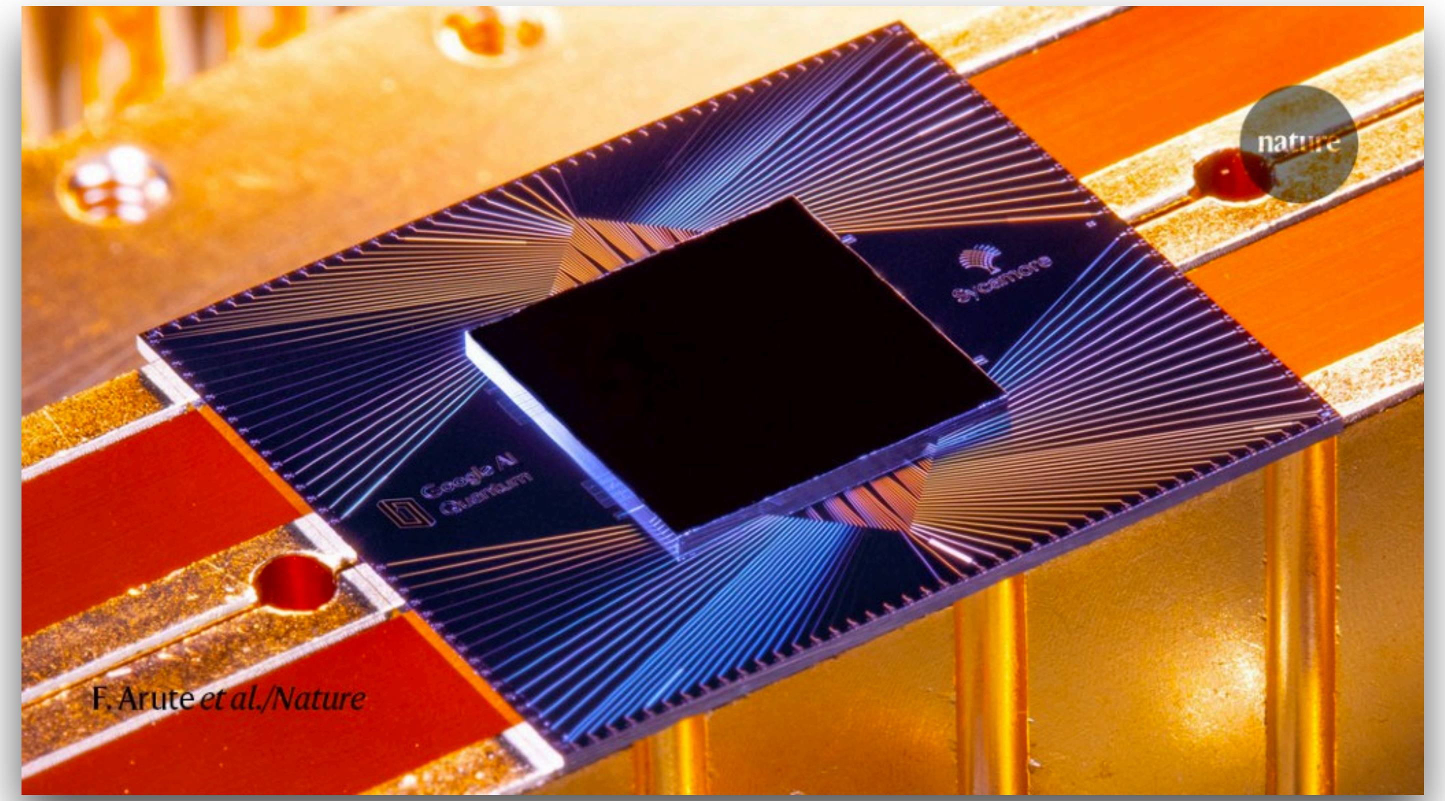

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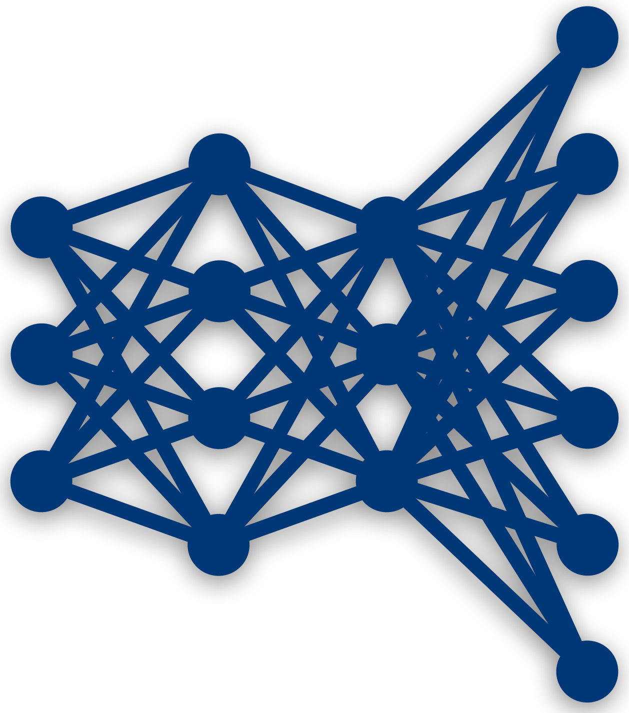
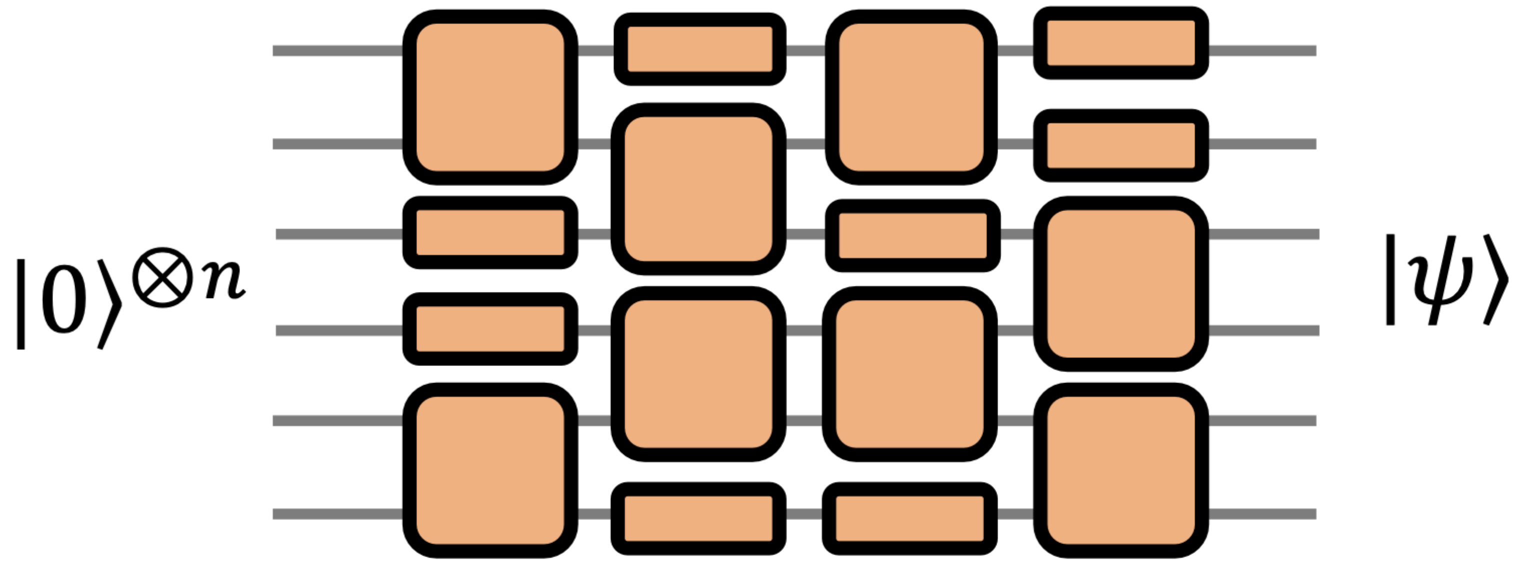


Example 3

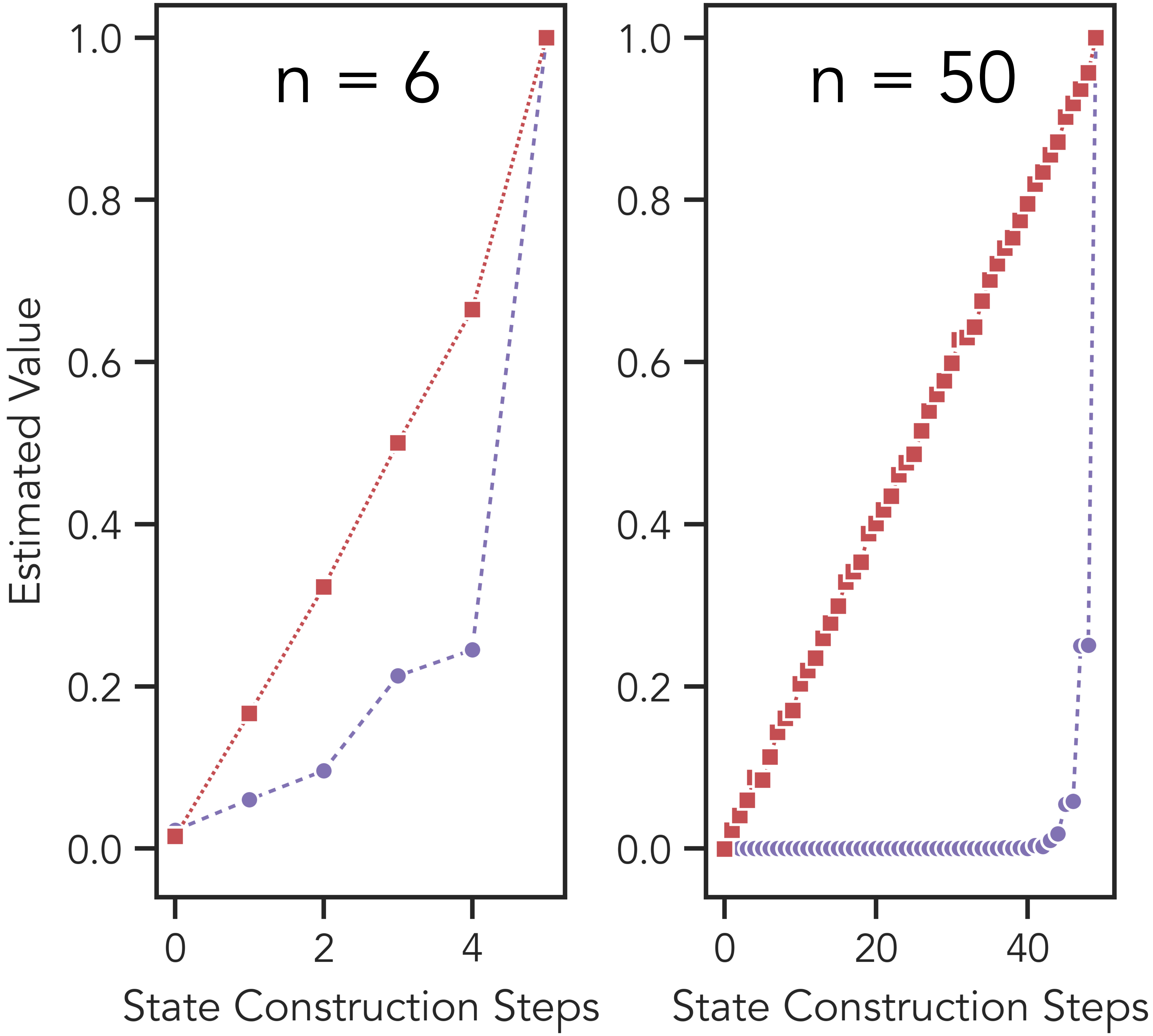
Optimizing circuits

To prepare a target state $|\psi\rangle$, we can optimize the circuit to max shadow overlap $\mathbb{E}[\omega]$



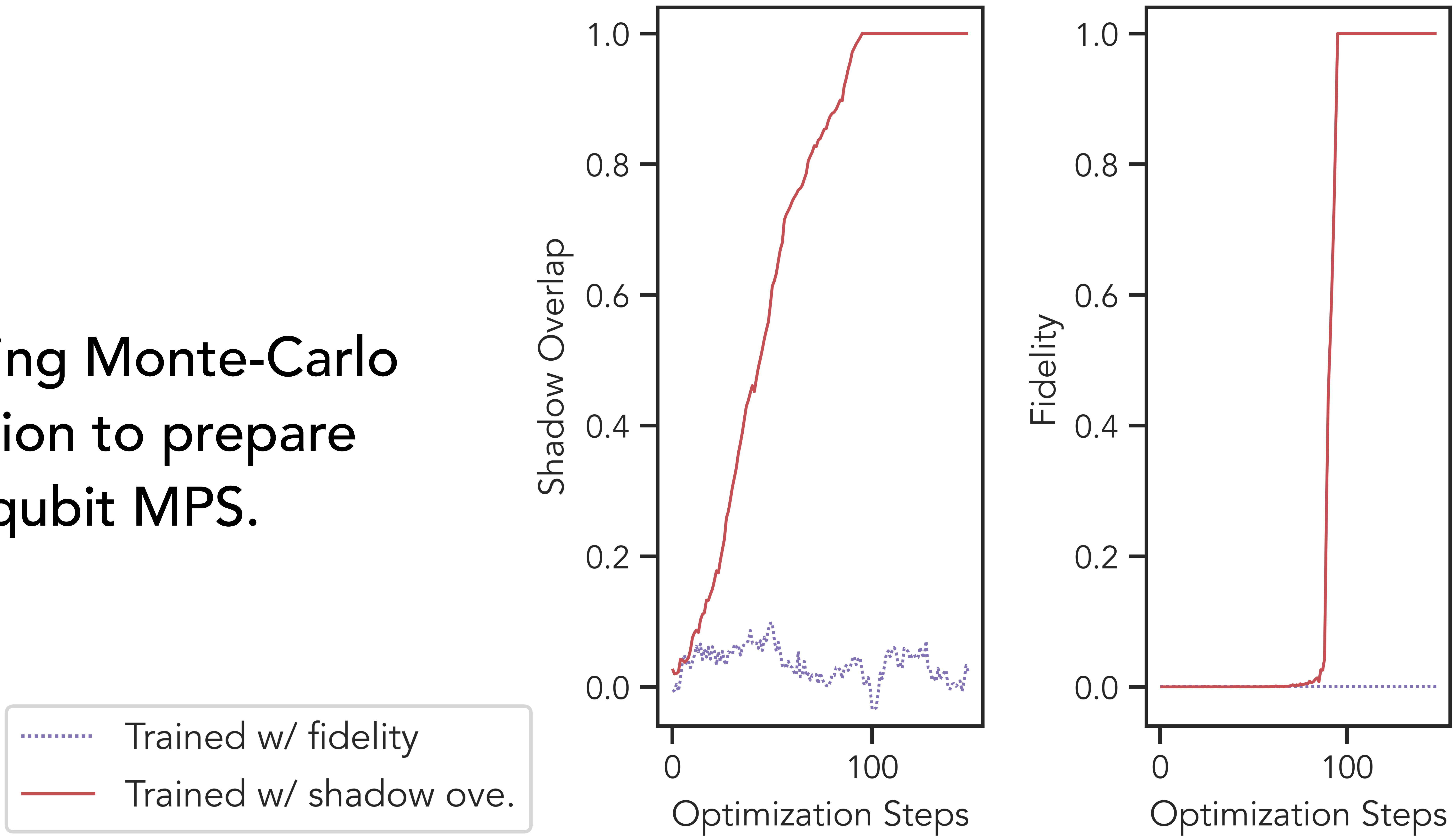
Optimizing state-preparation circuit

Constructing an n-qubit MPS with H, CZ, T gates.



Optimizing state-preparation circuit

Training using Monte-Carlo optimization to prepare a 50-qubit MPS.



Conclusion

- We prove that almost all quantum states can be efficiently certified from few single-qubit measurements.
- Are there states not certifiable with few single-qubit measurements?